



Sovereign default risk and debt limits: Case of Slovakia

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Working Paper No. 1/2017

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Sovereign default risk and debt limits: Case of Slovakia¹

Zuzana Múčka² and Ľudovít Ódor³

Abstract

We use a sovereign default model developed by Hatchondo et al. (2015) to study the implications of adopting constitutional debt limits. It can be shown, that for a benevolent government issuing long-term debt it is welfare-enhancing to introduce credible fiscal rules to mitigate the so called "debt dilution" problem. By calibrating the theoretical model to Slovak data, we estimate the optimal (net) debt brake threshold at 48 percent of the mean annual output. Compared to a no-rule economy, the introduction of a fully-credible debt limit represents a substantial decrease in average sovereign spreads (50 basis points). In the empirical part of the paper we find that the introduction of the constitutional Fiscal Responsibility Act in Slovakia in 2011 might have helped to lower sovereign spreads compared to euro area peers by 20-30 basis points.

Keywords: sovereign default risk, debt dilution, fiscal rules, debt limits

JEL Classification: H1, H63, H8

¹Useful comments and suggestions from Juan Carlos Hatchondo, Michal Horváth, Pavol Povala, Branislav Žúdel and participants of the CBR seminar are gratefully acknowledged. We would like to thank Juan Carlos Hatchondo for providing us with the Fortran code of their model. The authors alone are responsible for all views expressed.

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Introduction

The global financial crisis (GFC) and the subsequent European sovereign debt crisis pushed debt levels in advanced countries up by 40 percent of GDP compared to pre-crisis IMF forecasts. Greece, Portugal and Ireland lost market access and went through a series of fiscal adjustment programs to stabilize debt. Since measures taken by European leaders were unable to calm markets down ("too little too late" behaviour), the European Central Bank had to step in to prevent explosion of sovereign risk premiums in the euro area. Draghi's "whatever it takes" speech followed by the announcement of the Outright Monetary Transactions (OMT) program and the launch of quantitative easing (QE) put an effective cap on sovereign bond yields.

In this paper we argue that despite the substantial decline in sovereign risk premiums in the euro area (especially after the action of the ECB), better understanding and modeling of sovereign risk should be a high priority. In our view, there are at least four important reasons for putting sovereign spreads under more scrutiny. First, debt levels are still at peacetime record highs and substantial upward pressures lurk on the horizon because of population ageing (European Commission (2015)). Second, inflating away debt (or substantial part of it) is unlikely in the euro area and therefore debt denominated in euros is *real* debt for Member States (like external debt in emerging markets). Third, currently there is no political support for an "ever closer Union" or debt mutualization. In such an environment, stronger ex ante sovereign and banking resolution schemes are necessary to mitigate the moral hazard problem (Wyplosz (2017)). Making the no-bailout clause stronger will inevitably lead to more sensitive market pricing of sovereign debt to fundamentals, including fiscal positions. Fourth, with the end of quantitative easing, pressures on sovereign debt markets might reappear.

Cutting back sovereign debt to less dangerous levels (without the help of high inflation) brings fiscal and structural policy issues to the forefront¹. It is well understood that firmly anchored fiscal expectations and credible fiscal frameworks might make the public sector deleveraging process less costly. As Ódor and P. Kiss (2017) argue, one-size-fits-all fiscal rules are suboptimal in a diverse currency union. They propose strong country-specific (and probably time-varying) national fiscal rules: constitutional debt limits (targets) as anchors and expenditure ceilings as operational targets. In this paper we derive an optimal debt brake rule for a small euro area country (calibrated to Slovak data). The implications of the theoretical model are then compared to the empirical effects of the introduction of the constitutional Fiscal Responsibility Act in Slovakia in 2011.

The theoretical part of the paper rests on the Eaton and Gersovitz (1981) framework developed further by other researchers in order to make the model more realistic (Aguiar and Gopinath (2006); Arellano (2008); Hatchondo and Martinez (2009)). In this model, issuing long-term debt generates a deficit bias through time inconsistency. The so called "debt dilution" problem arises: when a government issues new debt, it does not take into account the loss it inflicts on existing creditors. Eventually it leads to over-borrowing and high risk premiums on government

¹Some researchers has proposed an alternative solution, which uses future monetary income to decrease legacy debt levels (Paris and Wyplosz (2014); Corsetti et al. (2015)).



bonds. But investors are well aware of this negative externality and therefore are willing to buy sovereign debt only with a higher discount already in the *current* period (to cover higher expected losses). Credible fiscal anchors, which limit the borrowing of future governments, might thus generate a welfare gain for the economy.

We calibrate the model of Hatchondo et al. (2015) to Slovak data and show that the value of the optimal (net) debt limit is at 48 percent of the mean annual output. When we introduce this debt brake into the model², average sovereign risk premiums fall substantially compared to a no-rule economy (50 basis points).

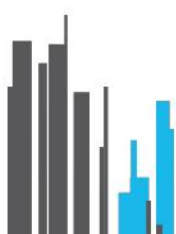
The empirical part of the paper looks at the introduction of the Fiscal Responsibility Act (FRA) in Slovakia in December 2011. The FRA was approved by constitutional majority with the unanimous support of all political parties in the Parliament. It introduced a constitutional gross debt ceiling at 60 percent of GDP, implemented various other fiscal rules³ and established the independent Council for Budget Responsibility. While it is extremely difficult to estimate the effect of the FRA on sovereign risk premiums, our simple empirical investigation identifies a relative drop of spreads by 20-30 basis points compared to other euro area countries. As we show, this decline cannot be explained by relative changes in economic fundamentals⁴ nor with different effects of QE on less liquid bond markets.

The first part of the paper offers a brief overview of the relevant literature. The second section describes the theoretical modeling framework, which closely follows Hatchondo et al. (2015). The third part calibrates the model to Slovak data, while the fourth section presents our results. The fifth section contains a simple empirical investigation of the evolution of spreads on Slovak government bonds. The last section concludes and presents several avenues for further research.

²We assume that the government can credibly commit to this rule.

³With the exception of expenditure ceilings. The FRA assumes their implementation, but without a deadline.

⁴We approximate fundamentals by realized GDP growths, forecasts of potential GDP, actual and projected gross debt figures.



1 Related literature

For many years, sovereign defaults have been studied mainly in the context of emerging markets (especially in Latin America). The renowned interest of policymakers in fiscal matters after the Great Recession has also increased the number academic papers focusing on fiscal sustainability issues. D'Erasmus et al. (2016) critically review traditional methods and recommend three alternative approaches to find out "What is a sustainable public debt?" The first one is a method based on the estimation of fiscal reaction functions (Bohn (1998) and Bohn (2008)). Ghosh et al. (2011) use this approach to calculate debt limits and measure fiscal space in advanced countries.

The second approach recommended by D'Erasmus et al. (2016) is a structural model based on a dynamic general equilibrium framework with a fully specified fiscal sector. A promising avenue of research is the concept of a debt limit, first used in Bi (2012). The fiscal limit arises endogenously from the peak of the Laffer curve, distribution of economic shocks and expectations about future policies (transfer regimes). Mucka (2015) calculates fiscal limit distributions for Slovakia using country-specific TFP shocks and transfer regimes.

In this paper we focus on the third approach recommended by D'Erasmus et al. (2016), namely strategic defaults. This strand of the literature assumes that governments cannot commit to repay debt and can thus optimally decide to default. Models of strategic default were pioneered by the seminal contribution of Eaton and Gersovitz (1981). These models feature endogenous sovereign spreads, a welfare criterion and endogenous borrowing policies. Before the GFC, strategic default models were extensively studied in the context of emerging markets.

Aguiar and Gopinath (2006) developed a quantitative model of debt and default in a small open economy, where defaults occur in equilibrium. Their model was able to match several emerging market empirical regularities: counter-cyclicality of interest rates and net exports and positive correlations between interest rates and current accounts. However, they used only one-period debt and simulated debt and spread levels were low compared to their empirical counterparts.

With the aim of building more realistic models, several additional features were introduced into the basic framework. Arellano (2008) added non-linear income cost of defaulting, while Hatchondo and Martinez (2009) introduced the possibility of issuing long-term debt. Chatterjee and Eyigungor (2012), Hatchondo et al. (2016) and Aguiar et al. (2016) all emphasize that the presence of long-term debt in the model is crucial in order to match empirical regularities in the data. However, once long-term debt is embedded into the model, debt dilution arises almost automatically, because existing sovereign debt contracts do not address this externality. In other words, if one wants to replicate realistic sovereign debt and sovereign default premiums, the problem of debt dilution cannot be avoided.

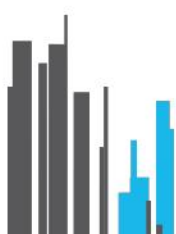
How significant is the debt dilution problem empirically? Hatchondo et al. (2016) show that debt dilution accounts for 78 percent of the default risk in a model calibrated to Spanish data. Therefore, eliminating the problem might generate substantial welfare gains. There are several possibilities how to eliminate or mitigate the debt dilution problem. One obvious solution is to issue one-period debt only. But shortening the maturity structure can be very costly, because it



increases the exposure to rollover risk (sudden stops). Another option is the introduction of a seniority structure or various covenants in order to compensate existing bondholders when new debt is issued (Hatchondo et al. (2016)). In this paper we focus on a third possibility: credible fiscal frameworks, which limit the borrowing capacity of future governments and thus mitigate the debt dilution problem.

There is an extensive literature on fiscal frameworks in general and on the euro area framework in particular. There seems to be a relatively wide consensus that the euro area is a "partially-finished house" (Five Presidents' Report, Juncker et al. (2015)) and substantial institutional reforms are necessary to make the common currency more resistant to future shocks. Fault lines in the initial institutional setup, lack of enforcement of existing rules and procedures together with the "too little, too late" crisis response by governments were identified as the most significant mistakes (Kopits (2017)). As far as the European fiscal framework is concerned Wyplosz (2017) and Ódor and P. Kiss (2017) call for a more decentralized framework of fiscal responsibility. They propose the introduction of country-specific debt limits (targets) at national level and a common spread brake at the level of the euro area⁵. Hatchondo et al. (2015) show that common spread limits generate larger average welfare gains across countries than common debt brakes.

⁵Defined either as a maximum spread over eurobonds (issued by a central authority) or above the average yields paid by the three best performing countries (like in case of the Maastricht criteria).



2 Modeling framework

Our modeling framework is inspired by recent work of Hatchondo et al. (2015), Hatchondo et al. (2012a) and Hatchondo et al. (2012b) using the approach of Eaton and Gersovitz (1981) with long-term debt. In a small open economy environment aggregate output is determined by a stochastic idiosyncratic technology process and labour services supplied by households which make their optimal consumption-labour decisions. Government uses labour taxes and long-term bonds to finance its consumption. Furthermore, under the assumption of non-complete asset markets and a single constant international real interest rate, these non-state contingent defaultable bonds are traded with risk-neutral competitive foreign investors. The aim of the benevolent fiscal authority is to maximise the utility of households. Each period, the government makes two decisions: whether to default on previously issued debt and how much to borrow or save⁶. Sovereign default has two consequences in our model: exclusion from bond markets for a stochastic number of periods (after which the debt is restructured) and an output loss modeled as lower productivity during the exclusion. Furthermore, foreign lenders consider the possibility of future default when they price current debt issuance. This way they can penalize the government for excessive debt by charging an extra risk premium above the international risk-free rate.

In this framework, the debt dilution is a time consistency problem. Decline in the value of existing sovereign debt occurs due to issuance of new debt, which increases the probability of default. As pointed out by Hatchondo et al. (2015) the sovereign debt dilution problem arises because (i) the current government cannot control debt issuances by future governments, (ii) governments issue long-term debt, and (iii) bonds are priced by rational investors⁷. Hence, the current government could benefit from imposing restrictions on future borrowings as this constraint can raise the price of bonds currently issued.

First, we study the benchmark model without rules. Later, we extend it by introducing a simple debt-brake rule that restricts the action of the government and thus prevents the debt to exceed a given threshold. In order to avoid negative welfare consequences stemming from the introduction of a debt limit in an already indebted economy, we also discuss the possibility of a time delay between the date of announcement and actual implementation of the debt rule. Finally, we describe the method we use to evaluate the welfare gain arising from the commitment of the fiscal authority to a debt limit.

Following Hatchondo et al. (2015), Hatchondo et al. (2012a) and Hatchondo et al. (2012b), the model has the following building blocks. We assume that firms produce a homogeneous good,

⁶This is different compared to the standard approach à la Lorenzoni and Werning (2014). They assume that first, the government determines the proceeds needed from debt issuances (in order to cover consumption expenditures, transfers etc.) and then, lenders decide about the interest rate. In contrast, our setting assumes that the fiscal authority chooses the level of debt it wants to issue. As noticed by Calvo (1988), Hatchondo et al. (2015) and Lorenzoni and Werning (2014) this approach eliminates the possibility of multiple equilibria.

⁷Rational investors anticipate that additional borrowing by future governments will increase the risk of default on long-term bonds issued by the current government. Therefore, they offer a reduced price for these bonds already in the current period.



y , consumed by households, c , and government, g , employing a linear technology in labour h ,

$$y_t = e^{a_t} h_t,$$

with the country-specific technology shock a_t following a standard autoregressive process,

$$a_t = \rho_a a_{t-1} + (1 - \rho_a) \mu_a + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a^2). \quad (1)$$

However this idiosyncratic shock should be understood in a broad sense as the model abstracts from explicit treatment of real exchange rates or terms of trade. Therefore, we assume that changes in a implicitly incorporate fluctuations in relative prices. Next, following Cuadra et al. (2010) and Hatchondo et al. (2015) households decide about their consumption and labour supply by maximising their utility à la Greenwood et al. (1988)⁸ augmented by public goods consumption supplied by the government (free of charge), subject to the following budget constraint,

$$\begin{aligned} \max_h \quad u(c, g, h) &= \frac{\pi}{1 - \sigma_g} g^{1 - \sigma_g} + \frac{1 - \pi}{1 - \sigma_c} \left[c - \psi \frac{h^{1 + \omega}}{1 + \omega} \right]^{1 - \sigma_c}, \\ c &= (1 - \tau) e^a h. \end{aligned} \quad (2)$$

Hence, households consume all their after-tax labour income and the optimal labour supply, $h^* = h^*(a, c, g, \tau)$ arises as the solution to (2), i.e. at any time $t \geq 0$

$$h_t^* = \left[\frac{1 - \tau_t}{\psi} e^{a_t} \right]^{1/\omega}, \quad (3)$$

for any labour tax rate τ_t and technology shock a_t .

