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Case of Slovakia

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Abstract

Following the tradition of Aiyagari and McGrattan (1998) we use a heterogeneous agent economy model with incomplete insurance markets to determine the optimal quantity of public debt. The canonical model is augmented to capture the role played by productive government investment. Calibrating the model to key aggregate and distributional moments of the Slovak economy, we show the sensitivity of the optimal level of public debt to the presence of public investment, the inclusion of transitionary dynamics between stationary states into the welfare analysis and the fiscal reaction function. We find that our theoretical model implies a substantially lower optimal level of public debt for Slovakia than the actual level of indebtedness of the public sector.

Keywords: Infrastructure, public investment, heterogeneous agents, public debt, welfare, transitional dynamics.

JEL Classification: E2, E6, H3, H4, H6

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Introduction

There has been a considerable effort to better understand the role of government debt in the economy since the Great Financial Crisis (GFC) and the subsequent euro area sovereign debt crisis. Debt levels in advanced economies are at peacetime record highs and countries has been reforming their fiscal policy frameworks in order to put public finances on a more sustainable footing. In this context the question of prudent or sustainable debt levels naturally arises, which is usually studied by modelling sovereign debt market reactions to policy choices. Concepts like fiscal space (Ghosh et al. (2013)) or fiscal limits (Bi and Leeper (2013)) are frequently used for this purpose. Another strand of literature is focusing on the question of optimality of risk-free public debt. Following the seminal contribution of Aiyagari and McGrattan (1998), simple theoretical models of optimal debt help us better understand some of the channels through which public debt is influencing the economy. In this paper we pursue this latter approach.

What is the optimal level of public debt? In a traditional representative agent framework the answer is simple: as long as the government does not run a Ponzi scheme against the private sector, the level of public debt is irrelevant and the Ricardian equivalence holds. The result, however, is different when we switch to a heterogeneous agents and incomplete markets set up. As Aiyagari (1994) and Aiyagari and McGrattan (1998) show, uninsurable idiosyncratic productivity shocks and restricted borrowing give rise to non-trivial effects of public debt on the economy. On the one hand, higher public debt can relax borrowing constraints of households by increasing liquidity and thus facilitating consumption-smoothing. On the other hand, rising public debt crowds out private investments and therefore lowers wages and consumption in equilibrium. A priori it is not clear which effect is stronger. In their seminal paper Aiyagari and McGrattan (1998) found that the optimal level of public debt in the US was about 2/3 of GDP in 1997.

Subsequent research showed that the degree of inequality implied by the model is crucial for the welfare effects of government debt. Rohrs and Winter (2017) concluded that by calibrating the stochastic productivity process to realistic wealth and earnings inequality, the optimal government debt for the US is negative. Chatterjee et al. (2017) study the role played by productive government investment. They show that the introduction of public investment into the aggregate production function fundamentally alters the calculation of optimal public debt: it is optimal to accumulate a larger government asset position compared to the benchmark specification. Peterman and Sager (2016) depart from the standard incomplete markets model along a different dimension. They abandon the assumption of an infinitely lived agent and use an explicit life-cycle model instead. Peterman and Sager (2016) show that introducing a life cycle leaves less role for a government to improve equilibrium allocations through the insurance channel. On the contrary, the wealth accumulation phase of the life cycle drives government policy in the other direction, toward public savings instead of public debt.

The aforementioned extensions to the standard incomplete markets model determine optimal public debt based on welfare comparison between stationary equilibria. However, Desbonnet and Weitzenblum (2012) show that the welfare effects of the transition path between stationary equilibria are substantial. As compared to models that rest on steady-state analysis, they show that the welfare gains of a public debt increase are substantially higher when transitional

dynamics are accounted for. The additional debt issue allows for a temporary reduction in the income tax rate, which stimulates labor supply and generates an overshooting of the interest rate. Debt increases continue to generate welfare gains even when debt is considerably higher than its long-run optimal level. Rohrs and Winter (2017) and Chatterjee et al. (2017) find similar effects.

Our paper is closely related to that of Chatterjee et al. (2017). We augment the canonical incomplete market set up to account for productive public investment (such as infrastructure) and also consider transitional dynamics when calculating the level of optimal debt. Public investments play an important role as they generate positive spillover effects in the private sector by boosting the productivity of labor and capital. This reduces the precautionary savings motives for households, as they can rely more on labor income. Transitionary welfare effects work differently. Reduction in public debt leads to a reduction of the tax rate in the long run. However, debt reduction requires an increase in the tax rate in the short run, which has a negative effect on welfare. As Rohrs and Winter (2017) show the transitional welfare costs in the short run more than offset the long-run benefits of a stationary equilibrium with a lower debt level. This explains why are debt reduction so politically painful.

Our contribution is threefold. First, in contrast to Chatterjee and Eyigungor (2013) we use a different solution technique – endogenous grid method – which is faster and more efficient than value function iteration (Villemot (2012)). Second, we use different fiscal policy reaction functions and study the utility-enhancing effect of public goods on the optimal debt level. Third, we calibrate our model to Slovak data, which is an economy with significantly less inequality than the US. Our findings are the following.

From a long-run perspective - when stationary equilibria are compared only - it is optimal for the government to accumulate surpluses (assets). Moreover, this surplus is substantially larger when we include public infrastructure which has positive spillover effects on production. The situation becomes fundamentally different when transitionary welfare effects are considered. Regardless of the inclusion of public infrastructure it is now optimal for the government to have a positive amount of debt. The welfare gain induced by changing the benchmark debt level to the optimal level of 27 percent of GDP (in case of the model with public infrastructure) is only 2.27 percent, which is considerably smaller than in case of long-run gains. The model-implied optimal level of public debt is substantially lower than the benchmark debt/GDP ratio of 50 percent. This indicates that even when the short-run welfare consequences are taken into consideration Slovak economy and households can benefit from lower debt (and lower tax rate in the equilibrium) despite some temporary losses induced by a short-lived tax hike.

Our findings of the optimal level of debt are relatively robust (at least qualitatively) to the following experiments: when the government adjusts transfers instead of tax rate the optimal debt level is 45 percent whereas when different model calibration with lower target return on capital is assumed, the optimal debt level attains 35 percent of GDP. However, in both cases welfare gains are smaller than in the benchmark model. Moreover, the robustness of our results is tested and confirmed on the extended model that includes a richer tax structure (capital tax and labour income tax): when the government adjusts capital tax rate the optimal debt level is 25 percent of GDP while with adjustment in labour tax rate the optimal debt level becomes slightly higher, 30 percent of GDP. Furthermore, our conclusions stay valid even if initially wasteful government

consumption is included directly in the utility function of households.

To our knowledge, this is the first paper in Slovakia investigating the question of optimal public debt levels. There are related papers such as Mucka (2015) or Mucka and Odor (2017), which study prudent or sustainable debt levels in Slovakia. The former calculates a fiscal limit for Slovakia and estimates a prudent sovereign debt ceiling, while the latter investigates optimal constitutional debt limits based on the debt dilution literature.

This paper is organized as follows. In Section 1 we present the model and build the analytical framework necessary to study optimal debt levels. We demonstrate how to include productive public investment into the framework. In Section 2 we describe how the model is calibrated to match key characteristics of the Slovak economy and what techniques we employ in order to solve the stationary and transitional equilibria of the model. We also show how to calculate welfare gains associated with transition between equilibria. Section 3 presents our main results. We show the relationship between the debt/GDP ratio and the economy's key distributional and aggregate variables in stationary equilibria. Then using the welfare measure we characterize the optimal level of sovereign debt from a long-run persepctive and determine the optimal debt level when transitional paths are taken into account. In order to better understand the short-run implications of fiscal reforms we study the transition path under a policy experiment where the government gradually changes its initial debt/GDP ratio to the optimal one. At the end we carry out various robustness checks. The final section concludes.
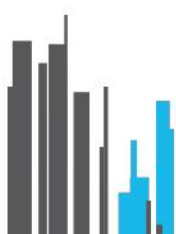
1   Analytical framework

Our analysis of the optimal level of public debt uses the standard incomplete markets framework with endogenous labour supply in the style of Bewley (1986), Huggett (1993), and Aiyagari and McGrattan (1998). We assume that ex-post heterogeneous households face idiosyncratic uninsurable earnings shocks when they make their consumption–labour decisions. However, for the sake of simplicity aggregate uncertainty is not considered[1]. Next, the production process is enriched by productive public investment (infrastructure) supplied by the government which contributes to reduction of the households' precautionary savings motive. Following Floden (2001) and Chatterjee et al. (2017) we employ the consumption-equivalent welfare criterion to evaluate the optimal debt level. Furthermore, in both cases we study the impact of public infrastructure on the optimal level of sovereign debt.

1.1   Model

Households.   There is a continuum of infinitely-lived households. They choose their consumption c, and labour supply, h, to maximise a per-period utility

$$U(c,h;G_c) = \frac{\left[c^{\eta}(1-h)^{(1-\eta)}\right]^{1-\sigma}}{1-\sigma} + \omega\frac{[\kappa G_c]^{1-\sigma_g}}{1-\sigma_g}. \tag{1}$$

Above, G_c represents the level of government consumption and κ is the share of government consumption that is productive from households perspective. In the benchmark model we do not assume that government consumption provides utility to households (i.e. its weight, ω, in households utility is set to zero). However, in Section 3.4 we relax this assumption to bring in government consumption as a welfare–enhancing public good[2].

The value of the parameter σ shapes the risk aversion of households. As noticed by Aiyagari (1994), Bewley (1986) or Chatterjee et al. (2017) individuals are highly risk averse, so they tend to accumulate larger buffer of savings. We emphasize the endogeneity of the household labour supply decision which creates a margin where distortions matters. This modelling aspect was missing in the original paper of Aiyagari (1994).

Although all households are identical ex-ante, impossibility of perfect insurance against the realization of an idiosyncratic labour productivity shock e (received at the beginning of each period) makes them heterogeneous ex-post. We assume that the household-specific productivity

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[1]Abstraction from aggregate shocks makes prices constant in stationary equilibrium. Referring to Guvenen (2011) a far more challenging problem with incomplete markets arises providing that the presence of aggregate shocks is assumed, since in that case equilibrium prices become functions of the entire wealth and income distributions. Krusell and Smith (1997) proposed an equilibrium solution that approximates the wealth distribution with a finite number of its moments and showed that the first moment of the wealth distribution was all individuals needed to track in this economy for predicting all future prices (then households can forecast how prices will evolve in the future as the aggregate state evolves in a stochastic manner). As noticed by Guvenen (2011), this approach makes feasible the solution of a wide range of interesting models with incomplete markets and aggregate shocks. On the other hand, it suggests that ex-post heterogeneity does not often generate aggregate implications much different from a representative-agent model.

[2]The functional form of the utility functions guarantees that provision of public goods (government consumption) has no impact on household's labour-consumption decision. It only provides extra welfare gain for them.

shock e evolves accordingly to a Markov process with a transition matrix π[3]. Households can partially insure against fluctuations in labour productivity by accumulating a stock of assets, a, comprised of private capital, k, and public bonds, b. We assume that households are indifferent between capital and bond stocks and their portfolio $a = k + b$ pay out a market-determined rate r. Recalling Rohrs and Winter (2017), market incompleteness generates a precautionary motive for savings[4] and leads to endogenous distribution of wealth across society.

Households make their labour-consumption decision subject to an intertemporal budget constraint

$$c + a' = (1 + r(1 - \tau))a + (1 - \tau)whe + TR, \tag{2}$$

where a' represents the household's stock of wealth in the next period, τ denotes the income tax rate, TR are flat lump-sum transfers supplied by the government and w is the gross real wage rate. Furthermore, households face a non-trivial exogenous borrowing constraint, $a' \geq \underline{a}$. Then the household's maximisation problem is given as:

$$V(a, e) = \max_{c, h, a'} \left[U(c, h; G_c) + \beta \sum_{e'} \pi(e'|e) V(a', e') \right]. \tag{3}$$

Note here an important point: households can both save and borrow for self-insurance. The availability of a generous borrowing limit $\underline{a}$ reduces substantially their need for precautionary saving.

As noticed by Chatterjee et al. (2017) in this type of model where some of the earnings risk is uninsurable, there could be scope for public insurance, i.e. government intervention through taxation and redistribution from those who are rich (or lucky) to poor (or unlucky) households. Therefore households pay a tax levied on their labour and capital income and receive flat social transfers, which is in fact the only source of income for the poor ones. Note that the tax scheme is progressive because the least productive or poorest agents pay lower taxes but get the same (nominal) transfer[5]. However the level of taxation/redistribution must be chosen very carefully since in the model economy with an endogenous labour supply decisions there is a trade-off between insurance and efficiency. Hence, if government uses only tax reforms to change the long-run debt/GDP ratio, the question on optimal tax level goes hand-in-hand with the question of the public debt level optimality.

Firms. The representative firm combines aggregate private capital stock, K (depreciating at a rate δ), aggregate labour supply, L, and stock of public infrastructure, K_g, to produce a homogeneous output, Y, using a standard neoclassical technology in a perfectly competitive environment. The optimality conditions arise from the profit maximisation problem:

$$\max_{K, L} \{ Y(K, L) - [wL + (r + \delta)K] \}, \qquad Y(K, L) = (K_g)^\phi K^\alpha L^{1-\alpha}, \tag{4}$$

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[3]We observed that Markov process approach (see Kopecky and Tauchen (1993)) can better approximate wealth and income inequalities than the autoregressive processes à la Tauchen (1986).

[4]Causing households to accumulate wealth during periods of high productivity, in order to compensate for low productivity periods.

[5]The average tax rate faced by a household is increasing in both labour and capital income.

This in turn defines the equilibrium[6] real wage w and return on capital r. Stock of public infrastructure, K_g, supplied by the government has a positive effect on firms' output, with an output elasticity ϕ.

In order to highlight the importance of productive public investment in the benchmark model we present an alternative model in which the public infrastructure channel is absent. In that case, the production function is given as $Y = \Phi K^{\alpha} L^{1-\alpha}$ with the scaling factor Φ representing an exogenously specified level of productivity[7].

Government. The government spends collected tax revenues on wasteful consumption, G_c, provides social transfers, TR, and invests[8], G_i, in the efficient stock of public goods infrastructure, K_g, which is supplied to firms. The stock of public infrastructure depreciates at a rate δ_k and evolves as follows:

$$K'_g = (1 - \delta_k)K_g + G_i, \tag{5}$$

where K'_g denotes the next-period stock of public infrastructure. Furthermore, the government issues or purchases one-period bonds, B. Government debt is an additional risk-free asset and, by no arbitrage, it must carry the same rate of return as capital in equilibrium, r.

The presence of sovereign debt has several effects on the model equilibrium. On the negative side, first of all, public debt crowds-out productive capital because some of the savings are shifted away from productive private capital into unproductive government debt. Next, debt service requires distortionary taxes. On the other hand, the presence of government bonds leads to looser borrowing constraint as the government provides an additional instrument for household to smooth consumption. Thus, higher debt level increases the return on capital and makes assets cheaper to hold. The higher the equilibrium rate, the lower the costs of excessive holding assets necessary for household consumption smoothing in the incomplete insurance market environment[9]

Next, we assume that each category of public expenditure is proportional to aggregate output, so

$$G_c = g_c Y\,, \qquad G_i = g_i Y\,, \qquad TR = g_{tr} Y\,, \qquad B = xY\,. \tag{6}$$

The government budget constraint satisfies the subsequent relationship:

$$B' = (1 + r)B + G_c + G_i + TR - \tau(wL + rK)\,, \tag{7}$$

where B' denotes the next-period stock of government bonds.

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[6]optimality conditions set real wage $w = (1 - \alpha)Y/L$ and $r = -\delta + \alpha Y/K$

[7]Value of the level of productivity Φ is chosen such that in the benchmark calibration both model specifications deliver the same level of output.

[8]In the alternative model specification without public infrastructure channel, we reduce government expenditures to social transfers and wasteful consumption only.

[9]The requirement on borrowing constraint exogeneity is crucial for this argument to hold. If we had assumed that agents can borrow up to the natural borrowing limit, higher return on capital would tighten the natural borrowing limit too – this is not true for an exogenous borrowing constraint.

1.2   Equilibrium solution

In order to determine and analyze the optimal level of sovereign debt, we consider both stationary and transitional equilibria. While comparing stationary equilibria is a useful exercise when addressing the long-run consequences of various fiscal policies, a careful analysis of transitional paths between two stationary equilibria is inevitable when short-run implications of fiscal reforms are studied.