Finally, the objective of the government at time t is to maximise the present value of the sum of expected discounted future utility of households,

$$\mathbb{E}_t \sum_{k=t}^{\infty} \beta^{k-t} u(c_k, g_k, h_k), \quad t \geq 0.$$

The government finances its consumption g by issuing defaultable debt b and collecting tax revenues levied on labour at a rate τ . Next, instead of issuing standard one-year bonds we allow for long-duration debt obligations. Similarly to Hatchondo and Martinez (2009) we assume that the government issues a bond in period t with a promise to pay an infinite stream of coupons which decreases at a constant rate δ ⁹. For simplicity, we assume that δ remains unchanged over time, and therefore it is not a choice variable. Its value is calibrated to fit Slovak data.

When the government defaults, it does so on all current and future debt obligations. There are two costs of defaulting – the government is excluded from capital markets and faces an income

⁸Monacelli and Perotti (2008) emphasize that the lack of the wealth effect in the consumption-leisure non-separable form of the utility function is helpful in explaining changes in behaviour induced by government consumption shocks.

⁹Thus, a bond issued in period t promises to pay one unit of the good in the following period and $(1 - \delta)^{s-1}$ units in period $t + s$, with $s \geq 2$.



loss. During the exclusion from markets, the government's payment obligations grow at the risk-free interest rate r . However, a return to markets after a default is possible. Referring to Hatchondo et al. (2015) and Hatchondo et al. (2016), the defaulting sovereign has an opportunity to regain the access to markets with probability ξ in each period after the default. Debt restructuring assumes a non-zero recovery rate for debt in default α^{10} . The defaulting country suffers a technology loss of ϕ in every period in which it is excluded from capital markets.

Chatterjee and Eyigungor (2012) and Arellano and Ramanarayanan (2010) suggest that due to nonlinear dynamics of sovereign spreads, a default in good times is proportionally more costly for the economy¹¹. Mendoza and Yue (2012) emphasize that such a behaviour is present in an environment in which a default affects the access of domestic firms to obtain foreign intermediate goods for their production. Following Chatterjee and Eyigungor (2012) and Hatchondo et al. (2015) we assume that the income loss due to default is independent of the size of a debt on which the country defaults and so we model the loss function during the country's default episode as:¹²

$$\phi(a) = \max\{\gamma_0 e^a + \gamma_1 e^{2a}, 0\}.$$

We emphasize that both γ_0 and γ_1 are country-specific and their values are calibrated to fit Slovak data¹³. These two parameters are connected to the average fraction of income loss during the default and the sensitivity of the fraction of income loss to income levels¹⁴.

Government bonds are priced in a perfectly competitive environment by risk-neutral foreign investors that discount future payoffs at the risk-free rate r . Therefore, bond prices are determined by the zero expected profit condition.

The timing of events within each period is as follows. First, at the beginning of each period t , the technology shock a_t is realised. Thus, the country learns the value of its income, y_t . Next, the government that has access to capital markets decides whether to default or not. On the other hand, a country suffering from an exclusion from capital markets chooses whether to end the default (if there is an opportunity to do so). Finally, in both cases the government determines

¹⁰In order to end the default, the government needs to exchange bonds that are in default with bonds promising to pay 100α percent of payments assumed by initial bonds. However, even though the country has an opportunity to return to capital markets it may decide to continue in default – in that case, its debt is still reduced to α fraction of its previous size. This way the government can obtain a lower recovery rate at the expense of longer default period.

¹¹Therefore, we require that $\phi(a)/e^a$ raises with the income shock realisation, a .

¹²Aguiar and Gopinath (2006) assumes that default cost is a constant fraction of income while Arellano (2008) prefers to describe the cost of default as a nonlinear and increasing function of income.

¹³As pointed out by Hatchondo and Martinez (2017) due to endogeneity bias present when calibrating the default costs parameters and modeling abstraction that may omit other possibly relevant costs of default, many studies in the field of sovereign defaults calibrate the parameters of the default loss function to match important data moments, such as the debt level. Furthermore, Chatterjee and Eyigungor (2012) show that having two-parameter cost function (quadratic in income) is sufficient to match exactly the targets for the country's average level of spread and debt.

¹⁴Hatchondo and Martinez (2017) notice that changes in the the average fraction of income loss during the default have a strong impact on the mean of the simulated debt level while changes in the sensitivity of the fraction of income loss to income level affect more the mean spread. Then, $\gamma_0 + \gamma_1$ determines the average fraction of income loss during the default while γ_1 drives the sensitivity of the fraction of income loss to income level, $\phi(a)/e^a$ and so defines the slope of the income loss function.



the level of its consumption g_t and tax rate τ_t . Furthermore, a government not in default makes also its decision about the amount of bonds to issue.

Given the government demand b_{t+1} , the supply of international investors is priced at $q(b_{t+1}, a_t)$. The implicit yield $v(b_{t+1}, a_t)$ implied by this long-term bond function and period coupon payment $(r + \delta)/(1 + r)^{15}$,

$$v(b_{t+1}, a_t) = \frac{\delta + r}{(1 + r)q(b_{t+1}, a_t)} - \delta,$$

exceeds the riskless rate r whenever investors consider a non-zero risk of sovereign default. Therefore, the associated annual default risk spread can be expressed as:

$$R_s(b_{t+1}, a_t) = \left[\frac{1 + v(b_{t+1}, a_t)}{1 + r} \right]^4 - 1.$$

We assume that the government cannot commit to future default and borrowing decisions. As pointed out by Hatchondo et al. (2016) it is possible to interpret this environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other players (governments) who will decide after time t . We focus on Markov Perfect Equilibrium and assume that the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables (in each period).

2.1 Recursive formulation of the model

For the sake of notation simplification, we drop the time subscript t in model variables. We denote b the number of outstanding coupon claims at the beginning of the current period. Next, we introduce b' the number of outstanding coupon claims at the beginning of next period and a' the next-period realisation of the technology shock. Let binary $D(b, a)$ represent the government default decision and associate $D = 1$ with their decision to default and $D = 0$ with their preference to repay debt obligations.

Bond price function. Recalling Hatchondo and Martinez (2017), Hatchondo et al. (2015) and Hatchondo et al. (2012a) the assumption of zero expected profit of foreign bondholders (operating in a perfectly competitive environment) implies the following:

$$(1 + r)q(b', a) = \mathbb{E}_{a'|a} \left\{ D(b', a') q_D(b', a') + (1 - D(b', a')) \left[1 + (1 - \delta)q(\hat{b}(b', a'), a') \right] \right\} \quad (4)$$

Above, \hat{b} denotes the debt policy rule and the price of a bond in default q_D can be expressed as:

$$(1 + r)q_D(b', a) = \mathbb{E}_{a'|a} \left\{ (1 - \xi)(1 + r)q_D(b'(1 + r), a') + \xi \alpha [D' q_D(\alpha b', a')] + (1 - D') [1 + (1 - \delta)q(b'', a')] \right\}, \quad (5)$$

¹⁵In order to simplify the algorithm, we assume a coupon payment $(r + \delta)/(1 + r)$ in order to achieve that the value of a risk-free bonds is always one, regardless of the value of delta.



where $D' \equiv \widehat{D}(\alpha b', a')$ represents the default policy rule and $b'' = \widehat{b}(\alpha b', a')$ is the debt policy rule evaluated for the end-of-period restructured debt. We emphasize that during the exclusion from markets debt obligations grow at the risk-free interest rate r and the country has an opportunity to regain its access to markets with a probability ξ and debt restructuring rate α in each period in exclusion.

Value function. Next, we introduce $V(b, a)$ the government value function evaluated at the beginning of the period t , hence before the default decision has been made by the government and $V_D(b, a)$, $V_R(b, a)$, respectively the continuation value when the government declares a default, and when it repays its debt obligations.

Then for each period t , the government value function satisfies the subsequent functional equation:

$$V(b, a) = \max\{V_R(b, a), V_D(b, a)\}. \quad (6)$$

Regardless of the default decision, the government has to determine the tax rate τ and the level of its consumption, g . However, provided that the government repays its current debt obligations, it also has to define the amount of bonds issued in the current period t . Then, under the Bellman's optimality principle, for any bond price $q(b', a)$, technology shock a and initial level of the liabilities b the value function V_R of the government that currently repays its liabilities is the following:

$$\begin{aligned} V_R(b, a) &= \max_{b' \geq 0, c \geq 0, g \geq 0, \tau \geq 0} \mathcal{V}_R(c, g, \tau, a, b'), \\ \mathcal{V}_R(c, g, \tau, a, b') &= u(c, g, h) + \beta \mathbb{E}_{a'|a} [V(b', a')], \\ \text{subject to} \\ c &= (1 - \tau)e^a h, \\ g &= \tau e^a h - b + q(b', a) [b' - (1 - \delta)b], \\ h &= [(1 - \tau)e^a / \psi]^{1/\omega}, \\ q(b', a) &\geq \underline{q}, \quad \text{if } b' \geq b. \end{aligned} \quad (7)$$

Above, $q(b', a)$ represents the bond price required by foreign investors given the amount of issued bonds b' . Therefore, $q(b', a) [b' - (1 - \delta)b]$ denotes the funds received by the government. Moreover, we assume that the government cannot issue bonds at a price lower than some fixed \underline{q} . This way we eliminate the possibility to issue large amounts of bonds before default.

When the government decides to default or remains in default it is excluded from capital markets, so it cannot issue debt obligations. Furthermore, it suffers from a loss in technology, $\phi(a)$. However, it is still eligible to levy tax and consume. On the other hand, the government may decide to end the default and restructure its debt at the recovery rate α .

Thus, for any level of liabilities b and technology shock a the value function $V_D(b, a)$ of the government in default solves the following problem:

$$\begin{aligned} V_D(b, a) &= \max_{c \geq 0, g \geq 0, \tau \geq 0} \mathcal{V}_D(c, g, \tau, a), \\ \text{subject to} \end{aligned} \quad (8)$$



$$\begin{aligned}\mathcal{V}_D(c, g, \tau, a) &= u(c, g, h) + \beta \mathbb{E}_{a'|a} [(1 - \xi)V_D((1 + r)b, a') + \xi V(\alpha b(1 + r), a')] , \\ c &= (1 - \tau)[e^a - \phi(a)]h , \\ g &= \tau[e^a - \phi(a)]h , \\ h &= [(1 - \tau)(e^a - \phi(a))/\psi]^{1/\omega} .\end{aligned}$$

In order to distinguish between the problem of the government that repays its debt and the government that defaults we denote c_R , g_R , and τ_R as rules associated with the problem (7) while c_D , g_D , and τ_D as decision rules associated with problem (9).

Markov perfect equilibrium of the benchmark model. We refer to the *Benchmark Model* \mathcal{B} as to a model defined by (4)–(9) in which the issuance of next-period debt is not limited by any constraint. Following Hatchondo et al. (2015) and Hatchondo et al. (2016) a Markov perfect equilibrium of the benchmark model \mathcal{B} is characterized by

1. rules for default \hat{d} , borrowing \hat{b} , tax rate $\{\tau_R, \tau_D\}$, government expenditures $\{g_R, g_D\}$ and private consumption $\{c_R, c_D\}$,
2. a bond price q ,

such that

- given a bond price q , the policy functions for default \hat{d} , borrowing \hat{b} , tax rate $\{\tau_R, \tau_D\}$, government expenditures $\{g_R, g_D\}$ and private consumption $\{c_R, c_D\}$ solve Bellman problems (6), (7) and (9); and
- given the policy functions for default \hat{d} and borrowing \hat{b} , the bond price function satisfies (4).

2.2 Debt rules

In order to mitigate the debt dilution problem, we introduce a debt rule that limits the budget balance in order to prevent the debt from exceeding a given constant threshold μ . When the government decides about the amount of the debt to issue in the next-period b' , it cannot go beyond this threshold. So to formulate the problem of a country with the debt rule, we extend (7) by the following constraint¹⁶:

$$b' \leq \max\{\mu, b(1 - \delta)\} . \quad (10)$$

The debt ceiling commitment brings a fundamental change in the government strategy. Indeed, its presence forces the government to reduce its consumption even for the debt levels signifi-

¹⁶As suggested by Hatchondo and Martinez (2017) restricting the next-period debt level b' by the payoff $b(1 - \delta)$ prevents from forcing the government to buy back its debt, which is particularly important when the initial level of the debt exceeds the threshold μ .



cantly lower than the debt limit in order to avoid it. However we emphasize that this change in the behaviour is not caused by the option to default, but by the presence of the rule itself¹⁷.

We call the *Debt Rule Model* \mathcal{R}_μ the original model \mathcal{B} (defined by (4)–(9)) augmented by the restriction on the level of debt liabilities (10). We express this debt ceiling as a percentage of the mean income of the benchmark no-rule economy¹⁸.

Instead of limiting the debt level, Hatchondo and Martinez (2009), Hatchondo and Martinez (2017) and Hatchondo et al. (2015) describe another alternative to mitigate the debt dilution problem with a fiscal rule. They study the effect of a spread-brake rule that imposes a ceiling on the fiscal budget balance that prevents the government from increasing its debt level to push the sovereign spread beyond a given threshold. Constraining the sovereign spread is equivalent to imposing a minimum sovereign bond price. They show that for a set of heterogeneous economies, a common spread brake generates larger average welfare gains than a common debt brake.

2.3 Problem solution

As in Hatchondo and Martinez (2017) we solve the model numerically, using the value function iteration approach, numerical integration and various interpolation schemes. Specifically, we employ linear interpolation for technology levels and cubic spline interpolation for asset (debt) positions. The algorithm determines the equilibrium of the finite-horizon version of the economy, and approximates the infinite-horizon economy by increasing the number of periods until value functions and bond prices are sufficiently close to each other in subsequent iterations. The details of the procedure are described in depth in Appendix A.

¹⁷Even in a simple model with no possibility to default the Kuhn-Tucker condition (10) constraints the consumption decision of the forward-looking government much earlier than the debt brake starts to bind. Therefore, cuts in government consumption prior the debt limit are necessary.

¹⁸However, an alternative formulation of the debt limit is possible - rather than imposing a constraint (10) on the next-period debt, a restriction on the next-period debt-to-income ratio can be used. So, one can modify (10) as:

$$b'/y'(b') \leq \mu, \quad \text{and} \quad b' \leq b(1 - \delta).$$

Above, y' denotes the next-period income which is for any fixed next-period realisation of the technology shock a' a function of outstanding coupon claims at the beginning of the next period, b' .



3 Benchmark calibration

Table 3.1 presents our benchmark calibration of parameters. A period in the model refers to a quarter. To calibrate this model we use data for Slovakia from 2009 to 2016¹⁹ and standard parameter values employed in the literature.

As emphasized in section 2 the technology process is defined in a broad sense, implicitly containing possible fluctuations in relative prices. The calibration of the technology process parameters need to reflect the volatility and autocorrelation of domestic output. Therefore, we estimated the technology process (1) using quarterly real GDP data for Slovakia in the period between the first quarter of 2009 and the third quarter of 2016. Our estimates of the technology persistence ($\rho = 0.7252$) and volatility ($\sigma_\varepsilon = 0.0167$) are consistent with observations of Neumeyer and Perri (2005) and Aguiar and Gopinath (2007) on the character of business cycles in less developed economies.

Regarding the parameters of the household utility function, the coefficient on public consumption risk aversion is taken from Hatchondo et al. (2015) ($\sigma_g = 3$). The inverse of the labour elasticity (ω) and the steady-state labour intensity (20%) are taken from Neumeyer and Perri (2005).

We assume an annual risk-free international rate of 4 percent, which is standard in the literature. However, since we need the model to generate realistic levels of debt, the time discount factor needs to attain lower values than those standardly used in the literature. Therefore, we rely on the empirical estimate of the discount factor from a recent study of Mucka (2015). The recovery rate of debt in default α is assumed to take a value of 0.35. This is the average recovery rate reported by Cruces and Trebesch (2013) for debt restructuring with a reduction in the face value. Following Dias and Richmond (2007) we assume an average exclusion from capital markets of three years after a default so we set the probability with which a government can exit a default ξ to 0.083.

There are five parameters that we calibrated to match five moments in the data for Slovakia between 2009 and 2016: the rate of decay of coupon obligations²⁰ (δ), the two parameters that define the productivity cost of defaulting (γ_0, γ_1), the risk aversion of households σ_c , and the weight of public consumption in the utility function²¹ π . These parameters are all calibrated to

¹⁹We do not include data before the introduction of the euro in Slovakia (January 1, 2009) as the Slovak government debt was denominated mostly in local currency before this date.

²⁰Long-term bond duration δ is calculated using the average spread \bar{R}_s , average bond duration \bar{T} and the international interest rate r as

$$\delta = -i + \frac{1+i}{\bar{T}}, \quad i = \frac{\bar{R}_s + 1}{r + 1} - 1.$$

Then the coupon is $\mathcal{C} = (r + \delta)/(1 + r)$.

²¹The weight of labour in the utility function ψ is derived from the first order condition of households, assuming mean debt-to-GDP ratio, 19 percent government consumption-to-GDP ratio and 20 percent labour intensity (as suggested by Neumeyer and Perri (2005)):

$$\psi = \left[1 - g/y - b/y \left(\frac{\delta}{\delta + r} - \mathcal{C} \right) \right] h^{-\omega}.$$



Table 3.1 : Benchmark Calibration

Parameter Description	Name	Value	Source
Time-discount factor	β	0.96725	Mucka (2015)
Private consumption - risk aversion	σ_c	2.1275	Calibrated to fit targets.
Public consumption - risk aversion	σ_g	3	Hatchondo et al. (2015)
Public consumption weight in utility	π	0.18	Calibrated to fit targets.
Inverse of labour elasticity	ω	0.6	Neumeyer and Perri (2005)
International interest rate	r	0.01	Standard Literature
Technology persistence	ρ	0.7252	NBS, real GDP (s.a.), (2009q1-2016q3)
Technology volatility	σ_ε	0.0167	NBS, real GDP (s.a.), (2009q1-2016q3)
Default loss function: average cost	γ_0	0.1115 - γ_1	Calibrated to fit targets.
Default loss function: slope parameter	γ_1	1.55	Calibrated to fit targets.
Duration of defaults	ξ	0.083	Dias and Richmond (2007)
Duration of the long-term bonds	δ	0.0279	Calibrated to fit targets.
Recovery rate of debt in default	α	0.35	Cruces and Trebesch (2013)

match: (i) the average duration of Slovak government debt, (ii) the level of government debt, (iii) the average long-term interest rate spread, (iv) the volatility of private consumption relative to the volatility of income, and (v) the ratio of government consumption to private consumption.

Parameter values are summarized in Table 3.1. Referring to recent annual reports of the Slovak Debt Management Agency (ARDAL (2014) and ARDAL (2015)) the average duration of government debt attains 6.15 years. Using monthly data on long-term government bond yields for Slovakia and Germany (collected by EUROSTAT between the January 2009 and December 2016) we calibrated the average annual spread at 1.35 percent.

To obtain the values of the last three parameters – the average level of government debt, the volatility of private consumption relative to the volatility of income, and the ratio of government consumption to private consumption – we use time series reported by the National Bank of Slovakia. Taking the series on net public debt we find that the average net government debt-to-GDP ratio was at 44 percent. Next, from data on government and final household consumption we get their ratio at 34 percent. Finally, the volatility of private consumption relative to the volatility of income achieves²² 0.95 using deviations from trends obtained by employing the Hodrick-Prescott filter with a smoothing parameter of 1600.

Hatchondo and Martinez (2017) pointed out that calibrating the parameters associated with the costs of defaulting, γ_0 and γ_1 is more difficult due to an endogeneity problem and the modeling abstraction (this framework does not describe other cost of default that may be very relevant). Therefore, many studies on sovereign defaults target these parameters to replicate data moments e.g. average debt or spread. To obtain the values of the loss function parameters we first solved the model using the approach explained in appendix A for various combinations of γ_0 and γ_1 , while keeping other model parameters²³ fixed. Next, we simulated 1000 samples for

²² Aguiar and Gopinath (2006) and Neumeyer and Perri (2005) observed higher numbers for emerging economies.

²³ However, we observed that more risk averse households (raising σ_c) tend to ask higher spreads. Likewise, the



each of these models for 500 quarters and we took last 100 periods without a default episode from each sample. This was used to compute the desired statistics – mean annual spread and the mean debt-to-income ratio. Our findings are consistent with the study of Hatchondo and Martinez (2017). The average debt is driven mainly by the average income loss parameter γ_0 , it raises (almost linearly) with γ_0 , while decreases with the slope parameter γ_1 . On the other hand, there is a dominant positive effect of the slope parameter (which measures the sensitivity of the fraction of income lost during default to the level of income) on annual spread, whereas the impact of γ_0 is negative and weaker. Therefore, by the implicit function theorem for a given set of remaining model parameters, for any reasonable desired target debt level and annual spread there exists a unique combination of default cost function parameters γ_0 and γ_1 such that these targets are achieved.

In order to check the robustness of results, we perform an alternative calibration (Table 3.2). For this purpose we chose three model parameters – the rate of decay of coupon obligations (δ) and the two parameters that define the cost of defaulting (γ_0 , γ_1) – to match the data for Slovakia from 2009 to 2011, thus after the introduction of the euro but before the Fiscal Responsibility Act was adopted²⁴. These parameters need to fit (i) the average duration of Slovak government debt, (ii) the level of government debt, and (iii) the average long-term interest rate spread. Based on the annual report of the Slovak Debt Management Agency (ARDAL (2014)) the average duration of government debt between 2009 and 2011 was set at 4.45 years. Using monthly data on long-term government bond yields for Slovakia and Germany between January 2009 and December 2016 we calibrated the average annual spread at 1.45 percent. Finally, during this period the mean net government debt/GDP ratio was 37.1 percent.

Table 3.2 : Alternative Calibration

Parameter Description	Name	Value	Source
Time-discount factor	β	0.96725	Mucka (2015)
Private consumption - risk aversion	σ_c	2.1275	Calibrated to fit 2009-2016 targets.
Public consumption - risk aversion	σ_g	3	Hatchondo et al. (2015)
Public consumption weight in utility	π	0.18	Calibrated to fit 2009-2016 targets.
Inverse of labour elasticity	ω	0.6	Neumeyer and Perri (2005)
International interest rate	r	0.01	Standard Literature
Technology persistence	ρ	0.7252	NBS, real GDP (s.a.), (2009q1-2016q3)
Technology volatility	σ_ε	0.0167	NBS, real GDP (s.a.), (2009q1-2016q3)
Default loss function: average cost	γ_0	0.105 - γ_1	Calibrated to fit 2009-2011 targets.
Default loss function: slope parameter	γ_1	1.5	Calibrated to fit 2009-2011 targets.
Duration of defaults	ξ	0.083	Dias and Richmond (2007)
Duration of the long-term bonds	δ	0.0433	Calibrated to fit 2009-2011 targets.
Recovery rate of debt in default	α	0.35	Cruces and Trebesch (2013)

Parameters in bold (γ_0 , γ_1 and δ) are set to different values in the benchmark calibration of the model.

more myopic the agents are (low β) the higher is the simulated mean debt level. However, there is no significant impact on the share of public goods in the household utility function (π) on the simulated debt or spread.

²⁴We decided to keep the remaining two parameters – the risk aversion of households (σ_c), and the weight of public consumption in the utility function (π) – unchanged as between 2009 and 2011 the private consumption is essentially less volatile than the income ($\sigma(c)/\sigma(y) = 0.49$) in compare to the benchmark calibration and the ratio of government consumption to private consumption is rather stable.



4 Results

This section is organized as follows. First, we show that the benchmark model (without rules) can mimic salient features of business cycles in Slovakia. Then, we demonstrate that a government can benefit from committing to a debt limit and that the gains from imposing fiscal rules may be even larger for indebted economies. Finally we present the results from the alternative calibration.

Table 4.1 : Simulations without a fiscal rule

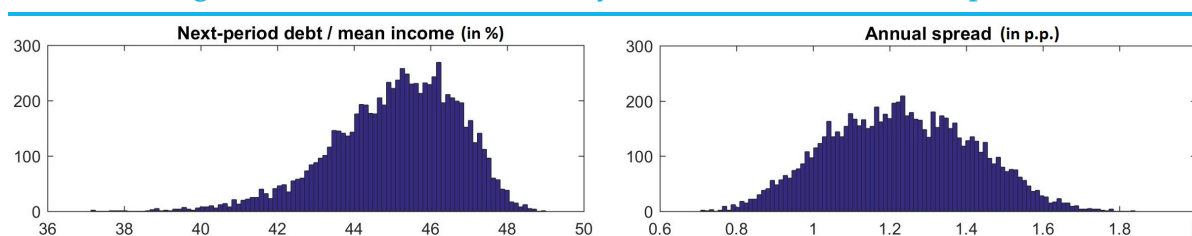
Target	Slovak Data	Benchmark Model
Average duration of government bonds	6.15 years	6.15 years
Average spread	1.35% p.a.	1.34% p.a.
Average debt	44.0%	44.8%
g/c	34.0%	32.4%
$\sigma(c)/\sigma(y)$	0.95	0.96

The standard deviation of a variable x is denoted by $\sigma(x)$. The second column is computed using Slovak data. The logarithm of private consumption c and income y were de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1600. We report deviations from trend. The debt level is calculated as the present value of future payment obligations discounted at the average risk-free rate. We report the annualized spread.

4.1 Simulations without a fiscal rule

Table 4.1 shows that the model without a fiscal rule well approximates the targeted data moments. Since there has not been a sovereign default in Slovakia, we report results for simulated sample paths without defaults. We report the mean value of each moment in 1,000 simulation samples. We take the last 74 periods (quarters) of samples in which no default occurs in the last 100 periods. Figure 4.1 illustrates the distributions of the simulated debt/mean income and the annual spread for zero income shock. It can be seen that the economy tends to accumulate debt quite close ²⁵ to the level at which it defaults as the debt distribution is right-skewed left-heavy tailed. On the other hand, spreads asked by investors are relatively disperse.

Figure 4.1 : Benchmark economy: distribution of debt and spread



Histograms of the simulated debt/mean income (in %) and annual spread (in p.p.) if zero income shock is assumed.