Stationary equilibrium.   Recalling Aiyagari and McGrattan (1998), Bewley (1986), Chatterjee et al. (2017), Guvenen (2011), and Domeij and Heathcote (2004) a stationary equilibrium in the economy is characterized by time-invariant decision rules, $c(a,e)$, $h(a,e)$ and $a'(a,e)$, a value function, $V(a,e)$, factor prices, r and w, fiscal policy variables x, τ, g_{tr}, g_i and g_c, a time-invariant joint distribution of individual states, $\Lambda(a,e)$, and a vector of aggregate variables A, K, L, K_g, Y, C, TR, B, G_i, G_c, such that:

1. Factor prices satisfy the firms' optimality conditions.

2. Given factor prices, the decision rules solve the household's problem.

3. The goods market clears,
$$Y = C + \delta K + G_c + G_i.$$

4. The asset market clears,
$$A = K + B.$$

5. The government budget constraint holds,
$$\tau(wL + rA) = rB + G_c + G_i + TR.$$

6. The joint distribution of individual states is stationary,
$$\Lambda(a',e') = \sum_e \pi(e',e)\Lambda((a')^{-1}(a',e),e).$$

7. Aggregate consumption, labour supply and assets are derived from individual decisions of households,
$$A = \sum_e \int_a a'(a,e)\lambda(a',e)da, \quad L = \sum_e \int_a eh(a,e)\lambda(a',e)da, \quad C = \sum_e \int_a c(a,e)\lambda(a',e)da.$$

Transitional equilibrium.   Following Bewley (1986), Chatterjee et al. (2017), Guvenen (2011), Rohrs and Winter (2017), and Domeij and Heathcote (2004) in the context of this study we characterize a transitional equilibrium in the economy by a sequence of time-varying decision rules,
$$\{c_t(a,e), h_t(a,e), a'_t(a,e)\}_{t=1}^T,$$

sequences of factor prices $\{r_t, w_t\}_{t=1}^T$, sequences of fiscal policy variables $\{\tau_t, g_t^{tr}, g_t^i, g_t^c\}_{t=1}^T$, sequences of aggregates
$$\{A_t, K_t, L_t, K_t^g, Y_t, C_t, TR_t, B_t, G_t^i, G_t^c\}_{t=1}^T,$$

and a sequence of time-varying distribution of individual states $\{\Lambda_t(a,e)\}_{t=1}^T$, such that given an exogenous sequence of the sovereign debt/GDP ratio over the transition between two stationary equilibria, $\{x_t\}_{t=1}^T$,

1. The economy at time $t = 1$ is consistent with a given initial stationary equilibrium.

2. The economy at time $t = T$ is consistent with a given terminal stationary equilibrium.

3. At each time period factor prices satisfy the firms' optimality conditions.

4. At each time period goods market clears,

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + G_t^c + G_t^i.$$

5. At each time period the asset market clears,

$$A_t = K_t + B_t.$$

6. At each time period the government budget constraint holds,

$$B_{t+1} = (1+r_t)B_t + G_t^c + G_t^i + TR_t - \tau_t(w_tL_t + r_tA_t).$$

7. At each time period the decision rules solve the household's problem.

8. The joint distribution of individual states follows

$$\Lambda_{t+1}(a',e') = \sum_e \pi(e',e)\Lambda_t((a')^{-1}(a'_{t+1},e),e).$$

9. At each time aggregate consumption, labour supply and assets are derived from individual decisions of households,

$$A_t = \sum_e \int_a a_t'(a,e)\lambda_t(a',e)da, \quad L_t = \sum_e \int_a eh_t(a,e)\lambda_t(a',e)da, \quad C_t = \sum_e \int_a c_t(a,e)\lambda_t(a',e)da.$$

1.3   Welfare gain evaluation

How to determine which value of the debt/GDP ratio is *optimal* for a society? We assume that the government keeps its expenditures constant relatively to the aggregate production. So, to accommodate the change in the level of sovereign debt, the government must adjust the income tax rate. Then, from a policy perspective it is crucial to know, how much agents and various subgroups of society gain or lose from the tax reform. To evaluate these welfare gains or losses we follow Aiyagari and McGrattan (1998), Floden (2001), and Rohrs and Winter (2017) and adopt the subsequent Benthamian social welfare function that puts equal weight to every household[10]:

$$\Xi = \int_a \int_e V(a,e)\lambda(a,e)da\,de. \tag{8}$$

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[10]This welfare measure can be interpreted as the welfare level of the average household in the economy.

Next, we express welfare changes in terms of consumption-equivalent variation. So, when long-run welfare implications of a particular tax reform are studied we quantify the units of consumption that need to be transferred between the initial and target stationary equilibrium such that the average individual remains indifferent between these two steady-states. In what follows we assume no provision of public goods for households, so there is no impact of government consumption on household utility[11]. Then, thanks to homogeneity of the utility function (1) in consumption, the definition of the long-run consumption-equivalent welfare change[12] $\Delta\Xi$ generated by switching from the pre-reform (with subscripts 0) to the post-reform (with subscripts 1) economy for the average household satisfies the subsequent:

$$\Delta\Xi = 1 - \left(\frac{\int_a \int_e V_0(a,e)\lambda_0(a,e)\,da\,de}{\int_a \int_e V_1(a,e)\lambda_1(a,e)\,da\,de} \right)^{\frac{1}{\eta(1-\sigma)}}. \tag{9}$$

Determination of the optimal level of debt by comparing welfare between stationary equilibria is not fully satisfactory. This approach can only be used to assess whether a household would prefer to live in the stationary equilibrium of an economy with debt/GDP ratio x_0 (and tax rate equal to τ_0) or in the stationary equilibrium of a different economy with debt/GDP ratio x_1(and tax rate equal to τ_1). Unfortunately, the welfare gain measure above ignores the short-run dynamics that occurs due to changes arising from the tax reform[13]. It is essential to consider these transitional dynamics in the welfare evaluation since adjustments in the debt level targeted by the government affect not only the the variables in the target stationary equilibrium but have impact on variables depending on the time horizon of adjustments[14]. Note that the transitional dynamics induced by the tax reform are deterministic.

To account for the transitional dynamics we compute the corresponding welfare gain as follows:

$$\Delta\Xi = 1 - \left\{ \frac{\int_a \int_e V_0(a,e)\lambda_0(a,e)\,da\,de}{\int_a \int_e \sum_{t=0}^{T} \beta^t U(c_t(a,e),h_t(a,e))\lambda_{t-1}(a,e)\,da\,de} \right\}^{\frac{1}{\eta(1-\sigma)}}. \tag{10}$$

Here c_t, h_t and λ_t denote the consumption and labour decision rules and the wealth density known at time t of the transitional path. So our aim is to determine how much to compensate the average household so that it will be indifferent between living through the tax reform and living in the pre-reform economy.

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[11]This assumption is revised in Section 3.4. When public goods provision enters household utility in additive form, utility function is no longer homogeneous in consumption and so relationships (9)-(10) cannot be used to evaluate welfare gain. For that purpose Section A.3 shows how to calculate welfare gain in case of general utility function.

[12]Agents would prefer being at the new stationary equilibrium without any compensation for change whenever $\Delta\Xi$ is positive. On the other side, household requires a compensation $-\Delta\Xi$ units of consumption to make it indifferent between these two stationary equilibria.

[13]In this case, we specify an exogenous path for the sovereign debt/GDP ratio $\{x_t\}$ and solve the resulting transitional equilibrium.

[14]Long- and short-run fiscal policies may differ significantly. For example, in the long-run the debt reduction is associated with a tax cut. However in the short-run – to pay down public debt – the government has to raise taxes. Therefore welfare implications may significantly differ in the short- and long-run.

2 Calibration and solution

In what follows we describe the calibration of the benchmark and the alternative models. We also show the computational techniques employed to solve the stationary equilibrium and describe the approach used to determine the transitional equilibrium path between two stationary equilibria.

2.1 Model calibration

We calibrate our model to match the key aggregate and distributional moments of the Slovak economy as summarised in Table 2.1 and Table B.1 in Appendix.

Concerning modelling targets, there are three key points that need to be discussed at this stage. Firstly, gross return on physical capital in Slovakia is considerably higher than in developed countries mainly because of higher share of the physical capital in the production process and lower level of the accumulated capital stock. Secondly, the degree of income redistribution in Slovakia implies a share of indebted households[15] that substantially lower than in most of the OECD countries[16] Finally, since a major share of households' wealth in Slovakia is allocated in durables (housing) and households hold only a very limited financial wealth we decided to target the distribution of the total net wealth[17] . It seems to be a better indicator of wealth distribution than the data on net financial wealth[18].

Table 2.1 : Calibration targets

Indicator	Target Value	Model Value
Indicator	Target Value	Model Value
gross return on physical capital	17.17%	17.18%
effective income tax rate	50.55%	50.52%
total capital/GDP	220%	220%
aggregate share of time allocated to work	29%	29.06%
share of households with negative net wealth	6%	5.86%
GINI total net wealth	0.49	0.50
GINI gross labour income	0.61	0.59

Calibration of the benchmark model is shown in Table 2.3 and Table B.1 in Appendix. Evidently, the benchmark model is able to match key indicators of Slovak economy well (Table 2.1). While values of the parameters located in the top part of Table 2.3 are set based on broad consensus of experts (production parameters) or arise from national accounts (fiscal parameters), values of parameters in the bottom part of Table 2.3 are set to match modelling targets. Since we study the model with two specifications, we first describe the parameter choices that are the same in both models. Depreciation rates of public and private capital (δ_k and δ, respectively) coincide and are set to a standard value of 10 percent. The share of the private capital stock in the production

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[15]Estimated using the Household Finance and Consumption Survey (HFCS hereafter) from 2014.

[16]During last decade GINI on disposable income in Slovakia has been close to 0.25 as reported by Murtin and d´Ercole (2015) and OECD (2012).

[17]Computed as the difference between all household's assets and liabilities, as reported in the recent HFCS survey.

[18]Although, from a theoretical perspective, mainly financial wealth is used for consumption-smoothing.

function (4), α, is 50 percent and reflects high proportion of manufacturing in Slovak production. The time-preference rate, $\beta = 0.9075$, is chosen such that the steady-state rental rate on capital[19], $r = 17.17$ percent is met. Concerning the utility function (1), the value of the inverse of the intertemporal elasticity of substitution $\sigma = 2.255$ has a strong impact on wealth distribution. We set the relative share of consumption in the utility, $\eta = 0.571$ to match the average aggregate share of time allocated to work, which is 29 percent. Next, by setting the borrowing constraint value $\underline{a}$ to -0.01 in the model equilibrium approximately 6 percent of households remain indebted, which corresponds to Slovak data. Finally, by calibrating the equilibrium debt/GDP ratio, x, to 50 percent we are able to get the target effective income tax rate, $\tau = 50.55$ percent.

Table 2.2 presents the setup of the household's idiosyncratic income shock process e and and the associated Markov transition matrix π. Using the methodology[20] introduced by Kopecky and Tauchen (1993) we set values of these parameters such that the model wealth and labour income distributions approximate well the corresponding observed distribution for Slovakia (see Table 2.1).

**Table 2.2 : Calibration of income process parameters**

Description	Parameter	Value
idiosyncratic labour income shock	e	$(0.105, 0.295, 0.965, 1.292, 2.815)$
transition matrix	π	$\begin{pmatrix} 0.800 & 0.140 & 0.060 & 0.000 & 0 \\ 0.100 & 0.810 & 0.070 & 0.020 & 0 \\ 0.025 & 0.045 & 0.900 & 0.030 & 0 \\ 0.015 & 0.040 & 0.055 & 0.840 & 0.050 \\ 0.030 & 0.030 & 0.025 & 0.025 & 0.890 \end{pmatrix}$

Some adjustments regarding fiscal variables are needed in order to ensure comparability across our model specifications. First, in the benchmark model with public infrastructure we set its corresponding output elasticity to 6.5 percent[21]. Next, in the alternative model without public infrastructure we set the technology parameter, Φ, to ensure that both model specifications yield the same productivity in the calibrated equilibrium. Furthermore, based on EUROSTAT database, in both models the share of public transfers, g_{tr} is set to 14.1 percent. While in the benchmark model the share of government investment in the stock of public infrastructure attains 3.6 percent and the wasteful consumption is 19.3 percent, in the model without public infrastructure wasteful consumption is set to 22.9 percent to ensure that the total share of government spending in GDP is identical across these two model specifications. Finally, we choose the share of productive government consumption, κ, so that the productive government consumption matches the long-run share of transfers in kind, 5.9 percent.

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[19] Optimality conditions arising from firms profit maximization problem define the optimum return on capital, $r = -\delta + \alpha Y / K$. Given the data-implied share of private investment/GDP (I/Y) of 18.5 percent and depreciation rate, the target capital/GDP ratio satisfies $K/Y = (I/Y)/\delta$. So, $r^{tar} = \delta(-1 + \alpha/(I/Y))$. Notice that large capital intensity of production in Slovakia ($\alpha = 0.5$) implies a relatively high return on capital.

[20] An alternative approach of approximating autoregressive processes was presented by Tauchen (1986).

[21] Estimated value of the public infrastructure elasticity w.r.t. output is based on EUROSTAT database.
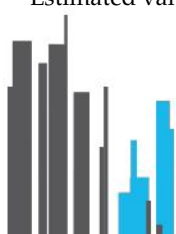
Table 2.3 : **Benchmark model calibration**

Description	Parameter	Value
private capital depreciation rate	δ	0.1
public capital depreciation rate	δ_k	0.1
public infrastructure elasticity w.r.t. output	ϕ	0.065
share of private capital in the production	α	0.5
social transfers/GDP	g_{tr}	0.141
government consumption/GDP	g_c	0.193
public investment/GDP	g_i	0.036
sovereign debt/GDP	x	0.5
government consumption weight in utility	ω	0
share of productive government consumption	κ	0.306
time-preference rate	β	0.9075
inverse of the intertemporal elasticity of substitution	σ	2.255
relative share of consumption in the utility function	η	0.571
borrowing constraint	$\underline{a}$	-0.01

2.2   Solution methods

In order to find the optimal debt level in long-run we need to calculate and compare the sequence of stationary equilibrium solutions. However, such solutions ignore short-run dynamics associated with transition from the previous state to the new one. Therefore to account for the short-run dynamics and its welfare implications, the associated equilibrium transition paths to different stationary solutions must be determined. Hence, in that case long-run views are considered jointly with short-run dynamics.

Stationary equilibrium.   To solve the stationary problem we follow an iterative fixed-point approach (see Villemot (2012)). First, we make guesses on aggregate labour supply, L, and aggregate assets, A, for which we find (by solving the problems of a representative firm and the government) an internally consistent set of aggregates. Then, we discretize the decision problem of households[22] and apply the endogenous grid method[23] (see Barillasa and Fernandez-Villaverdes (2007), Carroll (2011), Rust et al. (2012), or Villemot (2012)) to solve the optimization problem and to determine private agents' decision rules, c, h, and a'. Next, by discretization of invariant density functions (see Rios-Rull (1999)) using binning technique (see Castaneda et al. (2003)) we determine the invariant distribution of individual states[24], $\Lambda(a, e)$ and recover the new values

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[22]We use a large fixed grid with 500 unevenly-distributed points on the interval $[\underline{a}, \overline{a}]$ where $\underline{a}$ is the households' borrowing constraint and $\overline{a}$ is chosen large enough to ensure the existence of robust decision rules on a neighborhood of factor prices, tax rate and transfers that households take as given. To determine the invariant distribution of individual states we use an even finer grid of 200 unevenly distributed points on the same interval.

[23]Maliar and Maliar (2013) show that the endogenous grid method (EGM, hereafter) has similar performance as the envelope theorem approach. Furthermore, it can solve the non-surjective problems (like heterogeneous households labour-consumption decision problems) without additional approximation and penalisation techniques, which is not true when the widely used value function iteration (VFI hereafter) method is used. Moreover, VFI is significantly less efficient than EGM.

[24]Other techniques to determine the invariant distribution of individual states are possible, e.g. piecewise-linear approximation of invariant distribution function, eigenvalue method or Monte-Carlo simulations approach. For details see Heer and Maussner (2009) or Rios-Rull (1999).

of aggregate labour supply, $\tilde{L}$, and aggregate assets, $\tilde{A}$. Providing that initial and new values are sufficiently close, algorithm terminates. Otherwise, we update our initial guess and repeat the procedure until convergence is reached[25]. We solve the stationary equilibrium problem for various debt/GDP shares. Then by comparing the corresponding computed welfare gains we are able to determine the optimal level of debt/GDP in the long-run.

Transitional equilibrium.   Once the stationary equilibrium for different fiscal policies have been recovered, we can calculate the transition paths between the baseline policy with debt/GDP ratio of 50 percent, and various alternative options of fiscal policies with different long-run debt/GDP ratios. When computing the transition path between two stationary equilibria we make use of the methods outlined in Kirkby (2017), Trimborn et al. (2006), Heer and Maussner (2009), Bakis et al. (2015) and Domeij and Heathcote (2004). Concretely, we assume the economy starts at time $t = 1$ in the initial stationary equilibrium (consistent with a debt/GDP ratio of 50 percent). At time $t = 2$ the government changes its debt/GDP policy and so starts a transition to the new stationary equilibrium which converges to the new target stationary solution over time[26] T. Transition paths between two stationary equilibria is calculated similarly to the solution method utilized when stationary equilibrium is computed. Concerning the evaluation of transitional welfare necessary to evaluate transition welfare gain (10) we use a backward iteration approach[27]. Figures A.1–A.2 in Appendix illustrate procedures used to determine both types of equilibrium. We refer the reader interested in deep technical details to Appendix A.1.

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[25]Convergence of the method is guaranteed as it satisfies the contraction mapping theorem.

[26]The length of transition path and initial judgments are crucial. If the path is too long the solution does not exist due to non-convergence or numerical issues or is explosive. On the other hand side, when the path is too short solution is not stable, i.e. solution on a path longer by one year differs significantly. Technically, we are looking for the smallest possible length of path, that guarantees the solution existence and stability. The region of possible choices of transition path length is unknown ex-ante.

[27]It is possible to use brute force, i.e. Monte Carlo simulations. However, backward induction is more efficient.

3   Results

In this section we first present the relationship between the debt/GDP ratio and the economy's key distributional and aggregate variables in stationary equilibria. Next, using the Benthamian social welfare measure, we characterize the optimal long-run equilibrium level of the sovereign debt, provided that public infrastructure is included and compare our benchmark results with those obtained for the alternative model with no public infrastructure. However, by comparing the long-run welfare implications only we omit the welfare consequences that happen along the adjustment path. Therefore using the welfare measure we determine the optimal level of public debt that accounts also for transitional dynamics between stationary equilibria and discuss the impact of public infrastructure on it. Then in order to understand better the short-run implication of fiscal reforms we study the transition path under a policy experiment where the government changes its initial debt/GDP to the optimal one. Finally, we check the robustness of our results and study the impact of the inclusion of government consumption in households' utility.

3.1   Long-run equilibrium dynamics

First, we describe the stationary equilibrium relationship between the public debt/GDP ratio and key aggregate and distributional variables. Figure C.1 demonstrates how the share of sovereign debt in GDP – through changes in the income tax rate[28] – affects various equilibrium values, while Figure C.3 (left panel) illustrates the distributional impacts of different debt/GDP ratios. The blue line depicts equilibrium relationships for the benchmark model specification with public infrastructure while the red lines indicates the results obtained for the alternative model specification without public infrastructure.

The positive spillover effect of the public infrastructure on the economy is significant. The presence of public infrastructure supplied by the government increases both the marginal product of (private) capital and labour and so enables economy to achieve higher output. In both model specifications, private capital is crowded in whenever government cuts its equilibrium debt/GDP ratio. However, as illustrated on Figure C.1 presence of public infrastructure in the environment of debt reduction makes this phenomenon even stronger since in that case, both types of capital now crowd in: with declining debt level the gap between the flows of public capital and real wage under two model specifications increase. The positive implication of debt reduction for productivity is evident from Figure C.1. In the debt-reduction environment, government collects more taxes when it provides public infrastructure and since interest and tax rates coincide between these two models, it leads to higher output in the model with infrastructure in any debt-cutting situation. Furthermore, relatively higher production allows households to consume more in the benchmark model. However, this productivity enhancing feature of debt reduction is not present in the alternative model in which public infrastructure is absent. Indeed, as debt/GDP ratio declines, in the benchmark model public infrastructure increases productivity whereas the aggregate productivity element present in the model specification without public

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[28]We assume that in this exercise government expenditures remain constant with respect to GDP and are set according to Table 2.3. Therefore, adjustment in income tax rate (defined as the rate satisfying the government budget constraint) is the transmission channel used to implement fiscal policies targeting various long-run debt/GDP ratios.

infrastructure remains constant.

Next, distributional effects of debt reduction can be deduced from Figure C.3 (left panel). Both model specifications yield very similar levels of long-run wealth and labour income inequalities. All wealth and labour income groups benefit from tax-reduction reform. When the government decides to cut the debt, it tightens the household borrowing constraint and hence restricts a funding access for those who face the binding constraint. Their share in population raises. However, rich households enjoy public debt decline as it crowds in private capital for their portfolios despite lower return on capital. Therefore the wealth inequality raises with debt cuts. Different picture is obtained for the labour income inequality[29]. Increasing public debt lowers real wage (and aggregate labour supply) and by relaxed borrowing constraint provides additional insurance for low or medium income households which reduces their motivation to work even more. On the other side, households with high labour income tend to work relatively (even with lower wage) more because of lower capital income occurring due to a significant private capital crowding out. The middle income class is diminished while the gap between the top-earners and low-incomers grows with rising debt/GDP.

3.2 Optimal debt from different perspectives

In what follows we investigate the question of the optimal share of sovereign debt on in Slovakia and study how the presence of productive public investments affects its value. Furthermore, we analyze this issue from both short- and long-run perspective. We use the consumption-equivalent welfare measure and define the optimal debt as the one that leads to highest welfare gains. To accommodate the change in the level of public debt we adjust the income tax rate while government expenditures remain unchanged (compared to GDP). As can be seen from Figure D.1, the optimum quantity of sovereign debt is significantly affected by both the presence of public infrastructure in the model and by accounting for transitional dynamics between stationary equilibria.

Long-Run Optimum. First, in order to analyze the question of optimal debt level from a long-run perspective we use the relationship (9), to calculate welfare gains of different stationary equilibria relatively to the benchmark model with debt/GDP ratio set to 50 percent. Left panel on Figure D.1 illustrates the characterization of the long-run optimal share of sovereign debt for two stationary model specifications, i.e. for the benchmark stationary equilibrium model with public infrastructure and for the specification in which public infrastructure is absent. From Figure D.1 (left panel) it is evident that regardless of the presence of public infrastructure, in the long-run it is optimal for the government to be a net creditor. Running surpluses creates serious welfare gains[30]. However, the presence of public infrastructure affects the optimal level of

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[29]This is also different in comparison to most of the developed countries (see Murtin and d´Ercole (2015)). Usually, wealth is distributed more unequally among society than labour income and so in this type of models, income inequality is driven mainly by wealth inequality. However, this is not the case of Slovakia. According to the Household Financial and Consumption Survey (2013) almost all Slovak households possess some kind of asset and 96 percent hold real assets. The most common real asset (owned by 90 percent of households) is housing wealth. The share of home-owners in Slovakia by far exceeds the share in other euro area countries (60 percent).

[30]19 percent welfare gain in case of the model without public infrastructure and more than 35 percent increase when public infrastructure is present.

public debt substantially. When we do not include public infrastructure (red line on left picture on Figure D.1), it is welfare-enhancing for the government to accumulate a 130 percent surplus. In the environment with public infrastructure (blue line on left picture on Figure D.1) the welfare-maximizing surplus is much higher and exceeds 150 percent of GDP. Next, when public infrastructure is omitted the welfare gain path around the optimum is very flat which indicates that deviations from the long-run optimal debt levels in alternative model specification are not associated with serious welfare losses – this is not true for the benchmark model specification with public infrastructure included.

The intuition behind the differences in the long-run optimal public debt levels is the following: public infrastructure supplied by government to firms *for free* acts complementary to private capital in the production and so it increases factor prices. Hence, households receive more when provide their services to firms. This mechanism – absent when government does not provide public infrastructure – reduces the precautionary savings motive of households which in turn implies that it is optimal for households to be net debtors (and government, as a bond issuer, a net creditor in the long-run optimum). Therefore, if we do not consider the presence of public infrastructure, decreasing return to capital prevails for higher level of debt/GDP.

Next, welfare gains for different groups of society can be deduced from Figure D.2 (left panel). Considering wealth distribution, in the long-run all groups benefit from transition to the negative optimal debt because it is associated with higher consumption and production. However, poor individuals (those with negative assets) gain relatively more as increasing transfers are the most important source of their income. Furthermore, due to lower return on capital their debt service becomes cheaper. On the other hand side, rich households gain less due to decreasing return on capital and level of assets. However, for different labour income distributions the opposite is true. Those who do not work have a very limited benefit from lower taxes, while high earners enjoy lower taxation a lot.

Transitional dynamics.   Let us now discuss the optimal debt level that accounts also for intertemporal fluctuations in aggregates, household behavior, wealth distribution and welfare that occur as the economy gradually converges to the stationary equilibrium with different debt/GDP ratio. The right panel on Figure D.1 illustrates the optimal share of sovereign debt that accounts for the short-run dynamics derived either for the benchmark model with public infrastructure included (blue line) or the alternative model without public infrastructure. We see immediately that in both model specifications it is optimal for the government to be indebted – the opposite holds for the long-run optimal debt discussed earlier in this section. With the public infrastructure present in the model structure, the optimal debt/GDP ratio attains 27 percent while when the public infrastructure is ignored, the optimal debt level reaches 30 percent of the production. Interestingly, these results are quantitatively consistent with estimates on prudent debt levels in Slovakia (Mucka (2015)). Notice that temporal output loss leads to lower consumption of poor households and so they suffer by transition to the new equilibrium (see Figure D.2, right panel). But how different groups of society benefit or loose from transition to the optimal debt level? When short-run dynamics was ignored, all groups enjoyed welfare gains. However, by considering the welfare effects along the transition path, we need to account for short-run production losses, lower labour supply and investment activity, drop in consumption, transfers and temporarily higher taxes. The long-lasting decline of assets (resulting from lower

debt) tightens borrowing constraint and so reduces the ability of poor (indebted) households to smooth their consumption – lower transfers even worsen their situation. Therefore indebted households loose and wealth inequality increases.

The picture becomes different when benefits of labour income distribution groups are analyzed. The initial hike in real wage is offset by a substantial increase in tax rate so that temporal after-tax wage becomes lower than the initial one. This decreases the attractiveness of labour especially for non-high earners even more, so that labour income inequality increases in the short-run. However, since income tax stabilizes soon at lower-than-initial level and real wage is still kept high, low-earners start to benefit relatively more than high-earners from their participation on labour market. Therefore finally low-earners have relatively highest gains from tax reform (and lower debt) and labour income inequality becomes lower than the initial one.

Too much redistribution?   We emphasize that under both model specifications, the optimal debt level remains significantly below the actual debt level even if the transitional dynamics and its welfare consequences are taken into consideration. Our analysis that accounts for the tax reform induced transition path, indicates that actual public debt exceeds substantially its optimal level[31]. In other words, too generous redistribution is responsible for simultaneously inefficiently high consumption insurance and heavy distortions on labour and capital supply. From a technical point of view, the reason for which the model-implied optimal level of public debt is below the actual debt arises from the calibration of the labour income shock process: in order to generate the labour income and wealth distributions consistent with Slovak data (i.e. very high level of wealth equality and standard level of gross labour income inequality in the economy) the ex-post household wage is much less variable and less persistent than in most of the developed countries. Therefore individuals can largely self–insure against fluctuations in labour endowment and so the government provides in this sense *too much* public insurance[32].

Why is the optimal level of sovereign debt positive when transitional dynamics are taken into account? When comparing long-run equilibria, only the initial and target states are considered and the transition process is omitted. So the short-run implications of the aggregates and distributional effects are neglected. However, a reduction of public debt/GDP ratio forces the government to increase income tax temporarily, which directly lowers after-tax household income and creates downward pressure on consumption and production. Simultaneously, debt-reduction policy tightens the borrowing constraint and so reduces additionally the ability of poor households to smooth their consumption. These short run welfare losses accumulate gradually in transition, but must eventually trade-off against the long-run welfare gains from lower debt as the tax rate must be decreased in the long-run to sustain the lower debt level, and raising returns on production inputs prevail. However, short-run negative implications arising from a substantial tax hike erase almost all debt-cutting long-run gains, leading to an optimal level of debt that

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[31]For instance, similar analysis made in the environment the United States (U.S.) shows that if public infrastructure existence is assumed, transitional dynamics is taken into consideration and the income shock process is calibrated to match the U.S. wealth and earnings distributions the model-optimal and data-implied levels of public debt/GDP are very close.

[32]The excessiveness of public insurance and redistribution is even more amplified by the presence of the capital income tax (coincides with the labour income tax) which is much more distortionary and more progressive since the the model-implied capital/income ratio is higher than the labour/income ratio for rich households.

is positive (though still smaller than the actual debt) in equilibrium. Therefore the resulting welfare gains – only 2.27 percent in case of the benchmark model with public infrastructure and 1.91 percent in case of the model without public infrastructure – are substantially smaller than gains obtained from the long-run equilibria comparison. Next, likewise in case of long–run equilibria when public infrastructure is omitted the welfare gain path around the optimum is very flat which indicates that deviations from the optimal debt levels in alternative model specification are not associated with serious welfare losses – this is less evident for the benchmark model specification with public infrastructure.

Furthermore, when the public infrastructure is present in the model structure, the optimal debt level is reduced even when the transitional dynamics is taken into consideration. However, these differences are not fundamental in case of Slovakia, since the elasticity of public infrastructure with respect to production is very small (only 6.5 percent) and under both model specification the resulting optimal public debt levels are not extremely far from the actual debt level[33].

3.3 Understanding transitional dynamics

In order to better understand the short-run implications of tax reforms on economy that occur during the transition between stationary states, we now turn our attention to the transition paths taken by key aggregate and distributional variables in our model economy. Figure C.2 illustrates the transition paths under a policy experiment where the government[34] cuts instantaneously the debt/GDP ratio from its benchmark level 50 percent of GDP to the optimal level of 27 percent. Referring to the previous section, assuming that government keeps expenditures constant with respect to GDP, an increase of the income tax rate (from approximately 51 percent to almost 69 percent) is needed to finance the reduction of public debt. Increase in taxes and tighter borrowing constraint limit agents more, so they consume and save less and with significantly lower after-tax household income[35] the production and labour supply fall. However, after a very short time period tax fall below its pre-reform value very close to its long-run value associated with the new, low-debt equilibrium. Consequently, this supports capital accumulation and with more attractive labour the aggregate production, consumption, labour supply and private capital increase and gradually converge to the new low-debt equilibrium.

Concerning the evolution of income and wealth inequalities along the transition path, Figure C.3 (right panel) shows that immediately after the introduction of tax hikes both types of inequality rise: middle class shrinks, the share of poor (indebted) households increases while high income groups earn relatively more. Tight borrowing constraint restricts funding for households which are more taxed temporarily so their work and investment motivations are lower. After the fall of the tax rate wealth inequality is reduced but remains above the pre-reform level due to a tighter borrowing constraint. Similarly, after the income tax cut, labour income distribution becomes more egalitarian as in the environment of tight borrowing constraint and higher after-tax wages even poor households are working more. Therefore, labour income inequality converges to the level below its pre-reform counterpart.

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[33]The alternative model calibration ensures that productions induced by both model coincide at the actual/benchmark debt level 50 percent of GDP.

[34]For the sake of simplicity we assume the presence of public infrastructure in the model.

[35]After-tax wage instantly drops, which, in turns, lowers motivation of households to provide their labour service.

3.4   Robustness checks

We scrutinize the validity of our results from four different perspectives. First, we use a different fiscal reaction function – transfers instead of taxes – to satisfy the government budget constraint when moving from the initial debt level to the new one. The second robustness exercise changes model calibration such that the target return on capital is substantially smaller. The third approach extends the model to include a richer tax structure with capital tax and labour tax. The final experiment introduces government consumption as a utility-enhancing public good that can affect the welfare of the society.

Adjustment of transfers.   In Sections 3.1–3.3 we determined the optimal level of debt assuming that the government keeps public expenditures constant (relatively to GDP) and uses tax adjustment to satisfy the budget constraint. In this section the government changes transfers and keeps the income tax rate constant.

When short-run dynamics is ignored and government adjusts transfers to satisfy the budget constraint, welfare gains associated with various debt levels are similar to those of the benchmark economy with tax adjustments (see Figure E.4, left panel). Notice that the impact of changes in transfers on the economy is very small (Figure E.1) because transfers are identical across households. The reason is the following: although poor (or low income) households are very sensitive to the level of transfers, their impact on production is rather limited. On the other hand, transfers form only a small portion of income of wealthy households, therefore they are not very sensitive to their level. Indeed, the long-run profit of rich households from higher transfers is even negative due to losses on return on capital while poor households benefit more than in case of tax reduction (benchmark case). Therefore the overall long-run gain of the society from the optimal level of the debt is smaller than in case when government prefers to adjust income tax rate.

If short-run dynamics effects are also considered, the profile of welfare gain becomes substantially flatter for debt levels above the optimal value, which is in this case higher at 45 percent (see Figure E.4, right panel). Furthermore, welfare gains for the society arising from gradual transition to the optimal debt level are very small, less than 0.1 percent. This is caused by only subtle changes that occur in the economy during the transition process to the optimal debt level from the benchmark level of 50 percent. Short-run cuts in transfers (causing welfare loss of poor/low income households) are offset by higher labour and capital income and labour market, which in turns contributes to aggregate production growth as shown on Figures E.2–E.3.

So, when the government adjusts transfers instead of taxes, the conclusions about the long-run optimal level of the debt are similar to the benchmark case. However, changes in transfers reduce welfare gains and optimal debt levels are higher.

Lower interest rates.   One of the main drawbacks of this model is the lack of aggregate shocks that would enable us to separate the market-determined return on physical capital from the interest rate on government bonds. Since in the benchmark calibration we decided to target the return on physical capital[36] (see Table 2.1), in the equilibrium the same rate was used to price

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[36]target return on capital satisfies $\bar{r}^{tar} = \delta(-1 + \alpha/\bar{\iota}^{tar})$ where $\bar{\iota}^{tar}$ is the data-implied share of private investment to GDP.

domestic bonds. However, the target return on capital exceeds 17 percent (on annual basis) which is inconsistent with data on long-term yields of Slovak government bonds (see Figure E.5 in Appendix). Therefore in our alternative calibration (see Table E.1 in Appendix) we use a significantly lower return on capital. We set it at 5 percent annually – a value which is close to the average long-run rate on Slovak sovereign bonds – while keeping the remaining targets unchanged (see Table 3.1).

First, when short-run dynamics is ignored, the profile of welfare gains is similar to that of the benchmark-calibrated economy (see Figure E.9, left panel). However, as tax cuts in the low return on capital economy are relatively smaller[37], relative changes in aggregates are are also smaller as illustrated on Figure E.6. Therefore, the long-run welfare gains from being at the optimal level of public debt for the economy with lower target return on capital are smaller than for the economy with the benchmark calibration.

Next, if transitional dynamics is considered, the profile of welfare gains is still similar to that of the benchmark-calibrated economy (see Figure E.9, right panel). The optimal debt level for the alternatively-calibrated economy is at 35 percent of GDP[38]. This number is very close to the optimal debt level for the original economy.

Table 3.1 : Robustness check: alternative calibration targets

Indicator	Target Value	Model Value
gross return on physical capital	5%	4.92%
aggregate share of time allocated to work	29%	29.15%
share of indebted households	6%	5.76%
GINI total net wealth	0.49	0.48
GINI gross labour income	0.61	0.62

Two taxes. Our benchmark model assumes the existence of a unique, proportional tax which is applied on both labour and capital income. In this exercise we assume a richer tax structure with separate labour and capital[39] tax. Since government debt has an impact on the distribution of consumption via the composition of income[40], implementation of two taxes enables us to analyse different welfare effects when government adjusts only one of these taxes. Furthermore, we assume that the government does not collect capital taxes from households with negative net assets. Thus the household's intertemporal budget constraint satisfies the subsequent relationship:

$$c + a' = (1 + r(1 - \widetilde{\tau}_k))a + (1 - \tau_w)whe + TR, \qquad \widetilde{\tau}_k = \begin{cases} \tau_k, & a \geq 0, \\ 0, & a < 0. \end{cases} \tag{11}$$

When short-run dynamics is ignored and government changes either capital tax or labour tax, the profile of welfare gains is similar to that of the benchmark case (see Figure E.13, left panel).

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[37]Tax rate is less sensitive to changes in return on capital when return on capital is small.

[38]Optimal debt level for the alternatively calibrated economy with lower target return on capital is 35 percent and depends on whether public infrastructure is included or not.

[39]We apply capital tax on asset revenues.

[40]Households that receive capital income lose and households that mainly rely on labour income win when public debt raises.