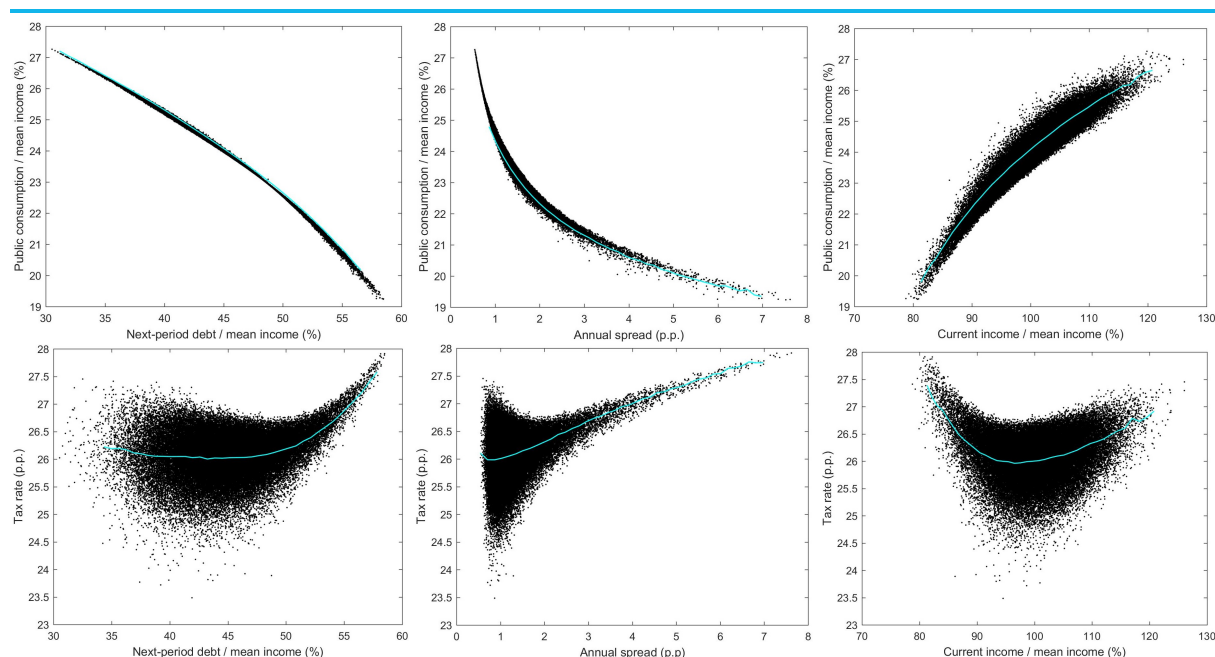
Figure 4.2 shows that it is optimal for the government to choose a pro-cyclical fiscal policy. That is, when aggregate output is lower, the tax rate tends to be higher, and the level of public good

²⁵With the steady-state technology (zero income shock) the government *space for manoeuvre*, measured as a standard deviation of the debt/mean income ratio attains 3.47 percent of annual income.



provision tends to be lower. When income is low, borrowing is more costly because it increases the probability of default (and future default decisions are not optimal from an ex-ante perspective). Thus, the government borrows less, increases the tax rate, and lowers expenditures. Furthermore, government reduces its consumption as a response to growing debt or spread. However, it tends to raise taxes only when debt is high, while prefers to borrow to finance its spending and debt service. On the other hand, it levies higher tax whenever the spread asked by investors goes up.

Figure 4.2 : Benchmark economy: pro-cyclical fiscal policy



Responsiveness of fiscal policy: government consumption (bottom panel) and taxes (top panel) to the debt/GDP ratio (left), annual spread (middle) and business cycle. Cyan curves represent the mean of the simulated response of the fiscal policy tool (tax rate, consumption) to debt/GDP, spread or business cycle.

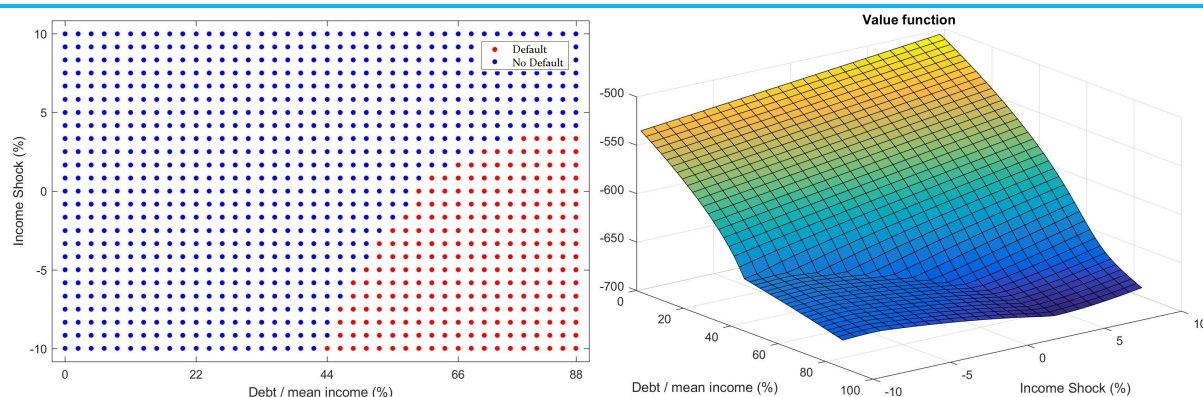
Figure 4.3 shows that capital markets and households are significantly more sensitive to changes in government debt than to purely exogenous income shocks. Furthermore, the decision to default on liabilities turns out to be optimal for moderate debt levels when the country is suddenly hit by a severe crisis. However, in that case the default decision is accompanied by a welfare loss and sharply rising interest rate.

4.2 Fiscal rules

In what follows we discuss the optimal debt-limit rule. That is, we search for the debt ceiling (expressed as a percentage of the mean output of the benchmark economy) that maximizes welfare when imposed on a no-rule economy with the benchmark parametrization. This limit constrains the government when issuing new debt obligations. In this section we assume that there is no initial debt and productivity attains its unconditional mean.



Figure 4.3 : Benchmark economy: optimal default decision and value function



Optimal default decisions and value function for various debt levels and income shock realizations in the benchmark model

Table 4.2 : Benchmark vs. optimal debt-rule model comparison

Target	Slovak Data	Benchmark Model	Debt Rule (48%)
Average Duration of government bonds	6.15 years	6.15 years	6.61 years
Average Spread	1.35% p.a.	1.34% p.a.	0.83% p.a.
Average Debt	44.0%	44.8%	47.2%
g/c	34.0%	33.4%	33.5%
$\sigma(c)/\sigma(y)$	0.95	0.96	0.99
Average annual default rate	-	0.68	0.57

The standard deviation of a variable x is denoted by $\sigma(x)$. The second column is computed using Slovak data. The logarithm of private consumption c and income y were de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1600. We report deviations from trend. The debt level in the simulations is calculated as the present value of future payment obligations discounted at the average risk-free rate. We report the annualized spread.

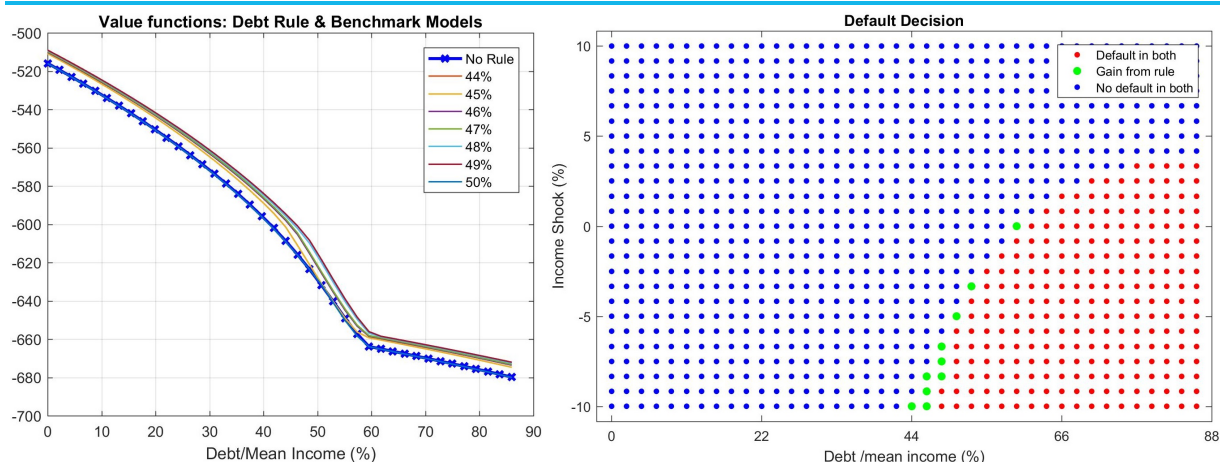
Obviously, the value of the debt limit has a significant impact on the utility of households and the price of debt. We find that the optimal debt limit is 48 percent of the benchmark no-rule economy mean income. Figure 4.4 shows that committing to the debt ceiling exceeding the optimal limit is futile as it has no influence on welfare and the price of debt. As illustrated on Figure 4.5, the government benefits from implementing a fiscal rule because it mitigates the debt dilution problem and with higher bond price it creates new borrowing opportunities. This is because lenders anticipate that future governments will choose a lower debt level and so the government pays a lower interest rate. On the other hand, it imposes constraint on the amount the government can promise to pay.

Notice that the optimal debt ceiling exceeds the simulated mean debt/mean income ratio (see Table 4.2 and Figure 4.1). On the other hand, the rule also limits future borrowing, enabling the government to pay a lower interest rate for any chosen debt level. Table 4.2 shows that the preferred debt limit reduces the default frequency and, consequently, the sovereign spread.

Figure 4.6 confirms the conclusion from the no-rule economy model - that it is optimal for the government to choose a pro-cyclical fiscal policy. That is, when aggregate output is lower, the

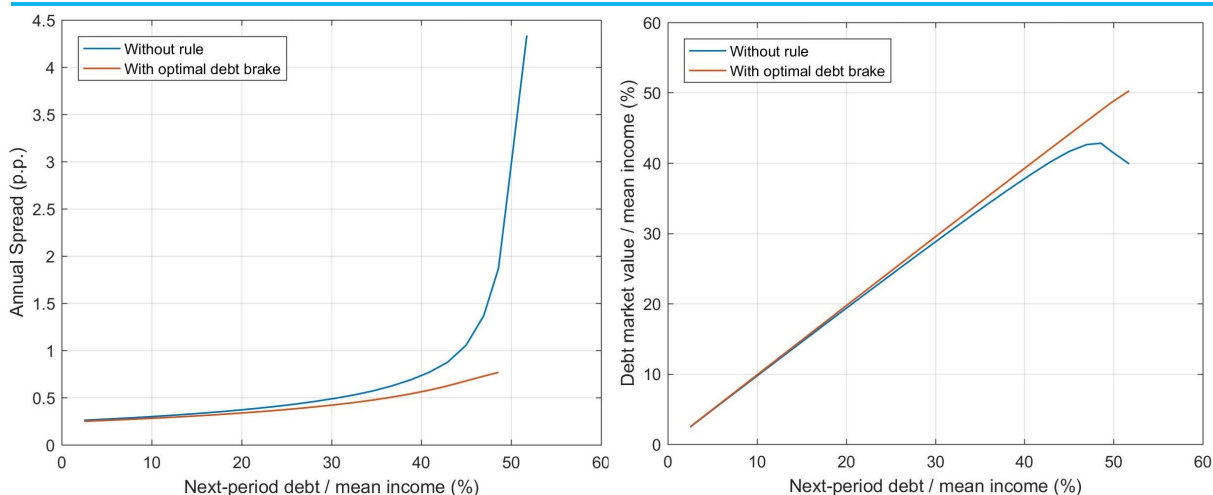


Figure 4.4: Debt rule economy: optimal default decision and value function



Left panel illustrates the value function calculated for zero technology shock under various debt limits imposed on the government. Right panel shows optimal default strategies for the benchmark model and the model with the optimal debt ceiling (48 percent of the mean output of the benchmark economy). The gain from the implementation of such debt ceiling (green dots) – no default under the debt-rule model while default in the benchmark economy – is evident especially if the country is hit by a crisis.

Figure 4.5: Borrowing opportunities



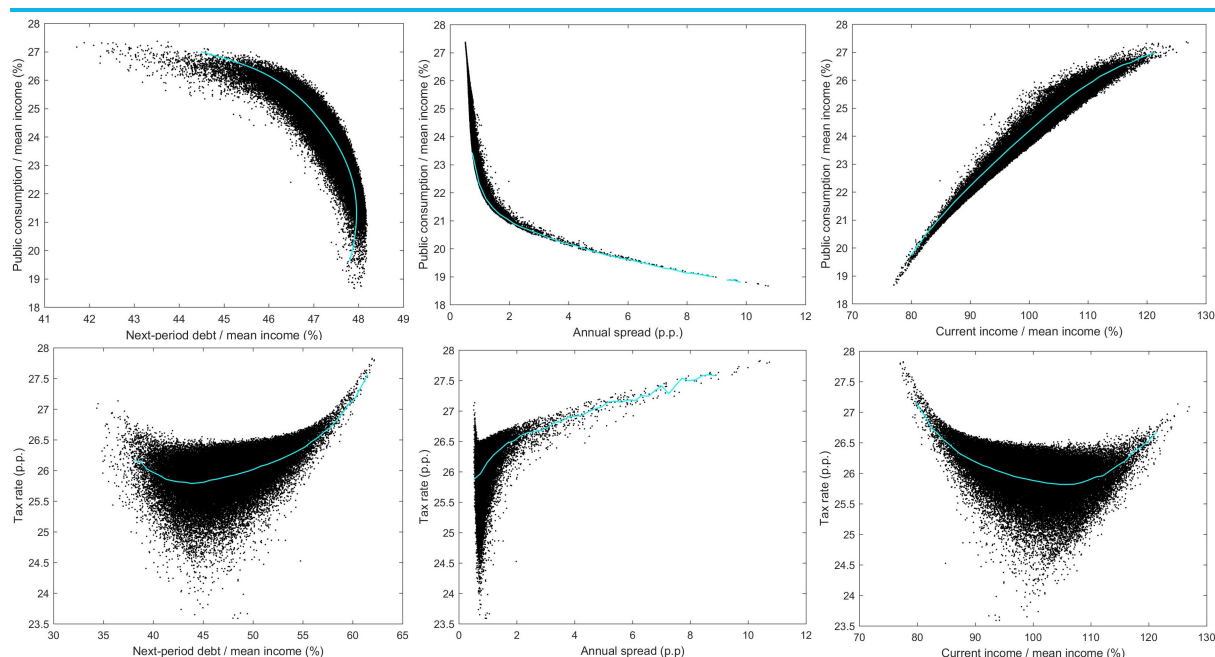
The left panel presents the annualized spread asked by lenders for different levels of debt. The right panel presents the market value of the debt stock (which represents the resources a government without debt could obtain from borrowing) for different levels of debt. Both figures assume zero technology shock.

tax rate tends to be higher, and the level of public good provision tends to be lower. Furthermore as debt approaches its ceiling, government has to dramatically cut its consumption and increase taxes to finance the debt.