Welfare gains are higher if the government prefers adjustments via lower capital tax, rather than labour tax. This difference comes from a significantly lower amount of leisure (see Figure E.10). Unsurprisingly, labour income inequality declines with decreasing debt/GDP thanks to higher labour market activity and real wages[41]. The opposite is true for wealth inequality, as due to lower capital tax saving decisions are less distorted and households who receive capital income gain more.

Next, if transitional dynamics is considered, the profiles of welfare gains are also close to that of the benchmark, one-tax economy (see Figure E.13, right panel). Assuming the presence of public infrastructure, the optimal debt level for the economy in which government adjust capital tax rate is 25 percent of GDP while approaching 30 percent of GDP if labour taxes are used. Notice that these numbers are very close to the optimal debt level for the original, one-tax economy (27 percent of GDP). Furthermore, welfare effects arising from the transition to the corresponding optimal debt levels are 2.83 percent in case of capital tax-adjusting economy and 1.65 percent in case of labour tax-adjusting economy. Lower welfare gains in case of the labour tax-adjusting economy are caused by a significant initial fall in household consumption (more harmful for low-incomers) and higher labour supply than in case of the economy in which government changes the capital tax rate (see Figures E.11–E.12).

Government consumption in the utility function   In the previous section we assumed that government consumption provides no utility to private agents[42]. Now we relax this assumption. By introducing government consumption as a utility-enhancing public good we can also study the impact of adjusting government consumption expenses (instead of tax rate or social transfers) on the welfare of the society. Hence, we assume that part of government consumption is beneficial from the household perspective. Notice that when government supplies transfers in kind it creates extra welfare for all households although this type of government expenditure is not present in the budget constraint of households.

Unsurprisingly, when government adjusts taxes or social transfers to satisfy the budget constraint, by providing public goods it increases welfare of households - this effect is significant if only long-run stationary solutions are compared. However, since debt reduction has some negative short-run consequences for the economy (loss in production, labour supply, household and government consumption, higher taxes) additional welfare gain implied by the supply of public goods is lower when transitional path is considered (Figure E.15). Furthermore, provision of public goods does does not change our findings about the optimal level of public debt and has only a very limited impact on welfare gains when transitional dynamics is taken into account.

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[41]Both after-tax wages and employment is higher when labour tax is lower.
[42]Results in Sections 3.1–3.4 were made under the assumption of $\omega = 0$.

Conclusions

In this paper we study the question of optimal sovereign debt in Slovakia. Our research fills a gap in this area since only a few studies have discussed this issue and none of them from a theoretical perspective. We conduct our analysis with the help of a workhorse incomplete markets framework à la Bewley (1986), Huggett (1993), and Aiyagari and McGrattan (1998) in which ex-ante homogeneous households face idiosyncratic uninsurable earnings shocks. Furthermore, inspired by Chatterjee et al. (2017) we enriched the model by public infrastructure that generates positive spillover effect on production and help reduce the precautionary savings motive in this type of models. We calibrated the benchmark model such that it matches key aggregate and distributional variables of the Slovak economy. Then, by employing the consumption-equivalent welfare criterion we evaluated the optimal debt level both in the long–run and also in the short–run when transitional dynamics is considered too.

Our findings are the following. From the long-run perspective when the stationary equilibria are compared only, it is optimal for the government to accumulate surpluses. Moreover, this surplus is substantially larger when include public infrastructure investment, which has a positive spillover effect on production. The situation becomes fundamentally different when we also account for the transition dynamics between stationary equilibria. First, regardless of the inclusion of public infrastructure, it is now optimal for the government to accumulate debt. Next, the welfare gain induced by changing the benchmark debt level to the optimal level of 27 percent of GDP is only 2.27 percent. Furthermore, we showed that the impact of public infrastructure on the level of optimal debt is less significant than in the long-run case.

The model-implied optimal level of public debt in Slovakia is substantially lower than the benchmark debt/GDP ratio of 50 percent. This indicates that even when the short-run welfare consequences are taken into consideration, the Slovak economy and households would benefit from lower debt (and lower tax rate in the equilibrium). This is true despite the temporary losses induced by a short-lived income tax hike. Furthermore, we showed that our results are robust to changes to the fiscal reaction function, level of interest rates, tax environment structure and the inclusion of public expenditures in the utility function of households.

There are several features that are not yet considered in our approach. First, the current model setup uses proportional tax rate. However a more realistic design of tax structure is needed since labour tax is usually progressive. Second, all households receive the same level of transfers, although in reality transfers are mean-tested. Third, currently the return on capital coincides with public bonds return which is not the case in reality. Therefore one might want to add an aggregate shock and explicit modelling of the household portfolio choice. Finally, the model can also be extended to incorporate features of a small and open economy.
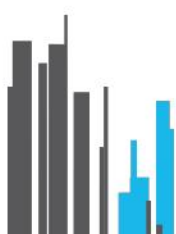
References

Aiyagari, R. (1994). Uninsured Idiosyncratic Risk and Aggregate Savings. *The Quarterly Journal of Economics*, 3(109):659–684.

Aiyagari, R. and McGrattan, E. (1998). The Optimum Quantity of Debt. *Journal of Monetary Economics*, (42):447–469.

Bakis, O., Kaymak, B., and Poschke, M. (2015). Transitional dynamics and the optimal progressivity of income redistribution. *Review of Economic Dynamics*, (3):679–693.

Barillasa, F. and Fernandez-Villaverdes, J. (2007). A generalization of the endogenous grid method. *Journal of Economic Dynamics and Control*, (31):26982712.

Bewley, T. F. (1986). Statationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers.

Bi, H. and Leeper, E. M. (2013). Analyzing Fiscal Sustainability.

Carroll, C. D. (2011). Solution Methods for Microeconomic Dynamic Stochastic Optimization Problems.

Castaneda, A., Diaz-Gimenez, J., and Rios-Rull, J. V. (2003). Accounting for the U.S. Earnings and Wealth Inequality. *Journal of Political Economy*, 111:818–857.

Chatterjee, S. and Eyigungor, B. (2013). Debt Dilution and Seniority in a Model of Defaultable Sovereign Debt. *Federal Reserve Bank of Philadelphia*, pages 13–30.

Chatterjee, S., Gibson, J., and Rioja, F. (2017). Optimal public debt redux. *Journal of Economic Dynamics and Control*, 83:162–174.

Desbonnet, A. and Weitzenblum, T. (2012). Why Do Governments End Up With Debt? Short-Run Effects Matter. *Economic Inquiry, Western Economic Association International*, 50(4):905–919.

Domeij, D. and Heathcote, J. (2004). On the distributional effects of reducing capital taxes. *International Economic Review*, 45(2):523–554.

Floden, M. (2001). The Effectiveness of Government Debt and Transfers as Insurance. *Journal of Monetary Economics*, (48):81–108.

Ghosh, A., Kim, J. I., Mendoza, E., Ostry, J., and Qureshi, M. (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *Economic Journal*, pages F4–F30.

Guvenen, F. (2011). Macroeconomics with Heterogeneity: A Practical Guide. *Economic Quarterly*, 97(3):255–326.

Heer, B. and Maussner, A. (2009). *Dynamic General Equilibrium Modeling: Computational Methods and Applications*. Springer Verlag, 2 edition.

Huggett, M. (1993). The risk free rate in heterogeneous-agent, incomplete-insurance economies. *Journal of Economic Dynamics and Control*, (17):959–969.

Kirkby, R. (2017). Transition paths for Bewley-Huggett-Aiyagari models: Comparison of some solution algorithms. *SEF Working PAper*, (1).

Kopecky, K. and Tauchen, R. S. (1993). Finite State Markov-chain Approximations to Highly Persistent Processes. *Review of Economic Dynamics*, 13:701–714.

Krusell, P. and Smith, A. (1997). Income and wealth heterogeneity, portfolio chioce and equilibrium asset returns. *Macroeconomic Dynamics*, (1):387–422.

Maliar, L. and Maliar, S. (2013). Envelope condition method versus endogenous grid method for solving dynamic programming problems. *Economics Letters*, (120):262–266.

Mucka, Z. (2015). Fiscal Limit: Case of Slovakia. *CBR Working Paper*, (2/2015).

Mucka, Z. and Odor, L. (2017). Sovereign default risk and debt limits: Case of Slovakia. *CBR Working Paper*, (1/2017).

Murtin, F. and d´Ercole, M. M. (2015). Household wealth inequality across oecd countries: new oecd evidence.

OECD (2012). Income distribution data review - Slovak republic.

Peterman, W. B. and Sager, E. (2016). Optimal Public Debt with Life Cycle Motives.

Rios-Rull, V. (1999). Computation of equilibria in heterogeneous-agent models. In Marimon, R. and Scott, A., editors, *Computational Methods for the Study of Economic Dynamics*. Oxford University Press, 1 edition.

Rohrs, S. and Winter, C. (2017). Reducing government debt in the presence of inequality. *Journal of Economic Dynamics and Control*, (82):1–20.

Rust, J., Schjerning, B., and Iskhakov, F. (2012). A generalized endogenous grid method for discrete-continuous choice. *Meeting Papers from Society for Economic Dynamics*, (1162).

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economic Letters*, 20:177–181.

Trimborn, T., Koch, K.-J., and Steger, T. M. (2006). Multidimensional transitional dynamics: A simple numerica procedure. *CESIFO Working Paper*, (1745).

Villemot, S. (2012). Accelerating the resolution of sovereign debt models using an endogenous grid method. *DYNARE Working Paper Series*, 17/2012.

Appendix A   Computational methods for stationary and transitional equilibrium

A.1   Stationary equilibrium solution algorithm

Figure A.1 :   Scheme of the stationary equilibrium calculation procedure
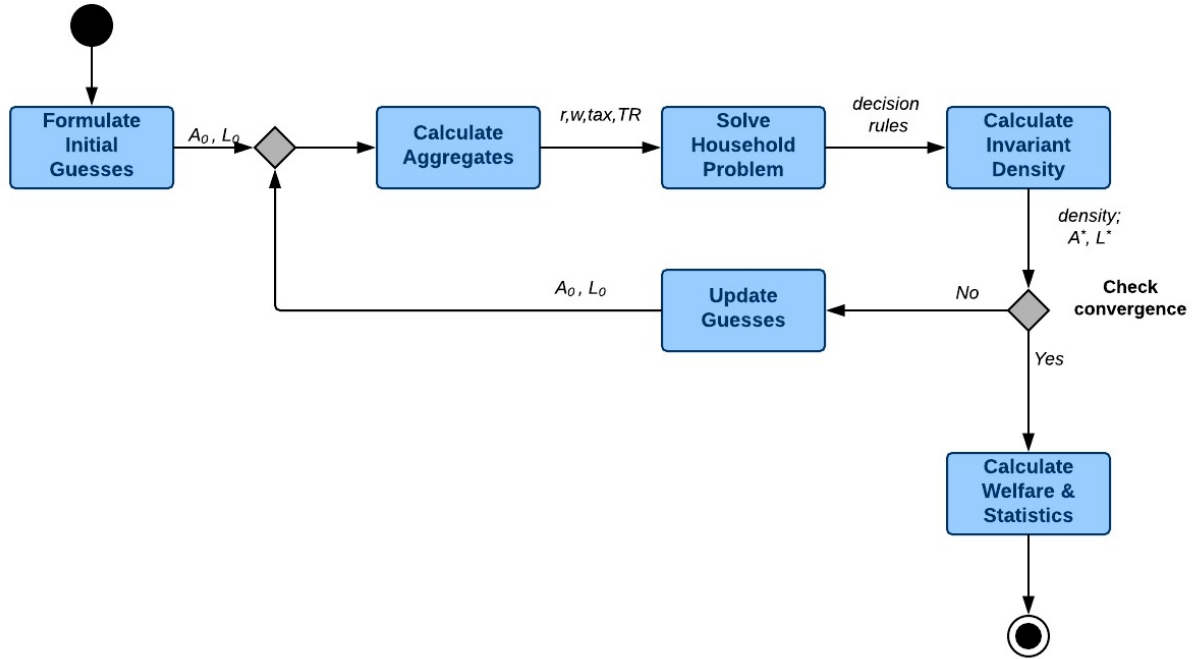
1. Define two grids of 500 and 2000 unevenly distributed points on an interval $[\underline{a}, \overline{a}]$.

2. Make initial judgements on aggregate stock of assets, A_0 and aggregate labour supply, L_0.

3. Solve the optimisation problem of a representative firm and government problem: find the set of aggregates, factor prices and fiscal variables consistent with initial judgements A_0 and L_0.

4. Solve the heterogeneous households' problem: find the optimal decision rules on consumption, labour supply and next-period asset holdings using the endogenous grid method.

5. Compute the invariant distribution Λ employing binning approach and piecewise-linear approximation and calculate new aggregate assets, $\tilde{A}_0$, and aggregate labour supply, $\tilde{L}_0$.

6. Check convergence: if both $|A_0 - \tilde{A}_0|$ and $|L_0 - \tilde{L}_0|$ are small enough, terminate.
    Otherwise, return to step 2 with new values $A_1 = A_1(A_0, \tilde{A}_0)$ and $L_1 = L_1(L_0, \tilde{L}_0)$ and repeat until convergence is obtained.

A.2   Transitional equilibrium solution algorithm

1. Solve initial and target stationary equilibrium problems, $\mathscr{S}_{ini}$ and $\mathscr{S}_{tar}$.

2. Make judgment on the length of the transitional path, T, and define the debt/GDP path $\{x_t\}_{t=1}^T$ such that $x_1 = x_{ini}$ and $x_t = x_{tar}$ for $2 \leq t \leq T$.

3. Make judgment on paths of aggregate assets $\{A_t\}_{t=1}^T$ and aggregate labour supply $\{L_t\}_{t=1}^T$.

Figure A.2 : **Procedure for finding the equilibrium transition paths between two stationary solutions**
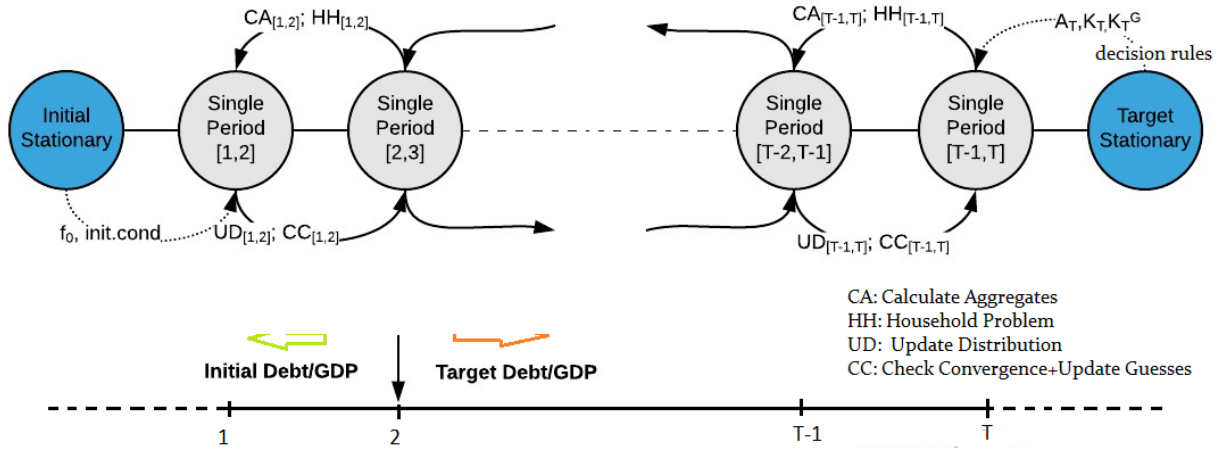
4. Iterate backward on periods $\{[t-1,t]\}_{t=T}^{2}$ to solve the problem of firms and government. At each time period $[t-1,t]$ combine the next-period (end-of-period) values of aggregate private capital stocks, K_t, aggregate assets, A_t, and public infrastructure stock K_t^g, with current (begin-of-) period judgments on aggregate assets A_{t-1} and labour supply L_{t-1} to produce the consistent set of aggregates, factor prices w_{t-1}, r_{t-1} and fiscal variables $(\tau_{t-1}, TR_{t-1}, \dots)$.
   Start with time period $[T-1,T]$ and for the next-period values of aggregates use target stationary equilibrium values K_{tar}, A_{tar}, K_{tar}^g.

5. Iterate backward on periods $\{[t-1,t]\}_{t=T}^{2}$ to solve the problem of households. At each time period $[t-1,t]$ combine the next-period consumption decision matrix $c_t(a,e)$ with current-period factor prices (w_{t-1}, r_{t-1}), tax rate τ_{t-1} and transfers TR_{t-1} to determine the current-period optimal decision rules on consumption, $c_{t-1}(a,e)$, labour supply, $h_{t-1}(a,e)$ and next-period asset holdings, $a'_{t-1}(a,e)$.
   Start with time period $[T-1,T]$ and for the next-period household consumption decision use target stationary equilibrium consumption decision $c_{tar}(a,e)$.

6. Iterate (simulate) forward on periods $\{[t-1,t]\}_{t=2}^{T}$ to update the distribution of individual states. At each time period $[t-1,t]$ combine the previous-period distribution with the current-period decision matrices to determine the current-period distribution of individual states.
   Start with time period $[1,2]$ and for the previous-period distribution use the invariant stationary distribution that corresponds to the initial stationary equilibrium.
   Furthermore, at each time period $[t-1,t]$ calculate the new values of aggregate labour supply, and aggregate assets and so derive the end-of-iteration paths $\{\tilde{L}\}_{t=1}^{T}$ and $\{\tilde{A}\}_{t=1}^{T}$.

7. Convergence check:

   - If both $||A - \tilde{A}||_\infty$ and $||L - \tilde{L}||_\infty$ are small enough, algorithm converges and solution $\mathscr{S}_{[0,T]}$ is found.
     Check the stability of the solution $\mathscr{S}_{[0,T]}$ of the length T: Find the solution $\mathscr{S}_{[0,T+1]}$ for the equilibrium path of length $T+1$ (return to step 2) and compare it with the solution for the equilibrium path of length T. If the maximal difference between them is small enough, the solution $\mathscr{S}_{[0,T]}$ is a stable solution of the transition equilibrium problem and procedure terminates. Otherwise increase T and proceed until stability is obtained.

   - Otherwise tune the initial judgments on paths using e.g. a simple dumping rule, so for each $2 \le t \le T-1$ set
     $$A_t^{new} = \phi_A A_t + (1-\phi_A)\tilde{A}_t, \quad L_t^{new} = \phi_L L_t + (1-\phi_L)\tilde{L}_t,$$

where $\phi_A, \phi_L \in (0,1)$ are dumping factors. Return to step 4 and repeat the procedure until the convergence is obtained.

A.3 Welfare gain calculation: general case

Following Rohrs and Winter (2017) consumption equivalent welfare change for the average household is defined as the percentage change in consumption that the household must incur in the old situation in order to be indifferent between staying in the old situation and being in a new situation. The consumption equivalent change for the average household, $\Delta \Xi_{old \to new}$ is

$$\int W_{old}(a,e;\Xi_{old \to new})d\theta_{old}(a,e) = \int W_{new}(a,e)d\theta_{new}(a,e)$$

$$\int \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left((1+\Delta\Xi_{old \to new})c_{t,old}^{\eta}(1-h_{t,old})^{1-\eta} \right)^{1-\sigma}}{1-\sigma} + \omega \frac{(\kappa G_t^c)^{1-\sigma_g}}{1-\sigma_g} \right] d\theta_{old}(a,e) =$$

$$\int \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_{t,new}^{\eta}(1-h_{t,new})^{1-\eta} \right)^{1-\sigma}}{1-\sigma} + \omega \frac{(\kappa G_t^c)^{1-\sigma_g}}{1-\sigma_g} \right] d\theta_{new}(a,e).$$

If household utility function is homogeneous in consumption (i.e. $\omega = 0$), the expression above can be simplified to relationships (9)-(10). Otherwise, more general approach has to be applied. Hence, to calculate welfare gain associated with the transition from the old state to the new one we firstly evaluate welfare for the old state (initial debt level) for a reasonable scale of consumption multipliers. Then we interpolate the target (new) state welfare on the consumption-scaled vector of initial welfare obtained in the first to get the welfare gain.

Appendix B Wealth and income distributions matching

Table B.1 : Calibration distributional targets

	Total Net Wealth		Gross Labour Income	
	Target	Model	Target	Model
GINI Index	0.49	0.5025	0.61	0.5945
Q1	1.3	1.74	0	0
Q2	8.1	8.53	0.6	0.48
Q3	13.4	14.02	15.0	15.05
Q4	19.1	20.22	30.6	29.80
Q5	58.1	55.49	53.8	54.67

Appendix C   Benchmark model

Figure C.1 :   Stationary equilibria: Adjustment in aggregate variables
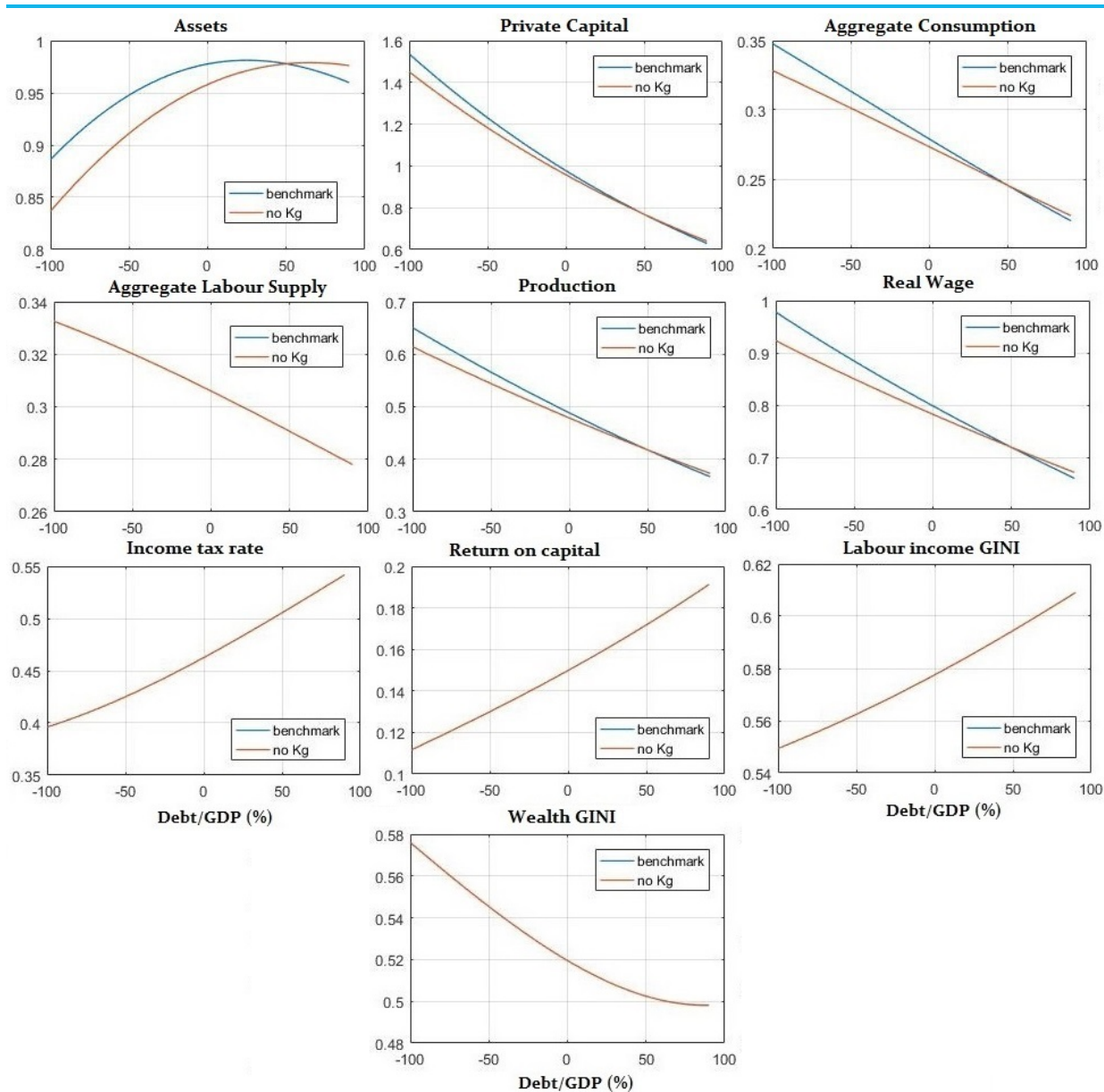
Adjustment in aggregate variables and GINI indices when transitional dynamics is ignored

Figure C.2 : Transitional Equilibrium: Adjustment in aggregate variables
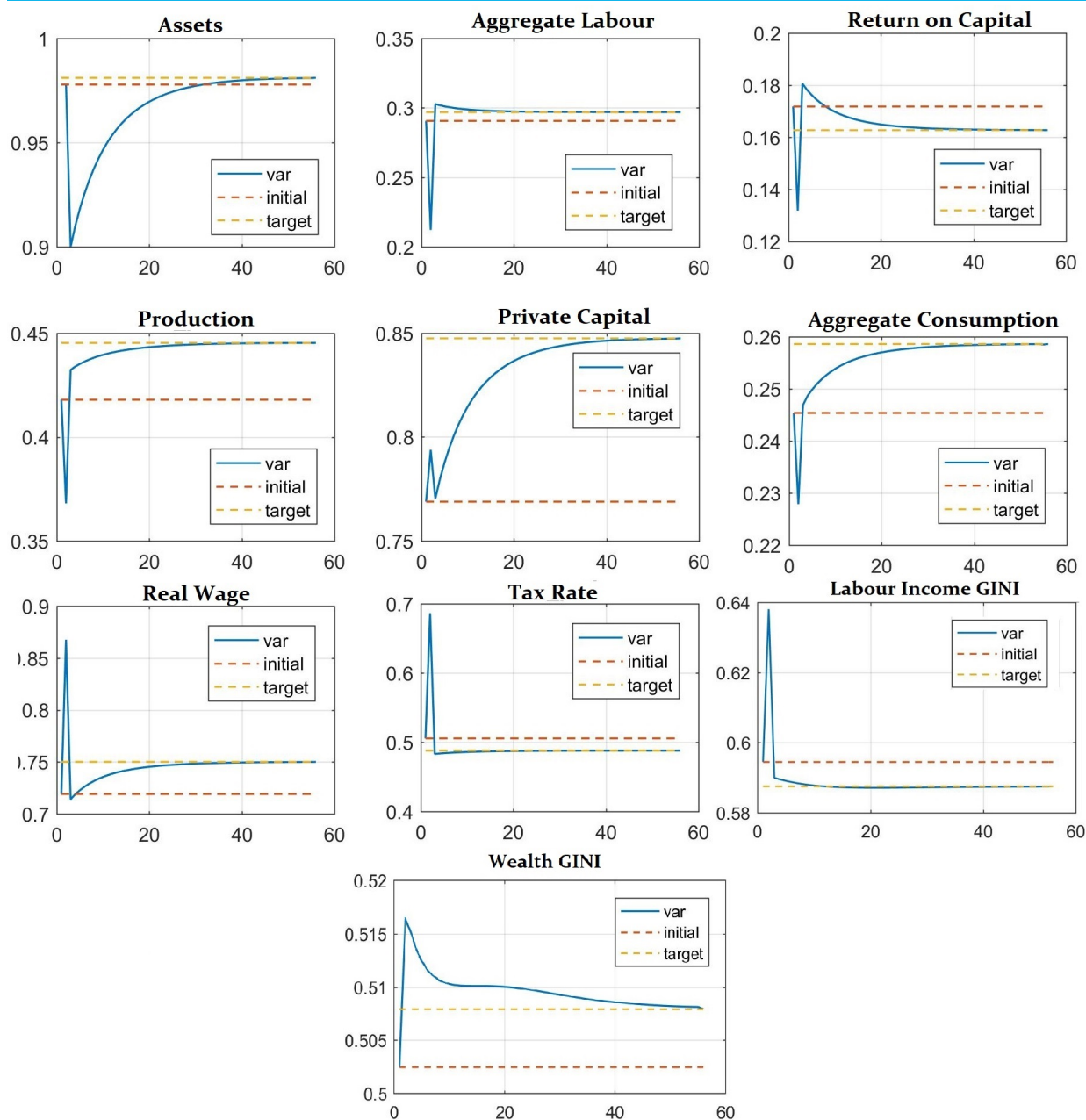
Equilibrium with transition dynamics: adjustment in aggregate variables and GINI indices when transitional dynamics between stationary equilibria is considered
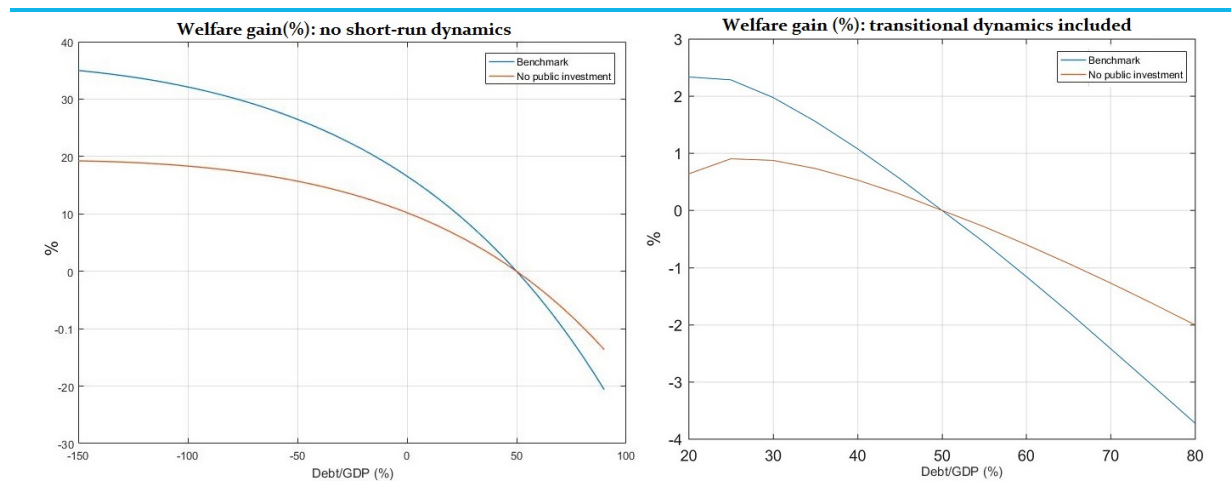
Figure C.3 : Adjustment in distributional variables
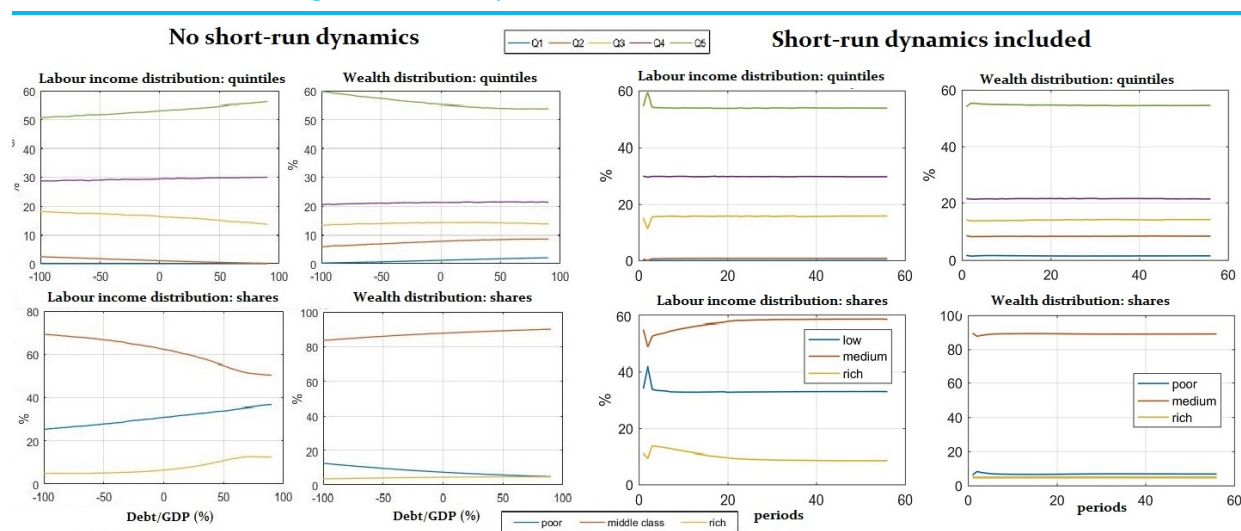
Adjustment in distributional variables (quintiles of distributions, population shares) when transitional dynamics between stationary equilibria is either ignored (left panel) or considered (right panel)

Appendix D   Welfare gains and optimal debt profiles

Figure D.1 :   Welfare gain comparison

Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right)

Figure D.2 :   Welfare gain comparison for specific groups
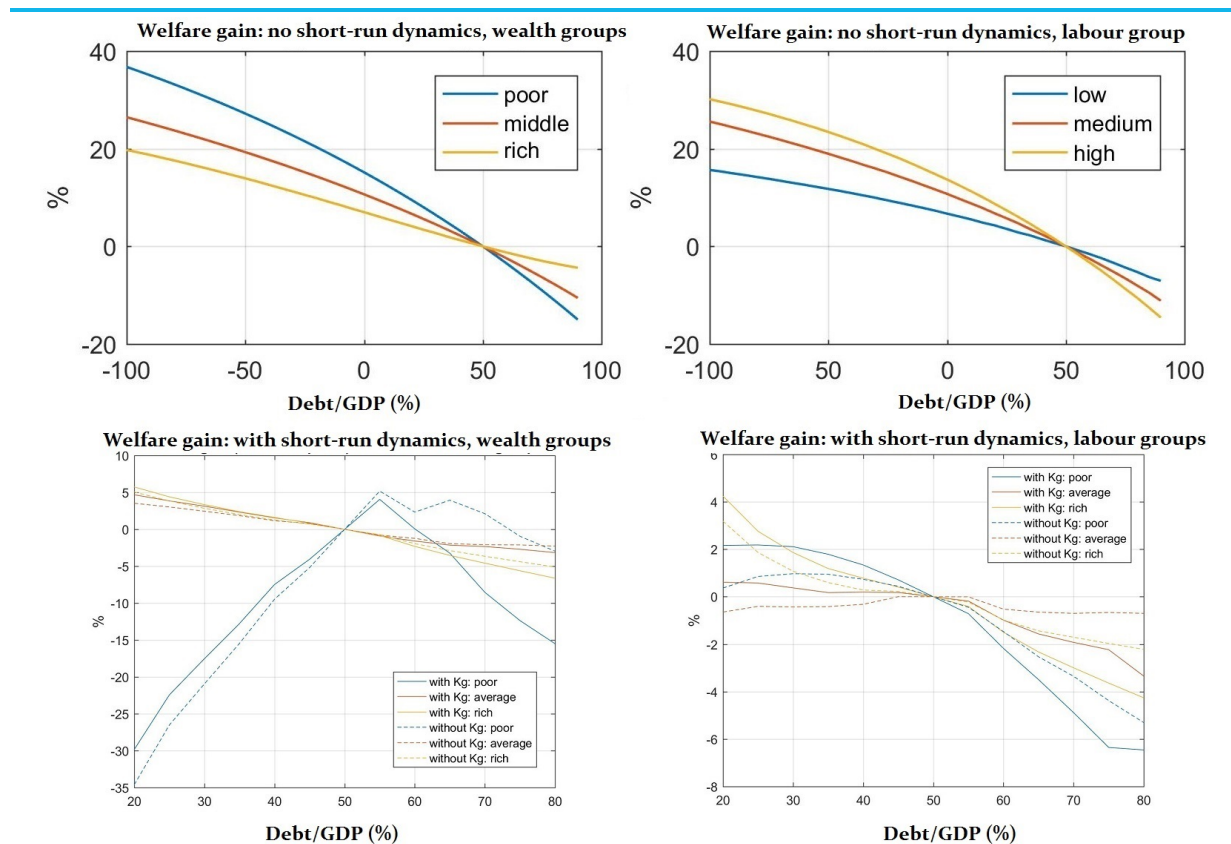
Long-run welfare gain comparison for different labour income and wealth groups: transitional dynamics is either ignored (up) or taken into account (down)