Figure 4.7 illustrates the benefits of anchoring expectations with the optimal fiscal rule. We present debt, spread, consumption, production, employment and tax rate paths obtained by simulating both the benchmark and the optimal debt rule economies for 80 quarters using the



Figure 4.6 : Optimal debt rule economy: pro-cyclical fiscal policy



Responsiveness of the fiscal policy (government consumption (bottom panel) and taxes (top panel) to the debt/GDP ratio (left), annual spread (middle) and business cycle. Cyan curves represent the mean of the simulated response of the fiscal policy tool (tax rate, consumption) to debt/GDP, spread or business cycle.

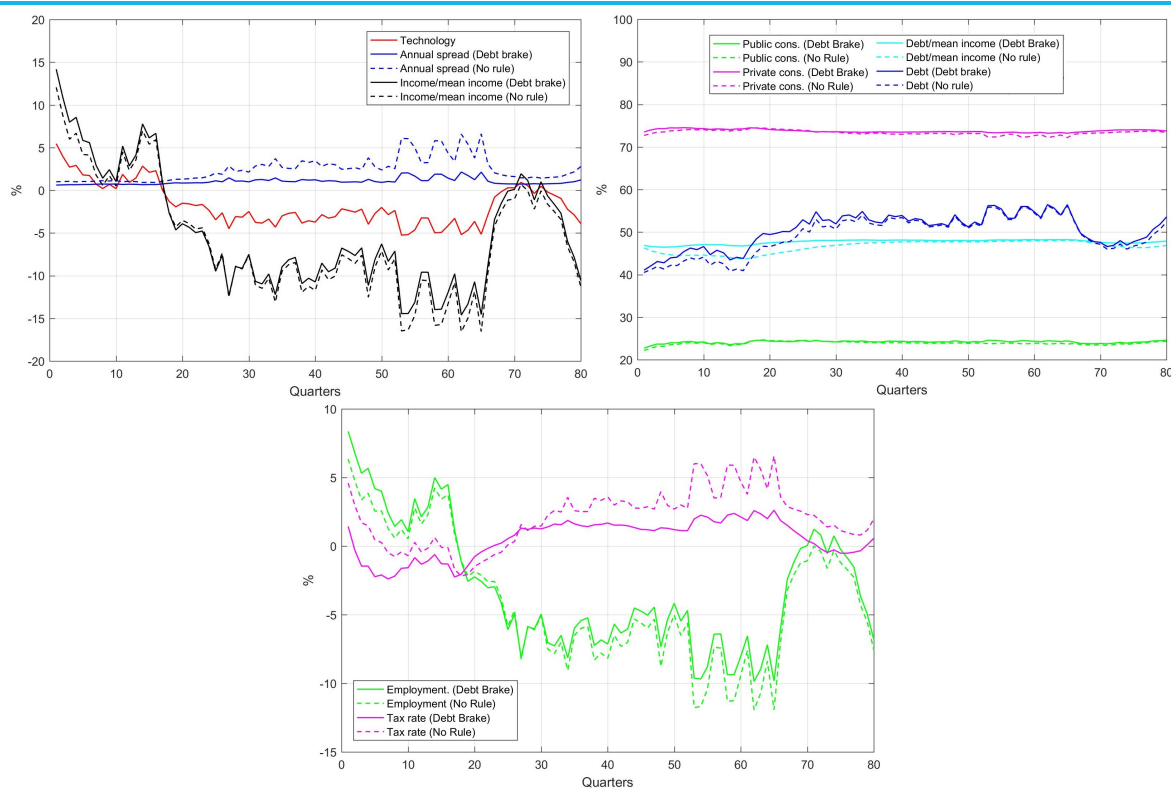
same technology shock. This figure demonstrates that the same series of productivity shocks induces significantly steeper and larger increases in sovereign spread in the benchmark model compared to those generated by the optimal debt rule model. Furthermore, the decrease of post-crisis spread is much faster in the presence of a debt brake in contrast to the benchmark model, where the spread remains relatively high after TFP recovers. This is the case because bad times lead to rapid increase of sovereign debts.

Rapidly growing sovereign spreads force the government in the benchmark economy to raise labour tax much more than the in the optimal debt rule economy. Moreover, the tax rate remains higher even after the recovery. The reaction of the labour market is also different, as in the benchmark economy the employment level remains essentially below its debt rule economy counterpart. Therefore the optimal debt brake society benefits also from higher aggregate output, whereas the no-rule economy is weaker.

When times are good and the initial debt relatively small, higher tax revenues collected in the benchmark economy keep the debt in the safe area and lower than in the debt brake economy with lower tax revenues. Nevertheless, the situation changes when both economies are hit by a crisis and the benchmark economy debt reaches the same level as in the debt rule economy. Observe that from Figure 4.7 it follows that faster recovery of the spread and stable debt-to-GDP ratio in the debt rule economy are not implied by any additional sacrifice of consumption required by the debt brake but from anchored expectations about future fiscal policy.



Figure 4.7: Simulations with and without a fiscal rule



Thick lines correspond to the paths of variables simulated using the optimal debt rule (48%) model while the dashed lines correspond to the paths of variables simulated using the benchmark no-rule model.

As explained in section 2.2 the introduction of a debt rule brings a fundamental change in the decision-making process of the government. From Figure 4.8 it follows that even in case when default is not allowed²⁶ the imposed debt limit affects essentially the government consumption behaviour for debt levels significantly below the debt ceiling. Hence, the presence of the debt rule forces the government to borrow less and so reduce its consumption.

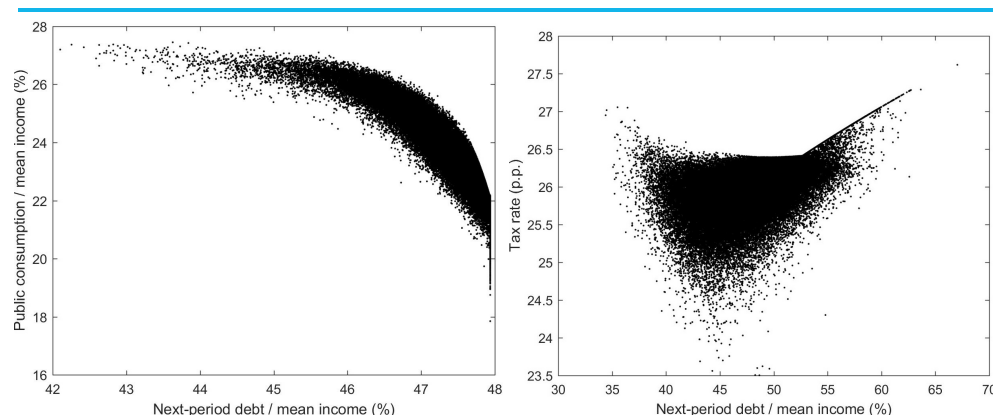
4.3 Optimal debt rule for an indebted economy

We study possible benefits from imposing a debt brake rule in countries with positive debt via allowing for smooth transitions towards lower debt levels. When the introduction of a debt ceiling is accompanied with a transition period between the rule announcement and implementation welfare will not suffer. Therefore, we assume that when the government introduces the debt brake it announces that the debt limit μ will constrain its decision in every period starting

²⁶For this purpose we developed a simplified no-default version of the debt rule model and simulated it using the benchmark calibration. Despite the missing option to default and trivial debt pricing function, government can still decide about its consumption, determine the tax rate and the next-period debt issuance. However, since in the no-rule no-default world the optimal strategy for the government is to have an infinitely large sovereign debt, in case that country cannot default on its liabilities there is *no* optimal debt limit.



Figure 4.8 : Fiscal policy with fiscal rule and no default



Variables simulated using the optimal debt rule (48%) in the model with benchmark calibration but no default option.

from period T . Following Hatchondo and Martinez (2017) the government's problem is not recursive until time T and we solve the problem backwards starting from the first period in which the government maximisation problem becomes recursive.

Our aim is to search for the combination of μ and T that maximizes welfare. For the purpose of this study we measure welfare gains as the constant proportional change in consumption of domestic agents that would leave a consumer indifferent between continuing living in the benchmark economy and moving to an economy with a debt rule. Appendix B contains further details about the welfare gain evaluation.

We assume that the initial debt level is at 44.8 percent of the average output in the benchmark no-rule economy (the average debt level for that economy) and we consider different levels of technology for the period in which the rule is introduced.

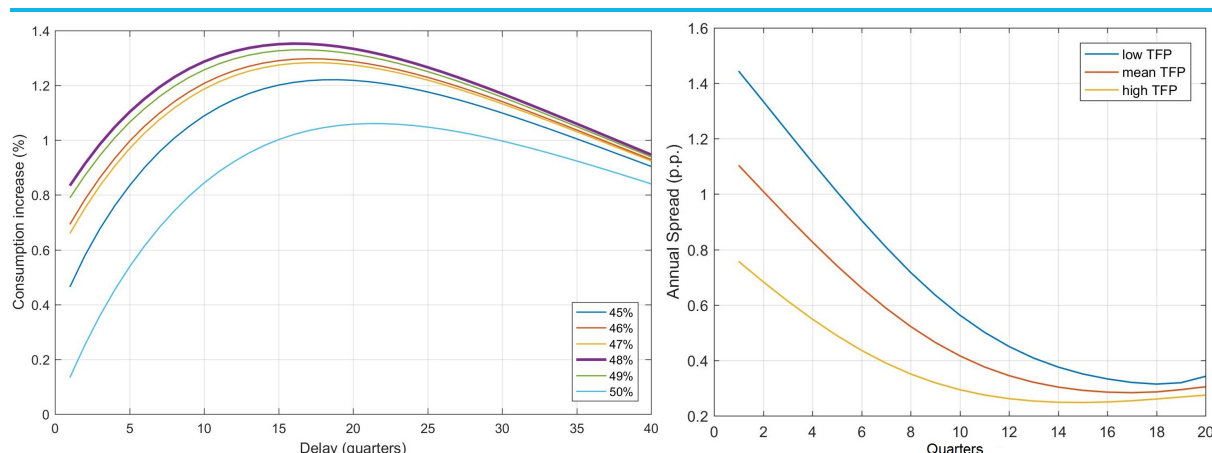
We find that the impact of the initial level of technology on the choice of the optimal debt ceiling is limited in all cases (low, average or high level of technology) and welfare is maximised with a debt brake at 48 percent of the mean output of the benchmark economy approximately 4 years after its announcement. The corresponding welfare gain achieves 1.25 - 1.45 percent, depending on the initial technology level²⁷. Left panel on Figure 4.9 illustrates the possible gains achieved for various debt ceilings and implementation horizons assuming the average initial level of technology.

The right panel of Figure 4.9 shows the mean spread level after the optimal debt rule is announced. We assume the average level of the debt and consider different levels of technology for the period in which the rule is introduced. The figure demonstrates that the commitment to the optimal rule implies a substantial and immediate decline of spreads regardless of the initial

²⁷In case of high initial level of technology (one standard deviation above mean) the optimal length of the transition period is 15 quarters (with the gain 1.45 percent) whereas if the initial level of technology is low (one standard deviation below its mean) it is optimal to implement the debt brake rule after 17 quarters (which generates the gain 1.25 percent). Assuming the average initial level of technology the highest welfare gain of 1.35 percent is achieved when the commitment to the debt brake starts to be active after 16 quarters



Figure 4.9 : Welfare gain and annual spreads



Left panel illustrates the welfare gain (measured as increase in household consumption, on quarterly basis) achieved for various debt ceilings and implementation horizon. The maximal gain is associated with the debt ceiling at 48 percent of the mean income (thick purple line) implemented with the delay of 4 years approximately. Right panel illustrates gradual fall of annual during transitions that follow the announcement of the optimal debt brake. We present the sovereign spreads calculated for the mean debt and mean (red curve), low (minus one standard deviation, blue curve) and high (plus one standard deviation, yellow curve) level of technology.

level of technology. As suggested by Hatchondo and Martinez (2017) this happens because part of the cost of defaulting is the loss of access to debt markets, and this cost is higher when debt markets are more attractive. Since the fiscal rule makes debt markets more attractive (by mitigating the debt dilution problem, and thus allowing the government to borrow at a lower rate), the rule increases the cost of defaulting, allowing the government to borrow more (for a given interest rate).

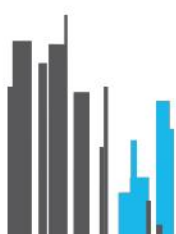
Table 4.3 : No-rule vs. optimal debt rule model under alternative calibration

Target	Slovak Data	Benchmark Model	Debt Rule (39%)
Average Duration of government bonds	4.45 years	4.45 years	4.73 years
Average Spread	1.45% p.a.	1.45% p.a.	0.31% p.a.
Average Debt	37.1%	37.0%	38.1%
g/c	34.0%	34.1%	35.0%
$\sigma(c)/\sigma(y)$	0.95	0.96	1.01
Average annual default rate	-	0.95	0.11

Both models are simulated using the alternative calibration (Table 3.2). The standard deviation of a variable x is denoted by $\sigma(x)$. The second column is computed using Slovak data. The logarithm of private consumption c and income y were de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1600. We report deviations from trend. The debt level in the simulations is calculated as the present value of future payment obligations discounted at the average risk-free rate. We report the annualized spread.