Appendix E   Robustness check
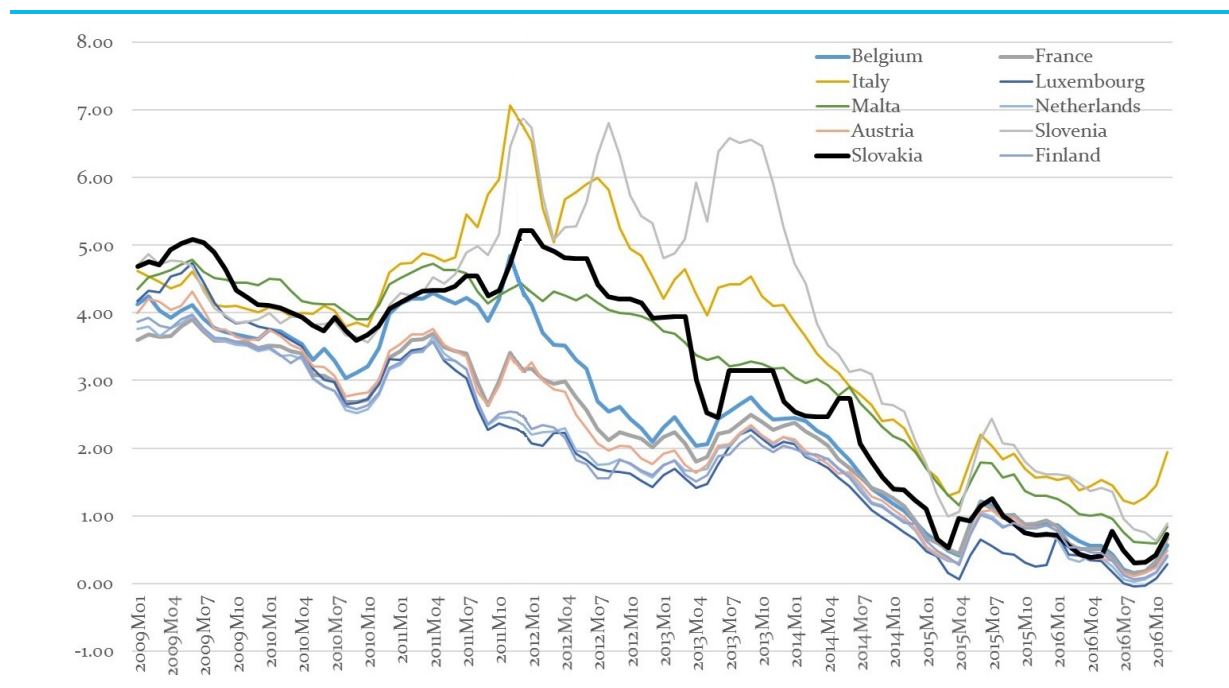
E.1   Transfers as policy instrument

Figure E.1 :   Stationary Equilibria (changes in transfers)
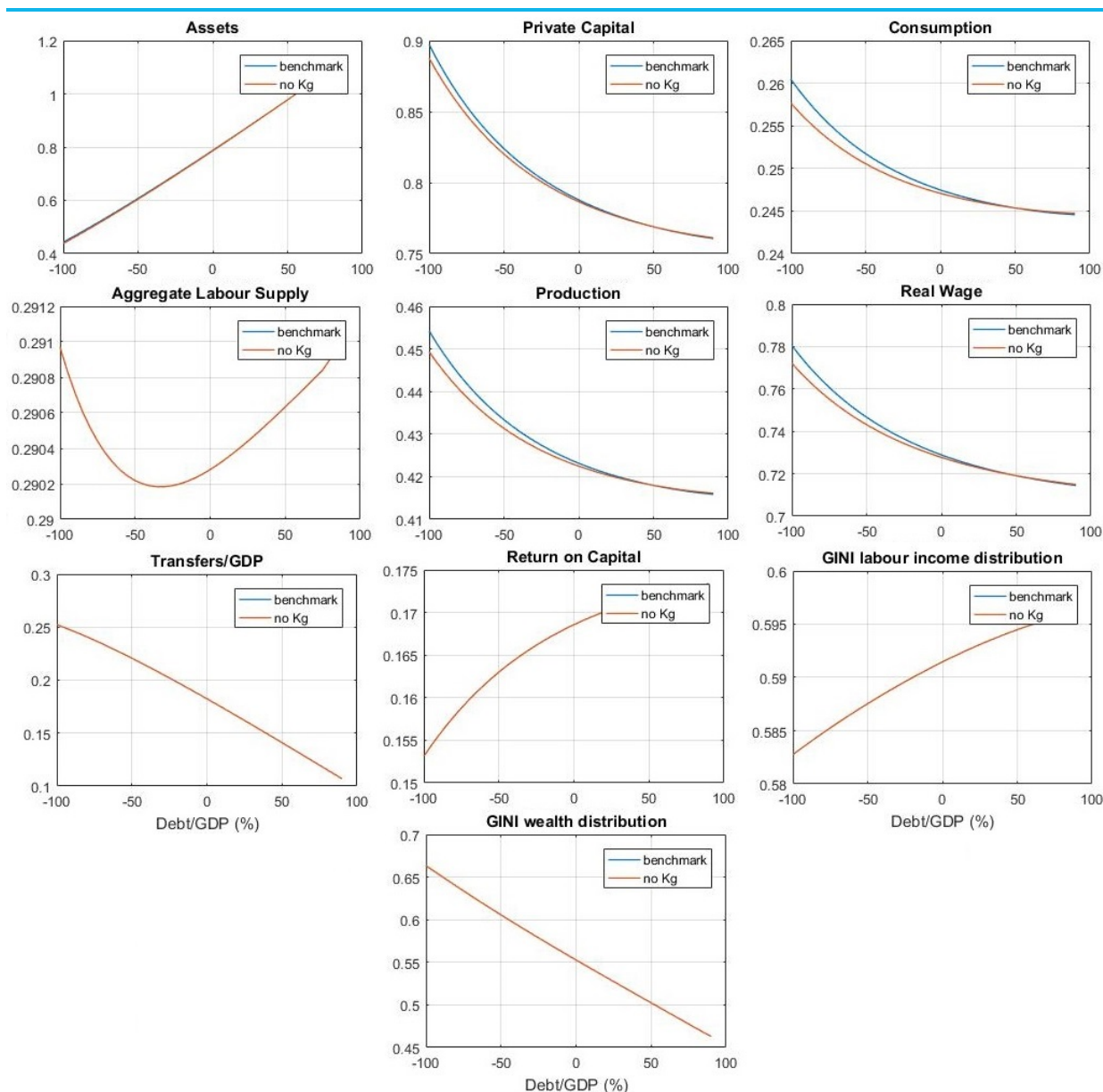
Adjustment in aggregate variables and GINI indices when transitional dynamics is ignored and government adjusts transfers instead of tax rate
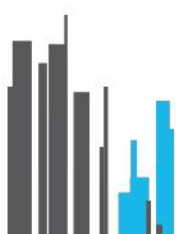
Figure E.2 :   Transitional equilibrium (changes in transfers)
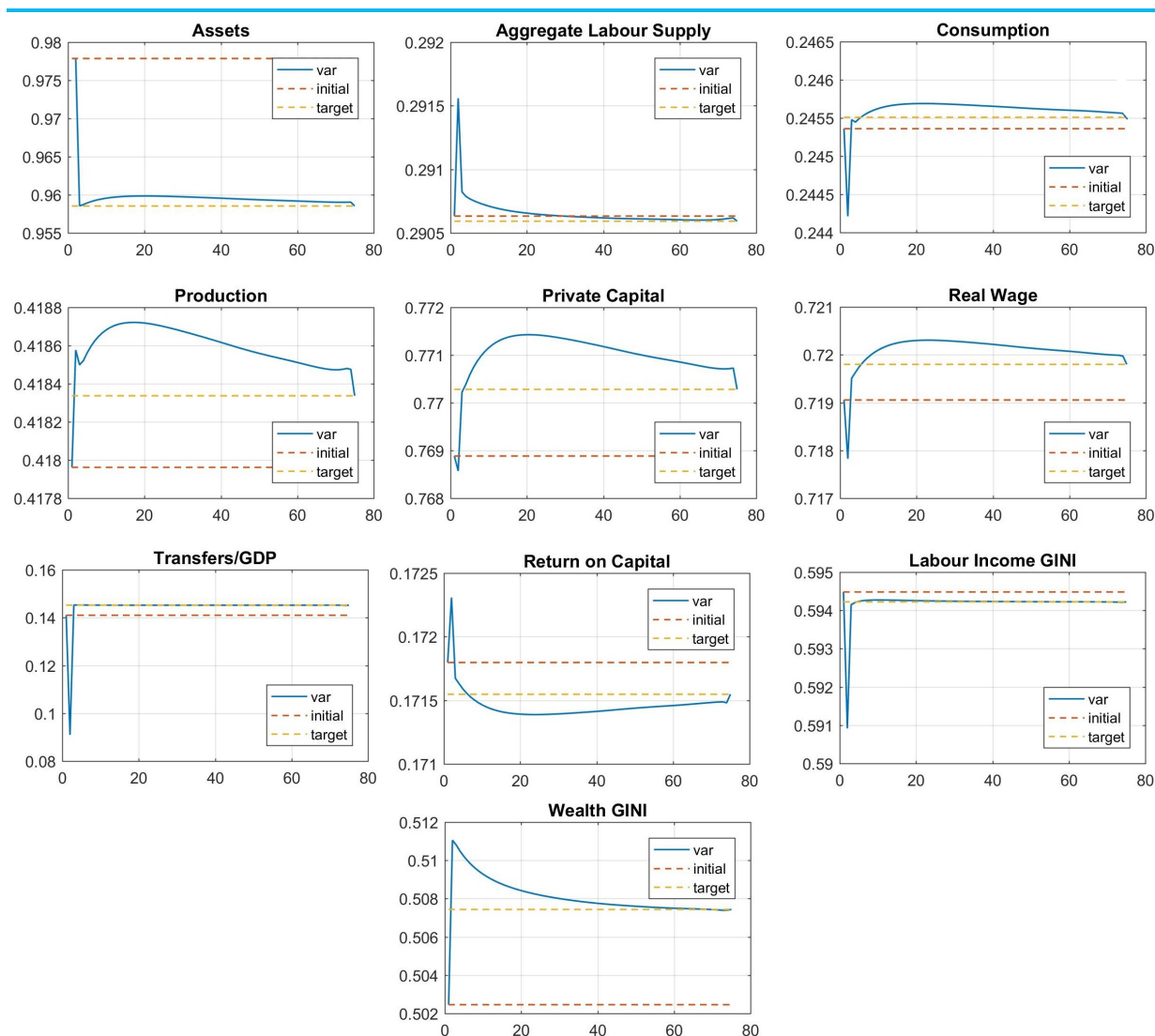
Adjustment in aggregate variables and GINI indices when transitional dynamics is considered and government adjusts transfers instead of tax rate

Figure E.3 : Adjustment in distributional variables (changes in transfers)
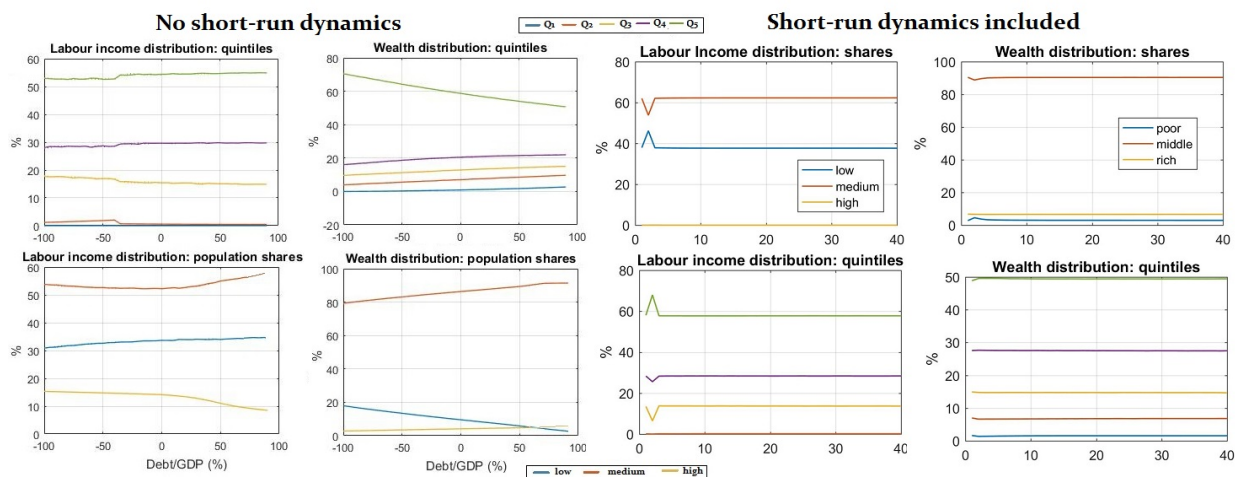
Adjustment in distributional variables (quintiles of distributions, population shares) when transitional dynamics is either ignored (left panel) or considered (right panel) and government adjusts transfers instead of tax rate

Figure E.4 : Welfare gain comparison (transfers changed)
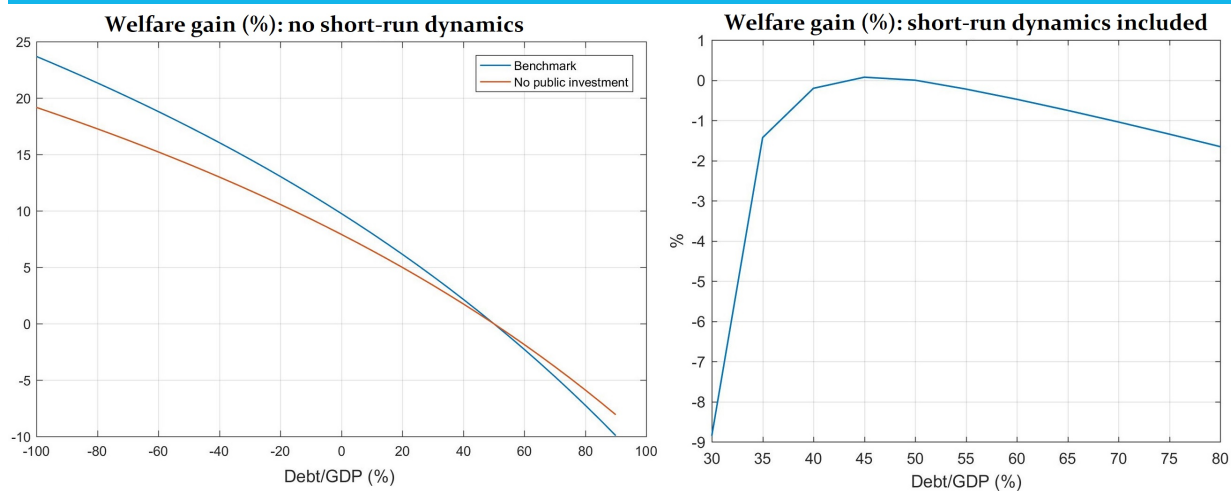
Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right) and government adjusts transfers instead of tax rate

E.2 Alternative calibration

Table E.1 : Alternative calibration model parameters

Description	Parameter	Value
private capital depreciation rate	δ	0.1
public capital depreciation rate	δ_k	0.1
public infrastructure elasticity w.r.t. output	ϕ	0.065
share of private capital in the production	α	0.5
social transfers/GDP	g_{tr}	0.141
government consumption/GDP	g_c	0.193
public investment/GDP	g_i	0.036
sovereign debt/GDP	x	0.5
government consumption weight in utility	ω	0
share of productive government consumption	κ	0.306
time-preference rate	β	0.975
inverse of the intertemporal elasticity of substitution	σ	2.65
relative share of consumption in the utility function	η	0.491
borrowing constraint	$\underline{a}$	-0.0175
idiosyncratic labour income shock	e	$(0.04, 0.48, 1.11, 1.23, 2.45)$
transition matrix	π	$\begin{pmatrix} 0.93 & 0.06 & 0.01 & 0 & 0 \\ 0.08 & 0.84 & 0.05 & 0.03 & 0 \\ 0.01 & 0.05 & 0.89 & 0.05 & 0 \\ 0 & 0.02 & 0.05 & 0.88 & 0.05 \\ 0 & 0.02 & 0.03 & 0.05 & 0.90 \end{pmatrix}$

Figure E.5 : Euro-Area bond yields (% ,p.a.)