4.4 Robustness check

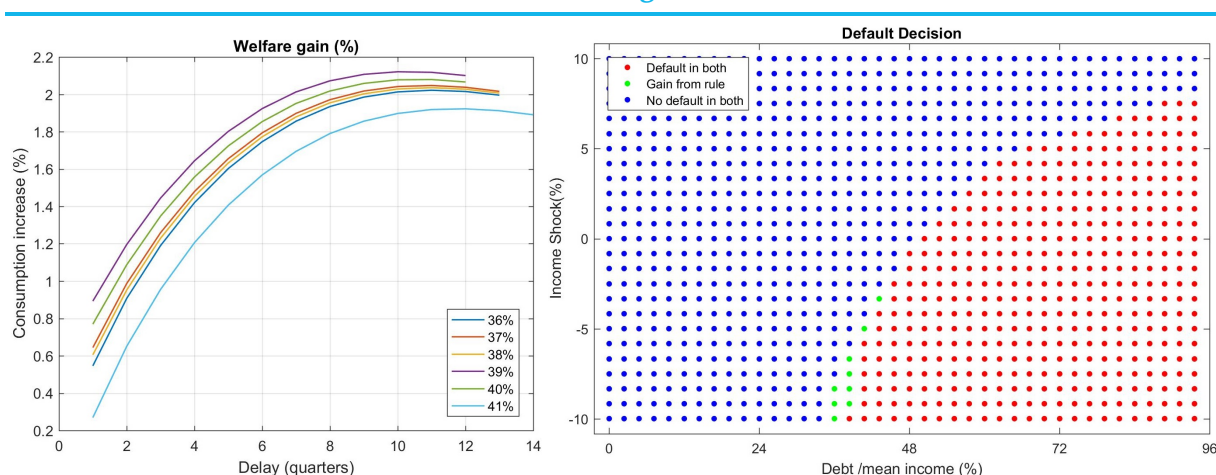
In our alternative calibration exercise we decided to use the duration of Slovak government debt and long-term interest rate spreads observed between 2009 and 2011 (see Table 3.2). On the one hand it corresponds to the period before the introduction of the FRA, while on the other



hand it does not take into account institutional changes in the euro area (including the OMT or ESM). As Table 4.3 shows, risk premium in this period were high already at relatively low level of government debt.

Figure 4.10 shows that in periods of high market sensitivity to debt, the government can greatly benefit from committing to a debt limit (in this case 39 percent of the no-rule economy mean income). Average spreads decline in this calibration by more than 1 percentage points. Furthermore, such a commitment is attractive also for indebted economies as after 10 quarters from its announcement it leads to 2.1 percent increase in consumption.

Figure 4.10 : Debt rule economy under alternative calibration: optimal default decision and welfare gain



Left panel illustrates the welfare gain calculated using the alternative calibration for zero technology shock under various debt limits imposed on the government. Right panel shows optimal default strategies for the benchmark model and the model with the optimal debt ceiling (39 percent of the mean output of the benchmark economy). The gain from the implementation of such debt ceiling (green dots) – no default under the debt-rule model while default in the benchmark economy – is evident especially if the country is hit by a crisis. Furthermore, it delivers increase in consumption by more than 2 percent after 10 quarters from its announcement.



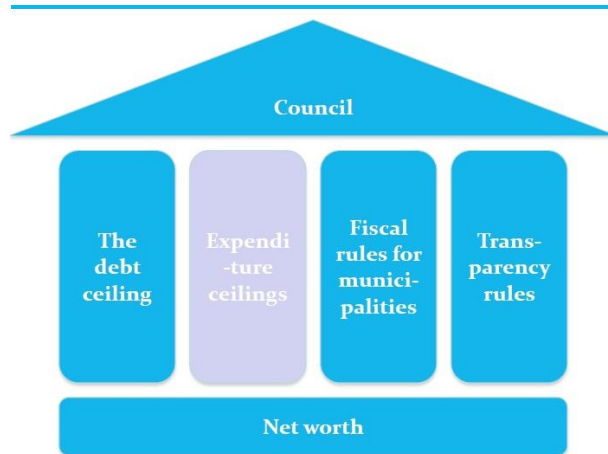
5 Debt ceiling in Slovakia

In this section we look at the evolution of sovereign risk premiums in Slovakia before and after the adoption of the FRA in 2011. First, we describe the main features of the constitutional law. Second, we discuss the importance of credibility. Third, we estimate the decline in the sovereign risk premium on Slovak government bonds, which cannot be attributed to global factors, domestic fundamentals and euro area specificities (including country-specific impacts of QE).

5.1 Fiscal Responsibility Act in 2011

The Slovak Fiscal Responsibility Act adopted in December 2011 was a completely home-grown initiative. The process started with a discussion paper (Horváth and Ódor (2009)) and culminated by the adoption of the constitutional law. The proposed framework is depicted on Figure 5.1. It combines three important ingredients: the concept of inter-temporal net worth, four types of fiscal rules and an independent fiscal watchdog. In this section we focus on the constitutional debt ceiling, which can be interpreted as the empirical counterpart of the debt limit introduced in Section 2.

Figure 5.1 : FRA framework



Expenditure ceilings have a different colour because these have not been implemented yet.

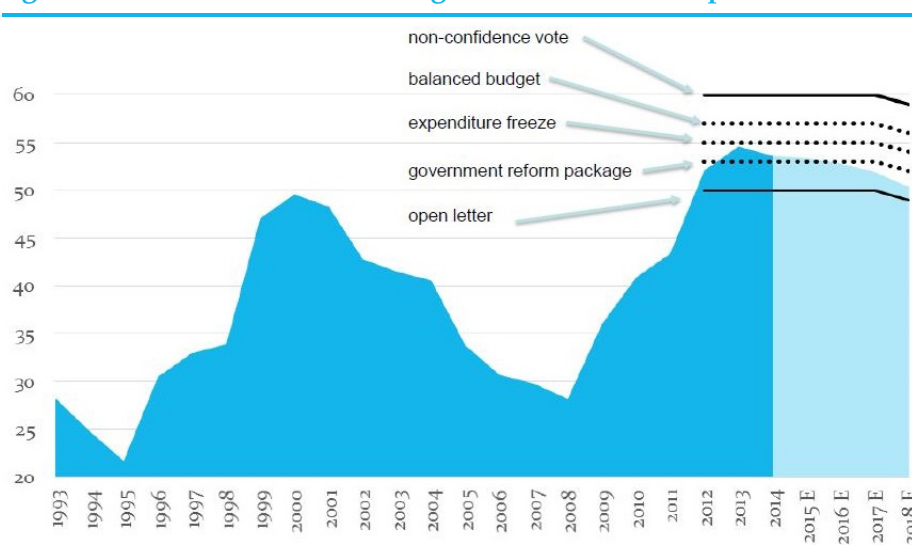
The constitutional debt limit was set at 60 percent of GDP and applies to *gross* general government debt (as published by the Eurostat). There is a gradual sanction mechanism attached to the debt limit (Figure 5.2). If the gross debt exceeds 50 percent of GDP, the minister of finance has to write an open letter to the parliament in which he explains the evolution of debt and outlines potential remedies. Breaching the 53 percent limit has a consequence of freezing the wages of ministers and the obligation of the government to adopt a debt reduction program. The next threshold is at 55 percent of GDP, above which the draft budget has to respect a zero nominal expenditure growth. Above 57 percent of GDP, the government has to approve a balanced budget.



At 60 percent of GDP a government non-confidence vote in the parliament follows. All these thresholds will gradually fall by 10 percent of GDP from 2018 onwards (1 percentage point per year).

In the model described in Section 2 a fully credible debt brake rule was introduced above which it was not possible to issue new debt (rollovers were possible). The closest empirical counterpart in Slovakia is the threshold defined at 57 percent of GDP (a balanced budget requirement). If one takes into account the value of liquid financial assets, the net threshold is around 52-53 percent of GDP. By coincidence, this is not far away from the optimal debt limit calculated in Section 4 (48 percent of GDP).

Figure 5.2 : Evolution of sovereign debt in Slovakia (percent of GDP)



The initial sanction thresholds were set at 50 percent, 53 percent, 55 percent, 57 percent and 60 percent.

5.2 Credibility of the framework

The theoretical model assumes that fiscal rules adopted will be fully respected by future governments and there are no escape clauses or creative accounting techniques. In reality, this is rarely the case. Of course, rules that change frequently and substantially are not immune to the debt dilution problem. The same applies if escape clauses are defined very vaguely. If rational investors realize that the commitment is weak or simply non-existent, there is no reason to expect different pricing of sovereign debt instruments compared to a no-rule economy.

Here we argue that the new fiscal framework in Slovakia is relatively credible compared for example to the Stability and Growth Pact. Therefore, one should expect lower relative default risk after the introduction of the FRA. Our arguments supporting this claim are the following:

- Strong political consensus: the FRA was endorsed by all political parties (146 from 147



MPs voted in favor²⁸).

- Constitutional law: the political scene in Slovakia is relatively fragmented and therefore it is not easy to change a constitutional law once adopted.
- Limited creative accounting: the presence of a very independent fiscal watchdog (financed primarily from the central bank) makes fiscal gimmickry (in case of debt figures) much harder.
- Respected thresholds: until now the government respected the thresholds and adopted measures to avoid the harshest sanctions (at 55 and 57 percent of GDP).
- Well-defined escape clauses: legislated in quantitative terms. Vague concepts were avoided.

5.3 Evolution of risk premiums in Slovakia

Is there any evidence of lower default premiums in the data after the adoption of the FRA? Or in other words: Was the introduction of the constitutional fiscal rules credible? These questions are hard to answer: time series are short and plagued with many turbulent events in the euro area. Moreover, the evolution of risk premiums is often driven by external (global) factors and not only domestic fundamentals (see Longstaff et al. (2011)). Using high frequency data (event studies) is of not much help, since it takes time to convince investors that the law has a real binding power and fiscal gimmickry is contained. Therefore, our investigation here is limited to a simple principal component analysis in the euro area and an effect calculated via the so called synthetic control method (SCM).

In this section we define a peer group of countries and compare the evolution of risk premiums relative to this benchmark. But how to select countries to the peer group?

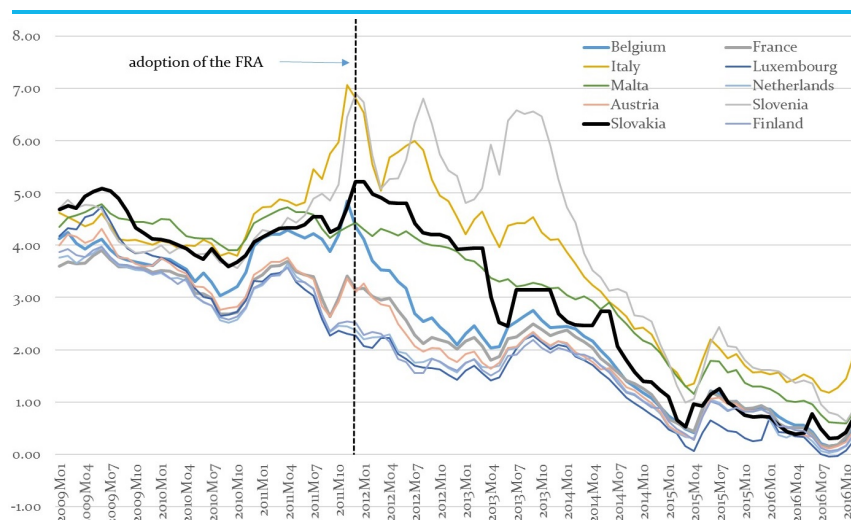
- First, we focus on euro area countries only to ensure that peers operate in the same institutional, legislative and policy environment. Therefore, common shocks can be filtered out.
- Second, we exclude crisis countries: Greece, Portugal, Ireland, Cyprus and Spain. Germany is also dropped, because of a special safe haven status. We approximate the risk free rate as yields on German bunds.
- For the remaining euro area countries, we look at potential changes in fundamentals: actual and expected growth rates, actual and expected government debt. This enables us to exclude those with very different dynamics in fundamentals compared to Slovakia.

Figure 5.3 shows the evolution of 10-year government bond yields since 2009, when Slovakia joined the euro area (countries which adopted the euro later are also excluded from the comparison).

²⁸The total number of MPs in the Slovak parliament is 150.

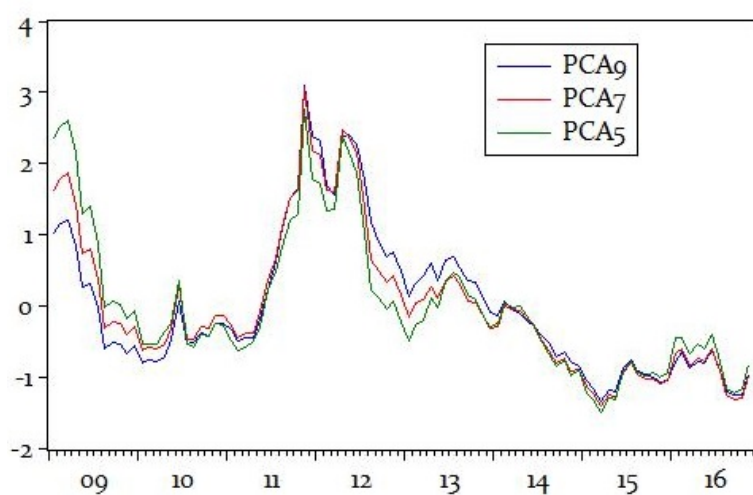


Figure 5.3 : Euro area bond yields (in percent)



The simplest option to measure the relative drop in risk premiums is to use the average spread of all the nine peer countries selected to establish a benchmark. To increase robustness, we form two other peer groups. In a seven-country case we further exclude Italy and Slovenia from the list²⁹, while in a 5-country case we also drop Malta (small size) and Belgium (high debt country).

Figure 5.4 : Euro area spreads: first principal component



PCAX denotes the first principal component of spreads in case of X countries.

Figure 5.4 shows the first principal component of euro area peers' spreads. The results are similar irrespective of the number of countries included in the analysis.

Using average spreads for euro area peers might help to filter out common factors. However, one should also check country-specific legislative changes³⁰ and fundamentals. In order to do

²⁹Countries hit by financial or structural turbulences.

³⁰No similar domestic fiscal responsibility acts were adopted in the peer countries.



so, we split the sample into two parts. The first period covers years 2009-2011, since the FRA was introduced in December 2011. The second interval (2013-2015) starts after the whatever it takes speech in order to filter out a period with erratic fluctuations and high uncertainty. Fiscal data are not yet available for 2016.