Figure E.6 : Stationary equilibria (alternative calibration)
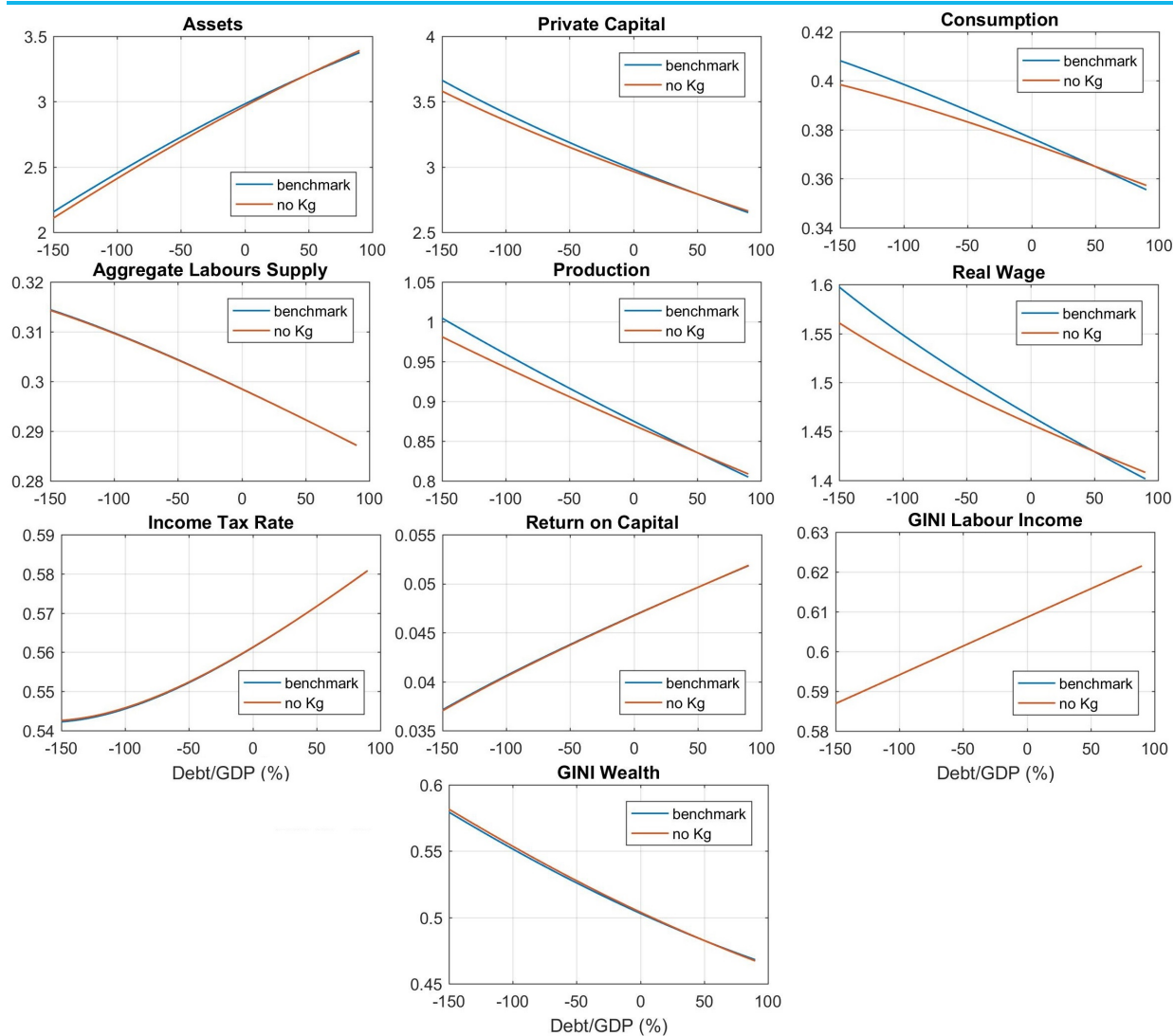
Adjustment in aggregate variables and GINI indices when transitional dynamics is ignored, alternative calibration (lower target return on capital) assumed.

Figure E.7 :   Transitional equilibrium (alternative calibration)
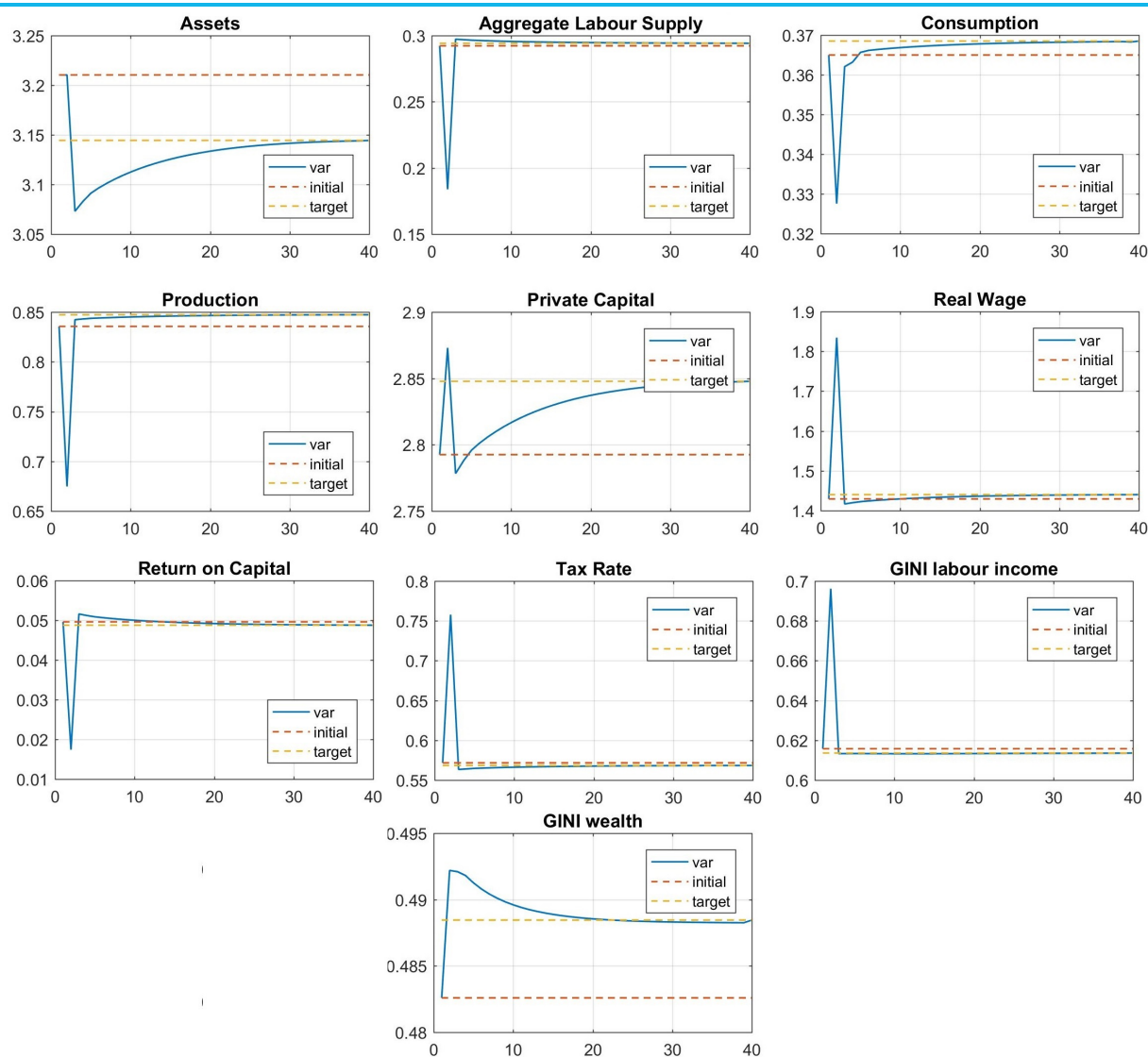
Adjustment in aggregate variables when transitional dynamics is considered, alternative calibration (lower target return on capital) assumed.

Figure E.8 : Adjustment in distributional variables (alternative calibration)
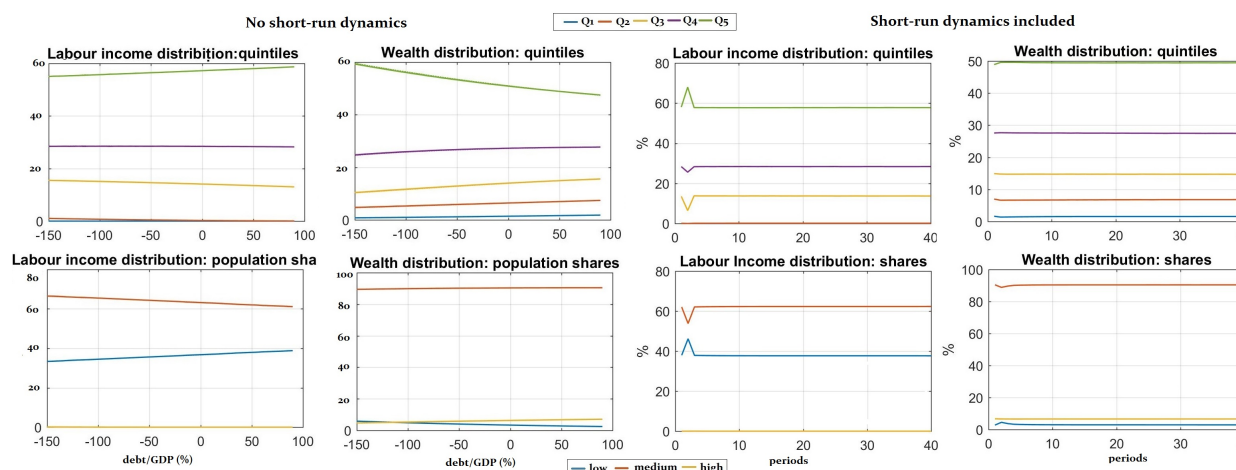
Adjustment in distributional variables (quintiles of distributions, population shares) when transitional dynamics is either ignored (left panel) or considered (right panel), alternative calibration (lower target return on capital) assumed.

Figure E.9 : Welfare gain comparison (alternative calibration)
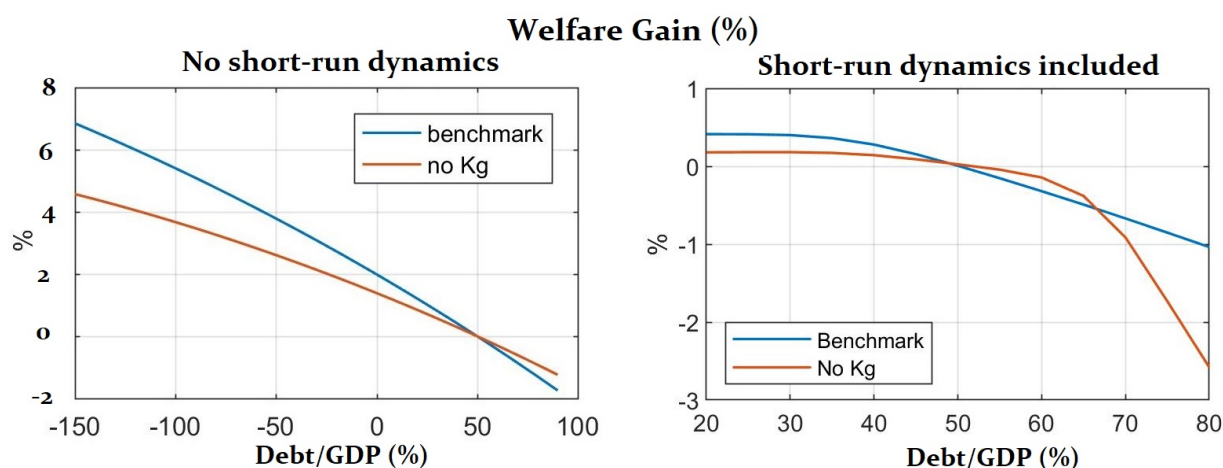
Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right), alternative calibration (lower target return on capital).
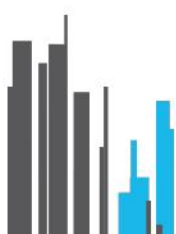
E.3 Two taxes

Figure E.10 : Stationary equilibria (labour vs. capital tax)
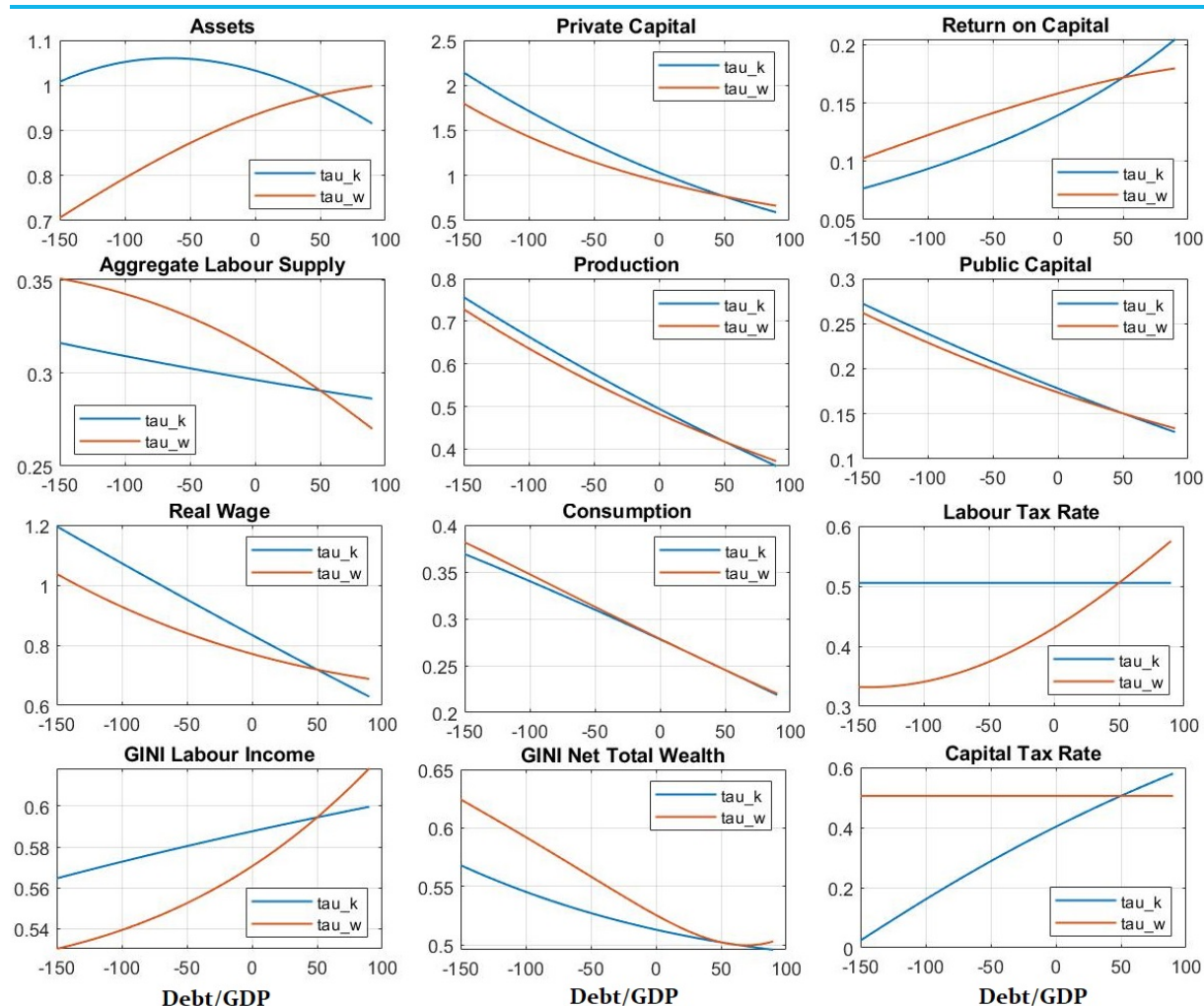
Adjustment in aggregate variables and GINI indices when transitional dynamics is ignored, government changes either capital or labour tax to satisfy budget constraint.

Figure E.11 : Transitional equilibrium (changes in capital tax)
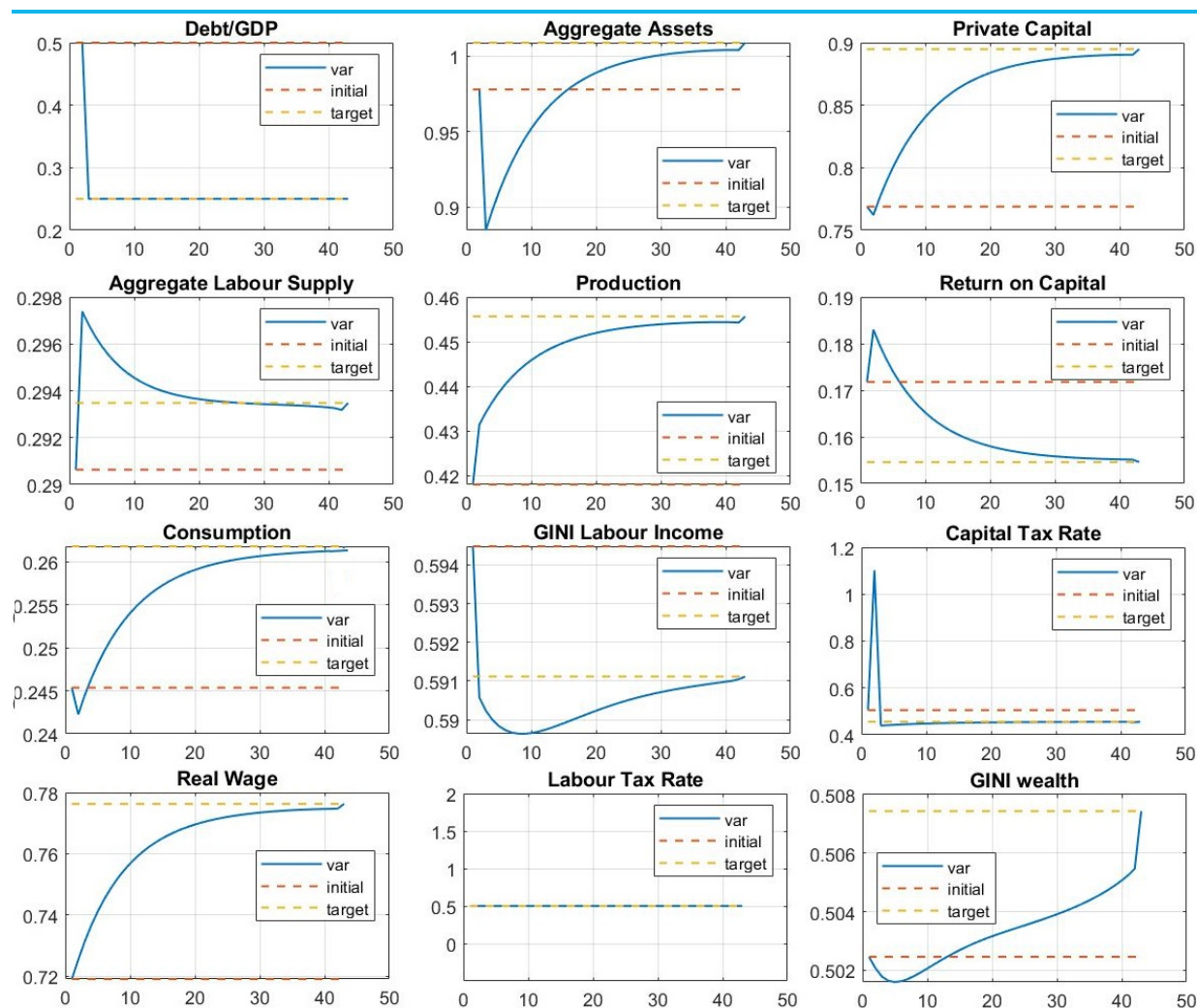
Adjustment in aggregate variables when transitional dynamics is considered, changes in capital tax rate are assumed (labour tax rate constant).
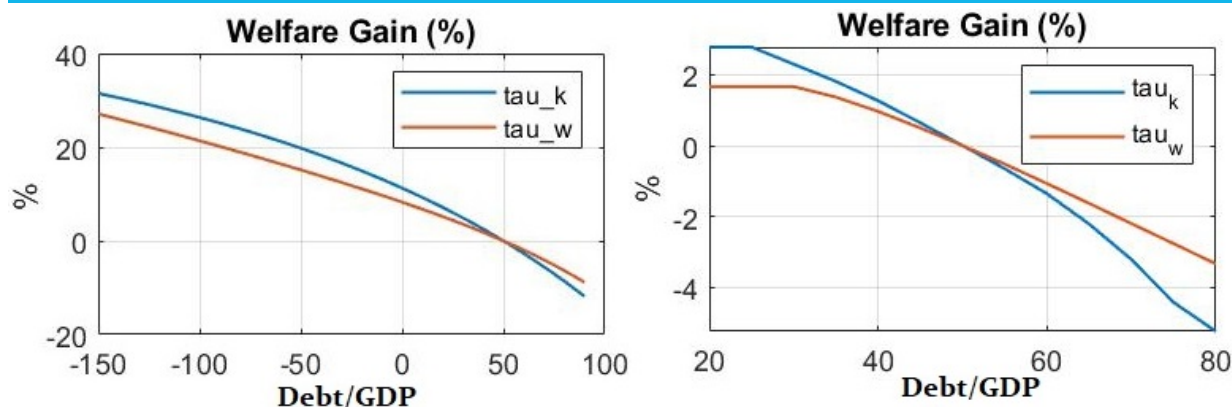
Figure E.12 : Transitional equilibrium (changes in labour tax)
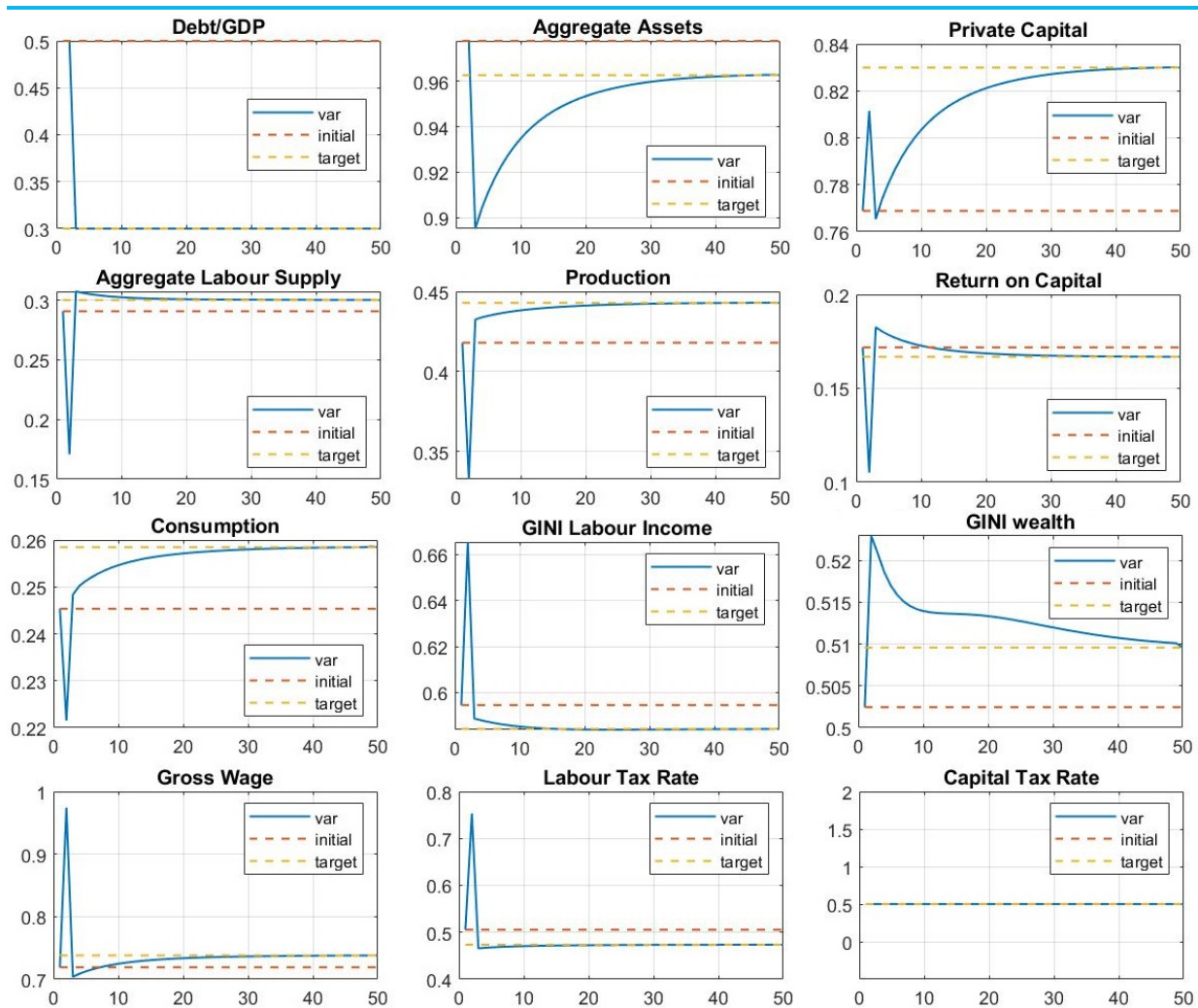
Adjustment in aggregate variables when transitional dynamics is considered, changes in labour tax rate are assumed (capital tax rate constant).
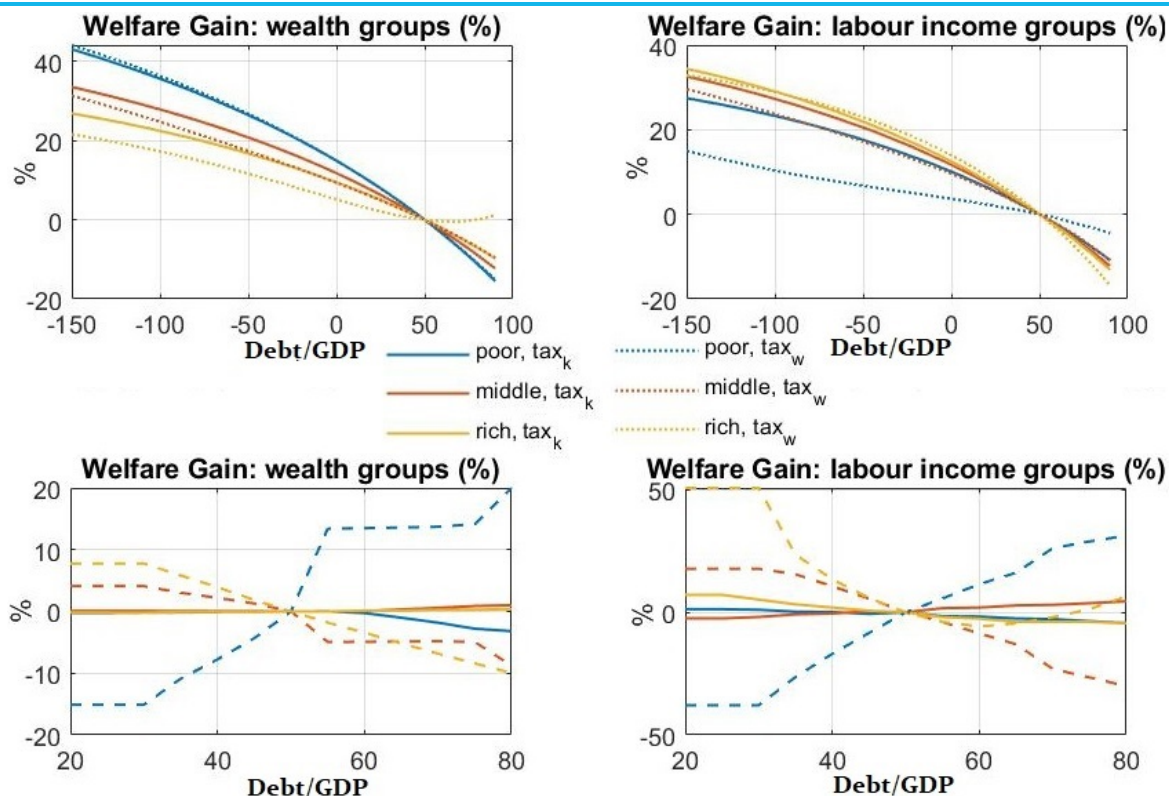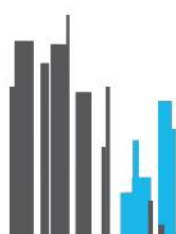
Figure E.13 : Welfare gain comparison (labour vs. capital tax)

Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right), government changes either capital or labour tax to satisfy budget constraint.

## Figure E.14 : Welfare gain comparison for specific groups (labour vs. capital tax)

Long-run welfare gain comparison for different labour income and wealth groups: transitional dynamics is either ignored (up) or taken into account (down), government changes either capital or labour tax to satisfy budget constraint.
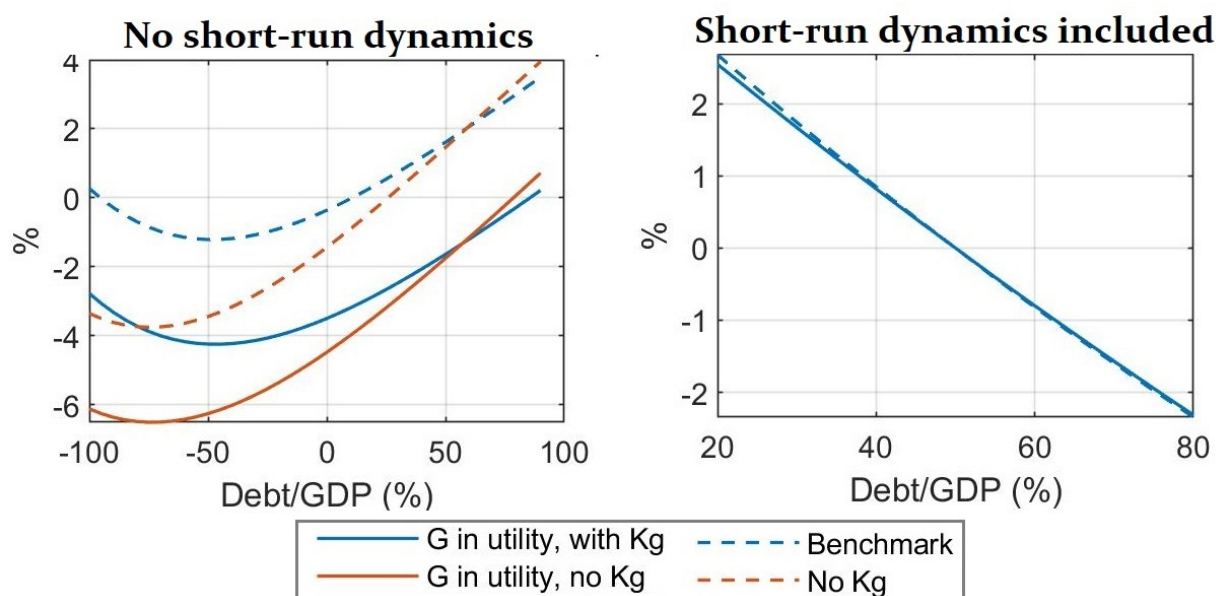
E.4 Government consumption in utility

**Figure E.15 : Welfare gain comparison (government consumption in utility, tax adjustment)**
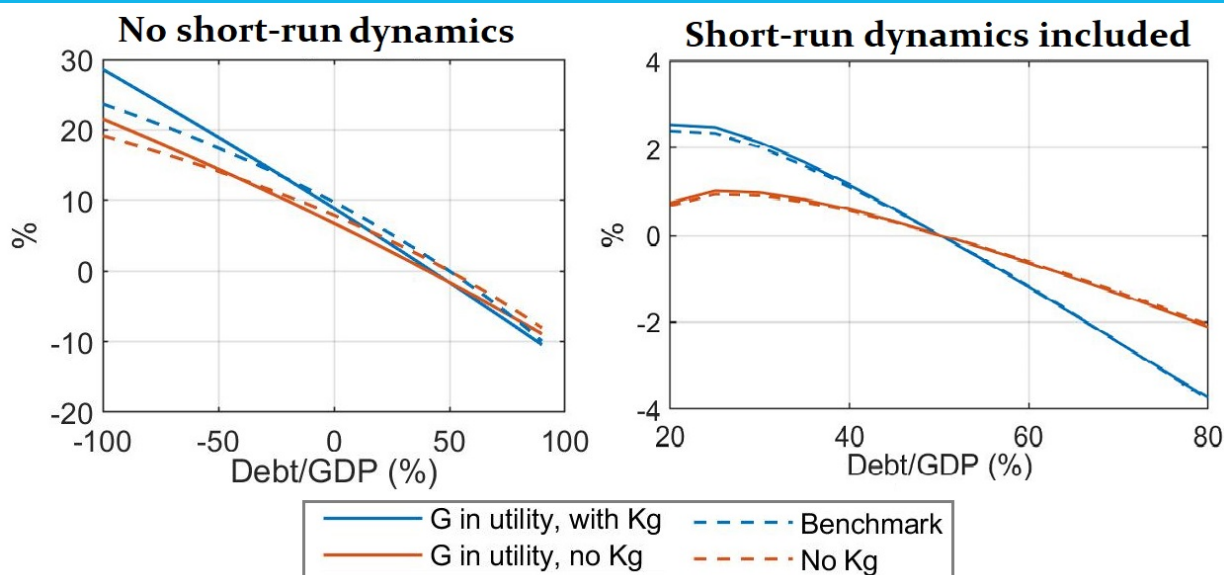
Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right), impact of government consumption (provision of transfers in kind) on households utility is assumed and government adjusts tax rate to satisfy budget constraint

Figure E.16 : Welfare gain comparison (government consumption in utility & adjustment)

Long-run welfare gain comparison: transitional dynamics is either ignored (left) or taken into account (right), impact of government consumption (provision of transfers in kind) on households utility is assumed and government consumption is changed in order to satisfy budget constraint
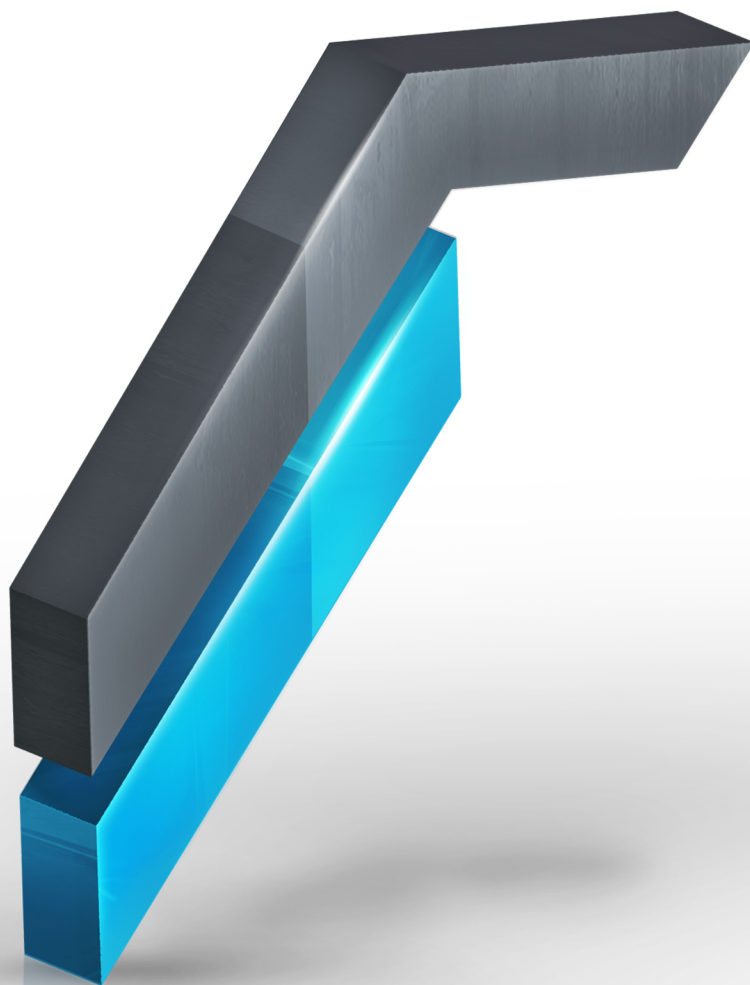
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