We focus on two sets of data: growth prospects and debt trends. At least in theory, these should be the most relevant determinants of future fiscal solvency. Table 5.1 shows average real GDP growth figures³¹ and potential GDP estimates. Potential output data are from the 2011 and 2015 Autumn Forecasts of the European Commission (production function approach). We can see that the improvement in growth prospects in case of the 7- and 9-country peer groups is very similar to the change in growth rates for Slovakia.

Table 5.1 : GDP growth and potential GDP estimates

Country	GDP Growth			Potential GDP Estimates		
	2009-2011	2013-2015	difference	2009-2011	2013-2015	difference
Belgium	0.7	1.0	0.3	1.3	1.3	0.0
France	0.4	0.8	0.4	1.1	1.1	0.0
Italy	-1.1	-0.3	0.8	0.3	0.4	0.1
Luxembourg	0.8	4.1	3.3	1.5	2.9	1.4
Malta	0.8	6.8	6.0	1.3	2.9	1.6
Netherlands	-0.2	1.1	1.3	1.3	1.1	-0.2
Austria	0.3	0.6	0.3	1.5	1.2	-0.3
Slovenia	-2.0	1.4	3.4	1.1	1.2	0.1
Finland	-0.9	-0.4	0.5	1.2	0.5	-0.7
Slovakia	0.8	2.6	1.8	2.4	2.6	0.2
average 5 countries	0.1	1.2	1.1	1.3	1.4	0.1
average 7 countries	0.3	2.0	1.7	1.3	1.6	0.3
average 9 countries	-0.1	1.7	1.8	1.2	1.4	0.2

Source: Eurostat, European Commission

Now we turn to gross debt data. It is important to note that none of the peer groups (calculated as average debt) experienced such a big increase in indebtedness as Slovakia did in 2013-2015 compared to the pre-crisis period. Forecasts of debt change are almost identical for peer groups and Slovakia.

Tables 5.1 and 5.2 show that there were no dramatic changes in economic fundamentals after the adoption of the FRA in Slovakia *relative* to euro area peers. Table 5.3 contains the average level of spreads in two periods: from January 2009 to June 2011 and from July 2013 to December 2016. We excluded the most turbulent period on euro area sovereign debt markets. Spreads on Slovak bonds compared to German bunds fell between the two periods by 59 basis points, while in peer groups only by 4-21 basis points. This indicates relative decline in spreads in Slovakia by 38-55 basis points.

We also estimated econometrically the relationship between Slovak yields and principal components of euro area spreads in the pre-crisis period. Figure 5.5 illustrates hypothetical scenarios,

³¹We assume no major differences in GDP deflators due to the low inflation environment.



Table 5.2 : Government debt and debt forecasts

Country	Government Debt			Debt Forecasts		
	2009-2011	2013-2015	difference	2015	2018F	difference
Belgium	100.5	105.9	5.4	105.8	106.4	0.6
France	81.9	94.6	12.7	96.2	97.1	0.9
Italy	114.8	131.1	16.3	132.3	133.1	0.8
Luxembourg	18.2	22.8	4.5	22.1	23.5	1.4
Malta	68.5	66.5	-2.0	64.0	57.2	-6.8
Netherlands	59.3	66.9	7.6	65.1	59.3	-5.8
Austria	81.8	83.7	1.9	85.5	79.2	-6.3
Slovenia	39.9	78.3	38.5	83.1	76.6	-6.5
Finland	45.8	60.1	14.3	63.6	68.1	4.5
Slovakia	40.4	53.6	13.2	52.5	51.5	-1.0
average 5 countries	57.4	65.6	8.2	66.5	65.4	-1.1
average 7 countries	65.1	71.5	6.4	71.8	70.1	-1.6
average 9 countries	67.8	78.9	11.0	79.7	77.8	-1.9

Source: Eurostat, European Commission

Table 5.3 : Slovak spreads relative to peer groups in percentage points

	2009M1-2011M6	2013M7-2016M12	difference
Slovakia	1.28	0.69	0.59
average 5 countries	0.77	0.73	0.04
average 7 countries	0.64	0.43	0.21
average 9 countries	0.46	0.28	0.18

proxies for Slovak yields in post-crisis period based on the estimated relationship between SK yields and principal components before the turbulences. The difference in the calm post-crisis period (from July 2013 to December 2016) compared to pre-crisis relationships is around 40 basis points.

Our second approach is based on the synthetic control method developed by Abadie and Gardeazabal (2003) and Abadie et al. (2010). SCM employs a data-driven control-group selection procedure. A synthetic control unit is defined as a weighted average of available control units that approximates the most relevant characteristics of the treated unit prior to the treatment. We use daily spreads for six control countries³² and two predictors: average ten-year spreads between January 2009 and June 2011 and the change in the debt-to-GDP ratio between 2010 and 2009.

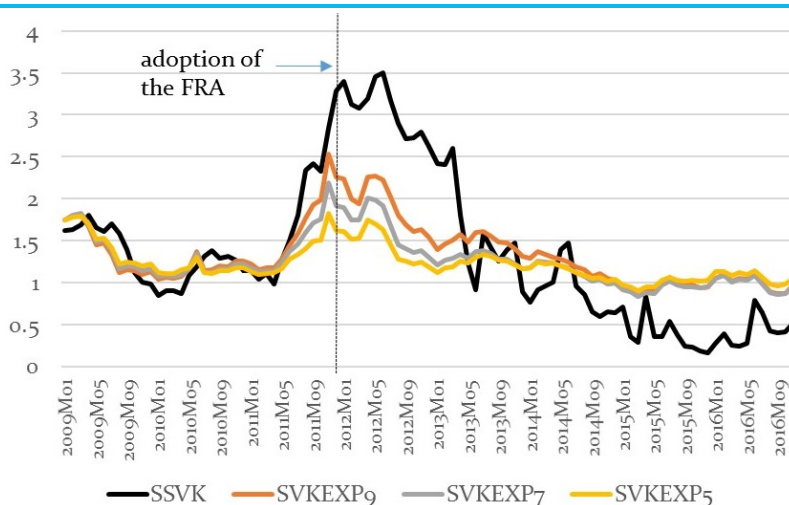
Figure 5.6 displays the differences between actual and synthetic ten-year yields. The average difference is around 80 basis points. We have tried different predictors in addition to the average spread, however the results were similar.

Our conclusion from this simple empirical investigation is that there has been a decline in Slovak ten-year sovereign spreads by at least 40 basis points, which is hard to explain by economic fundamentals or common factors in the euro area. We do not want to claim that this relative decline compared to peers is the result of the adoption of the FRA in 2011 only. One can imagine

³²Data for the three remaining peers are not available on a daily basis: Cyprus, Malta and Slovenia.

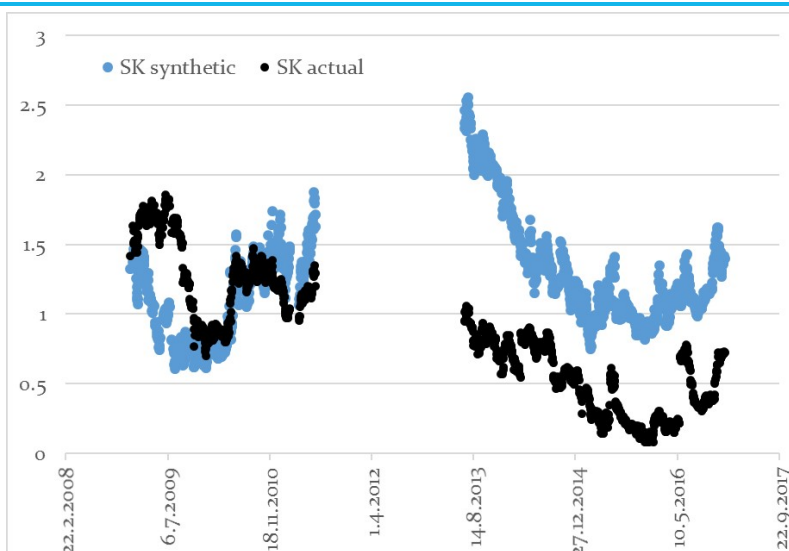


Figure 5.5: Proxies for Slovak yields in percent



SSVK = spread over bunds, SVKEXPY = spread implied by a model using Y-country principal components. The estimation period is between January 2009 and June 2011.

Figure 5.6: Actual and synthetic yields (in percent)



Based on a synthetic control method using daily spreads for six peer countries and two predictors: average ten-year spreads between January 2009 and June 2011 and the change in the debt-to-GDP ratio between 2010 and 2009.

other explanations like different effects of common shocks or changes in investors preferences (higher weight on low debt levels). We tested the possibility that, given the limited size of the Slovak government bond market, quantitative easing by the ECB might have resulted in a steeper decline of spreads than in case of countries with more liquid markets. We regressed "unexplained" spreads on monthly QE purchases available at the ECB website and found that 10-20 basis points can be explained (on average) by these unconventional policy actions. The unexplained part thus remains at around 20-30 basis points.



6 Conclusions and further research

Expectations play an important role in the world of financial markets. Therefore, credible commitments might be welfare improving. As we have shown, current institutional setups on sovereign debt markets together with long-term government bonds give rise to the so called debt dilution problem. Strict fiscal rules in the form of constitutional debt brakes or spread limits can generate non-negligible welfare gains. When calibrated to Slovak data, we found that the optimal value of the (net) debt limit is at 48 percent of GDP. Credible commitment to this debt limit improves welfare by 1.3-1.5% compared to a no-rule economy, lowers default frequency by 16 percent and cuts sovereign spreads by 50 basis points.

Slovakia introduced a relatively credible (compared to other frameworks in Europe) constitutional debt ceiling in December 2011. Our simple empirical investigation shows, that there has been indeed a relative decline of sovereign spreads on ten-year Slovak government bond compared to peers. After filtering out common shocks and idiosyncratic effects of QE, there remain approximately 20-30 basis points, which cannot be explained by changes in fundamentals.

We see three potentially fruitful areas for further research. First, the question of common spread limit in the euro area is promising, since current one-size-fits-all debt levels are clearly suboptimal in a diverse monetary union. Moreover, the SGP rests only on fiscal indicators and judgment, without incorporating price signals from financial markets. Second, the literature usually uses gross debt figures to calibrate models of sovereign defaults. In our view, incorporating liquid assets into the analysis might help to explain some of the cross-country differences. Third, since the problem of debt dilution can be mitigated without the use of fiscal rules, it might be interesting to discuss potential alternatives in the context of the common European currency. Or, for example, the question of state-contingent sovereign bonds and their role in risk sharing might be also analyzed.



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Appendix A Problem solution

Similarly to Hatchondo et al. (2015), Hatchondo et al. (2010), Arellano (2008) or Aguiar and Gopinath (2006) we employ a discrete state space method to solve the model. That is, we discretize the stochastic process for the technology shock a and allow the government to choose the borrowing b from a discrete set of points only. When interpolating between the evenly spaced³³ grid points $[b_1, b_{N_b}] \times [a_1, a_{N_a}]$. In order to increase the algorithm accuracy we employ cubic splines with the not-a-knot condition³⁴ for borrowing positions and linear interpolation in the income shocks domain. More precisely, when evaluating V_R at a point (b, a) we first interpolate over the debt level positions and calculate the vector $[V_R(b, a_1), \dots, V_R(b, a_{N_a})]$. Then we interpolate over the grid of income shock to determine $V_R(b, a)$.

Furthermore, as in Hatchondo et al. (2015) and Hatchondo et al. (2010), in order to decrease the computational time we use the one-loop³⁵ approach when solving the problem, i.e. within the algorithm we iterate simultaneously on the value function and the bond price function. We would like to emphasize that the continuity of the optimal bond price function is not required. However, within the procedure we assume that both value functions V_R and V_D are C_1 continuous.

Finding the optimal borrowing level goes hand in hand with determining the optimal the tax rate and the level of government consumption. In case of a default decision, the government cannot issue new debt. Thus, given the exogenous technology, household consumption-labour behaviour and government consumption are driven by the government objective function in default \mathcal{V}_D maximising tax rate that solves (9). However, if government decides to repay its liabilities, the optimal tax rate (and so the labour supply, private and public consumption) depends also on its decision about the amount of the debt issued for the next period, b' . Therefore, to find the optimal level of the next-period debt we evaluate the government objective function under repayment \mathcal{V}_R for all possible b' assuming that government sets the tax rate optimally for a given b' and then choose the one that delivers the highest payoff. In both cases the existence of a unique optimal tax rate $\tau_D, \tau_R \in (0, \omega/(1 + \omega))$ is guaranteed.

In order to provide any policy recommendation based on the simulated model we need to derive the Markov perfect equilibrium of this infinite-horizon problem. As noticed by Krusell and Smith (2003) the uniqueness of the equilibrium of such infinite-horizon problem is not guaranteed. Therefore, as suggested by Hatchondo et al. (2012a) to avoid this issue we solve for the equilibrium of the finite-horizon version of our economy, and we augment the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. The first-period equilibrium objects are

³³In this initial setup we prefer simple equally spaced grids in both domains although Hatchondo et al. (2010) shows that concentrating grid points in debt levels at which the bond price is more sensitive to the borrowing level, and in levels that are observed more often in the simulations leads to higher efficiency of calculations.

³⁴For further details see e.g. de Boor (1978) or Leader (2004).

³⁵It is usual in studies on debt default models solution techniques prefer the two-loop approach: the inside loop iterates on the value function while the outer loop iterates on the price function. Once the convergence on the inner loop is attained, using the optimal default decisions implied by the value function the bond price function is updated. The advantage in using one-loop approach becomes significant even more when we need to simulate the model using Monte-Carlo method in order to calibrate it properly.



then considered for the infinite-horizon-economy. The solution procedure can be described as follows:

1. As initial starting points for the value functions V_R and V_D we evaluate the household utility function over the grid of debt positions and income shocks, $[b_1, b_{N_b}] \times [a_1, a_{N_a}]$. Thus, for any grid point (b_i, a_j) we set

$$V_R^{(0)}(b_i, a_j) = u(y(b_i, a_j) + b_i), \quad \text{and} \quad V_D^{(0)}(b_i, a_j) = u(y(a_j - \phi(a_j))),$$

where u is the household utility function³⁶ (2) evaluated at a grid point $(b_i, a_j) \in [b_1, b_{N_b}] \times [a_1, a_{N_a}]$. Furthermore, for the sake of simplicity we set $q_R^{(0)}(b_i, a_j) = q_D^{(0)}(b_i, a_j) = \beta$.

2. Next, at iteration step k we need to determine the approximations of the value functions $V_R^{(k)}, V_D^{(k)}$ and price functions $q_R^{(k)}, q_D^{(k)}$ using the previous iteration approximations of the value functions $V_R^{(k-1)}, V_D^{(k-1)}$ and price functions $q_R^{(k-1)}, q_D^{(k-1)}$. It means that the optimization problem (4)–(9) must be solved for each grid point $(b_i, a_j) \in [b_1, b_{N_b}] \times [a_1, a_{N_a}]$ using the information from the previous iteration.

- (a) In order to be able to evaluate next-period continuation values, we update the corresponding cubic splines using the previous iteration approximations of the value functions $V_R^{(k-1)}, V_D^{(k-1)}$ and price functions $q_R^{(k-1)}, q_D^{(k-1)}$.
- (b) Notice that solving the objective functions (4)–(9) requires evaluation of the value function expectation $\mathbb{E}_{a'|a}[V(b', a')]$ and the next-period bond price $q(b', a)$. The k th iteration expectation in equations (7),(9) and the bond price function are computed employing Gauss-Legendre technique (see Abbott (2005) or Golub and Welsch (1969)). To obtain the continuation values of value functions and price functions we use the cubic splines derived from value functions $V_R^{(k-1)}, V_D^{(k-1)}$ and price functions $q_R^{(k-1)}, q_D^{(k-1)}$.
- (c) We determine the k th iteration approximation of the solution to optimization problem (4)–(9), i.e. the value functions $V_R^{(k)}$ and $V_D^{(k)}$ and price functions $q_R^{(k)}$ and $q_D^{(k)}$ for each grid point $(b_i, a_j) \in [b_1, b_{N_b}] \times [a_1, a_{N_a}]$. Since the governments objective function may not be globally concave, when we solve the model using interpolation methods, we first find a candidate value for the optimal borrowing level using a global search procedure³⁷. That candidate value is then used as an initial guess in a non-linear optimization routine. Each time when the objective functions are evaluated we use the expectations on value and price functions already determined.

³⁶Notice that there is a room for higher efficiency in algorithm implementation due to independence of the output on debt level in case of default.

³⁷When looking for the optimal borrowing level given the income shock a_j we evaluate the value function V_R for each grid point $(b_i, a_j), i = 1, \dots, N_b$. Within the evaluation we assume that government levies the tax optimally, i.e. for a given initial level of the debt b_i , and suggested next-period borrowings b'_i it chooses the tax rate that maximises the government objective function under repayment, $\mathcal{V}_R^{(k)}$ where expectations are approximated using $\mathbb{E}_{a'|a_j}[V(b'_i, a'_j)]$ and $q(b'_i, a_j)$. Then the level of the next period borrowings that delivers the highest payoff is used as an initial guess for the local non-linear optimization routine. This approach is simplified when finding the optimal tax rate under default as in that case government cannot issue new debt. Then given the income shock a_j we evaluate the value function V_D over the discrete tax grid. The tax rate τ_i delivering the highest value of the objective function \mathcal{V}_D is then used as a candidate in the local optimization procedure.



- (d) Finally, based on the k th iteration knowledge of value and price functions under default and repay we update the optimal default decisions and subsequently the bond price function $q^{(k)}$ and the value function $V^{(k)}$.
3. We calculate the maximal absolute updates of the value and price functions in the current iteration,

$$\Delta_V^{(k)} \equiv \max_{[b_1, b_{N_b}] \times [a_1, a_{N_a}]} |V^{(k)}(b_i, a_j) - V^{(k-1)}(b_i, a_j)|,$$

$$\Delta_q^{(k)} \equiv \max_{[b_1, b_{N_b}] \times [a_1, a_{N_a}]} |q^{(k)}(b_i, a_j) - q^{(k-1)}(b_i, a_j)|.$$

If both $\Delta_V^{(k)}$ and $\Delta_q^{(k)}$ do not exceed 10^{-6} the algorithm terminates and a solution, i.e. the value functions $V_R^{(k)}, V_D^{(k)}, V^{(k)}$ and price functions $q_R^{(k)}, q_D^{(k)}, q^{(k)}$ have been found. Otherwise, we increment the iteration counter and return to step 2 of this procedure.

Appendix B Welfare gain and fiscal rules implementation

We study the possible benefits from imposing a debt brake rule in an indebted country, when the introduction of a debt rule is accompanied with a transition period between the rule announcement and its actual implementation. Therefore, we assume that when the government introduces the debt brake it announces that the debt limit μ will constraint its decision in every period starting in period T . Our aim is to search for the combination of μ and T that maximizes the potential welfare.

Model with delay. For this purpose we formulate the *Model with Delay* $\mathcal{D}_{\mu, T}$ that assumes existence of a time delay between the announcement period and the period in which the debt ceiling μ starts to bind. Therefore, the model \mathcal{D} is defined recursively as a combination of the sequence of the debt rule model with the debt ceiling μ , \mathcal{R}_μ , and benchmark models indexed $\mathcal{B}_1, \dots, \mathcal{B}_T$ where T is the maximal considered delay. Then for any μ and T , the model equilibrium is determined as follows:

- The debt rule model (and the debt limit constraint) is active if there is no delay, $\eta = 0$. Then the equilibrium for zero delay coincides with the debt rule model equilibrium. We denote $V^{(0)}$ and $q^{(0)}$ the value function and price functions for the debt rule model, \mathcal{R}_μ .
- When calculating the equilibrium for the previous period (delay $\eta = 1$), value function and bond price function determined for the debt rule model for zero delay $V^{(0)}$ and $q^{(0)}$ are considered for the expected value function $\mathbb{E}V^{(1)}$ and expected price function $\mathbb{E}q^{(1)}$. Based on this, for $\eta = 1$ we calculate the equilibrium value $V^{(1)}$ and bond price $q^{(1)}$.
- When calculating the equilibrium for the k th period (delay $\eta = k$), value function and bond price functions determined for the debt rule model for $k - 1$ delay $V^{(k-1)}$ and $q^{(k-1)}$ are considered for the expected value function $\mathbb{E}V^{(k)}$ and expected price function $\mathbb{E}q^{(k)}$, i.e.

$$\mathbb{E}V^{(k)} = V^{(k-1)} \quad \mathbb{E}q^{(k)} = q^{(k-1)}, \quad k = 1, \dots, T.$$



Based on this, for each $\eta = k$ we calculate the equilibrium value function $V^{(k)}$ and the price function $q^{(k)}$.

Providing that the equilibrium of the debt rule model \mathcal{R}_μ is known, calculating the equilibrium of the model with the delay $\mathcal{D}_{\mu,T}$ is not time consuming and does not require us to employ the iterative process (used to find the equilibrium of the debt rule model) in each step if the recursive procedure.

Welfare gain measure. In what follows we clarify the method we use to determine the welfare gain associated with the introduction of a certain debt limit. We measure welfare gains as the constant proportional change in consumption of domestic agents that would leave a consumer indifferent between continuing living in the benchmark economy (without a fiscal rule) and moving to an economy with a fiscal rule. However, we assume the existence of a time delay between the announcement period and the period in which the ceiling starts to bind. Therefore our aim is to determine both the optimal debt limit and the associated delay in its announcement and implementation.

Our procedure is the following:

- First, we specify a fine consumption grid $\Lambda = \{\lambda_1, \dots, \lambda_{N_\Lambda}\}$. We substitute private consumption c for $\lambda_i c$ for all $i \in \{1, \dots, N_\Lambda\}$ within the benchmark model \mathcal{B} and calculate the corresponding equilibrium value function $V_{\lambda_i} = V_{\lambda_i}(b, a)$.
- Next, we calculate the equilibrium of the benchmark model \mathcal{B} and using it we simulate 1000 samples each of length 500 quarters. From these samples we extract those which have no default episode in last 100 quarters and determine the corresponding mean debt level \bar{b} and unconditional mean technology \bar{a} ³⁸.
- Then we define a set of reasonable debt limits $M = [\mu_1, \dots, \mu_{N_M}]$, delays $\tau = [\tau_1, \dots, \tau_{N_\tau}]$ and the associated models $\mathcal{D}_{\mu_j, \tau_k}$ for $j \in \{1, \dots, N_M^*\}$ and $k \in \{1, \dots, N_\tau\}$. We evaluate the equilibrium value functions of these models for each combination of the debt limit μ_j and delay in its implementation τ_k for the mean debt level \bar{b} and technology \bar{a} and denote the elements of the resulting grid $W_{M,T}$ as $\omega(\mu_j, \tau_k)$. In order to find these equilibria we need for each debt rule to calculate the equilibrium of the model with that rule and then recalculate recursively the equilibria of benchmark models linked with that model for each time delay.
- For each value function $\omega(\mu_j, \tau_k)$ we find $\lambda \in [\lambda_1, \lambda_{N_\Lambda}]$ for which $V_\lambda(\bar{b}, \bar{a}) = \omega(\mu_j, \tau_k)$. Then, the optimal debt rule (the one that delivers maximal increase in household consumption λ^*) is given by the pair (μ^*, τ^*) such that

$$(\mu^*, \tau^*) = \arg \max_{(\mu_j, \tau_k) \in M \times T} \{ \lambda \in [\lambda_1, \lambda_{N_\Lambda}] | V_\lambda(\bar{b}, \bar{a}) = \omega(\mu_j, \tau_k) \}.$$

³⁸In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths in which the last default was declared at least two periods before the beginning of each sample. Default frequencies are computed using all simulation data.



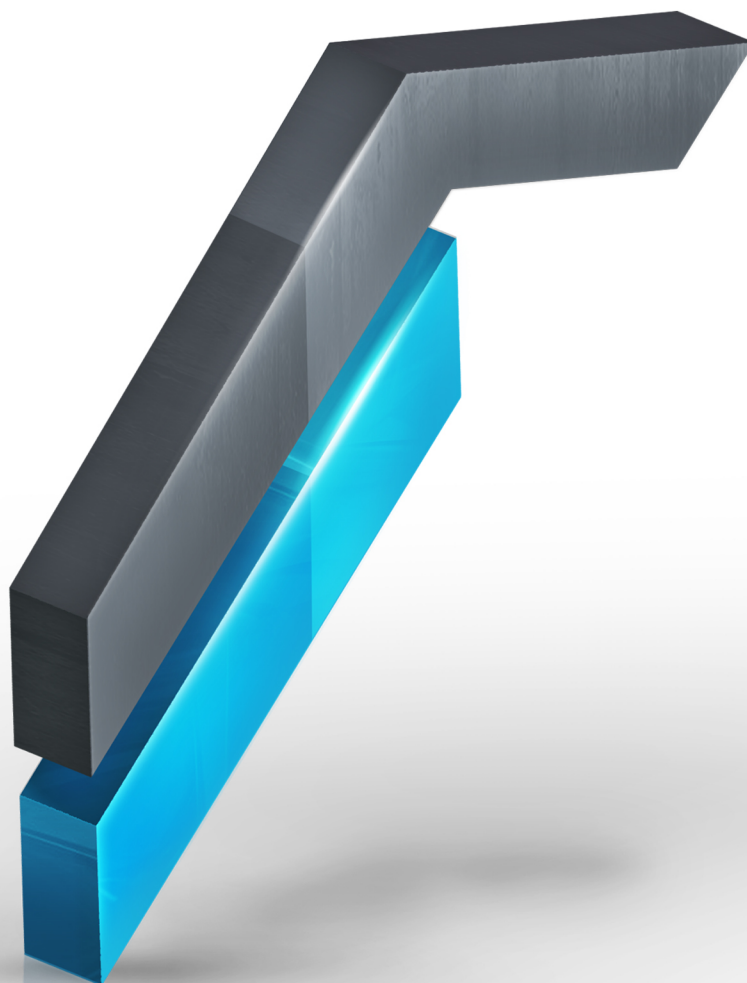
Appendix C Simulation results

Table C.1 : Benchmark vs. Optimal debt rule economies

Indicator	Benchmark Economy	Optimal Debt Rule Economy (48%)
Annual mean default rate	0.684%	0.569%
Duration	6.151 years	6.581 years
Mean debt/GDP	44.85%	47.19%
Mean public/private consumption	33.4 %	33.5%
Mean annual spread	1.340 p.p.	0.829p.p.
Mean labour tax rate	26.1%	25.9%
Mean employment	37.9%	38.1%
Mean annual output	1.52	1.52
Mean trade balance/GDP	2.2%	2.2%
$\rho(c,y)$	0.995	0.998
$\rho(tb,y)$	0.808	0.678
$\rho(R_s,y)$	-0.694	-0.752
$\rho(tax,y)$	0.525	0.083
$\rho(g,y)$	0.963	0.975
$\rho(R_s,tb)$	-0.323	-0.347
Mean $\sigma(tb/y)$	0.723	0.558
Mean $\sigma(c)/\sigma(y)$	0.958	0.986
Mean $\sigma(g)/\sigma(y)$	0.620	0.712
Mean $\sigma(R_s)$ (in %, p.a.)	0.481	0.278
Mean $\sigma(tax)$ (in %)	0.280	0.231
Mean $\sigma(y)$ (in %, p.a.)	1.143	1.120

The standard deviation of a variable x is denoted by $\sigma(x)$. The coefficient of correlation between variables a and b is denoted by $\rho(a,b)$. The second column is obtained using data from 1000 simulation samples of the benchmark model while the third column uses the optimal debt rule model (debt ceiling set to 48 % of the mean output of the benchmark economy). In both simulations take the last 74 periods of samples in which no default occurs in the last 100 periods.





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