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Zuzana Múčka[2]

Abstract

We study the interactions among fiscal policy, fiscal limits and sovereign risk premia. The fiscal limit, which measures the government's ability to service its debt, arises endogenously from dynamic Laffer curves and is a random variable. A nonlinear relationship between sovereign risk premia and the level of government debt then emerges in equilibrium. The model is calibrated to Slovak data and we study the impact of various model parameters on the distribution of the fiscal limit. Fiscal limit distributions obtained via Markov–Chain–Monte–Carlo regime switching algorithm depend on the rate of growth of government transfers, the degree of countercyclicality of policy, and the distribution of the underlying economic conditions. We find that it is considerably more heavy–tailed compared with the one usually obtained in the literature for advanced economies, and is very sensitive to the size and rate of growth of transfers. The main policy message is that the Maastricht debt limit is not safe enough for Slovakia: although in the equilibrium the chance of country default is 10 percent when the debt is 60 percent of GDP, it increases dramatically to approximately 40 percent in bad times (when productivity falls by almost 8 percent). A well-designed fiscal policy involving a deceleration in the growth of transfers can reduce the chance of default significantly.

Keywords: Simulation Methods and Modelling, Fiscal Policy, Government Expenditures, Debt Management and Sovereign Debt.

JEL Classification: C15, C63, E62, H5, H63.

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1 Introduction

The theoretical analysis of fiscal policy in advanced economies has traditionally abstracted from sovereign default risk. However, due to the recent financial crisis, and the resulting rise of public debt in developed countries, the importance of the debt sustainability and default risk took center stage. Moreover, age–related expenditures represent another source concern about the long-term sustainability of public finances. Therefore, it is essential to understand the interaction between sovereign default risk and fiscal policy and, furthermore, to discuss what kind of fiscal policies can contain the default risk. The Council for Budget Responsibility (CBR) evaluates the long-term sustainability of public finances. The analysis of the relationship between the fiscal policy and default risk represent a substantial part of the risk assessment building block in the CBR's toolkit[1].

In this paper we present one possible approach that enables us to study the relationship between the fiscal policy, and the resulting fiscal limit and the risk premium. Following Bi (2011) and Bi and Leeper (2013), we construct a simple real business cycle (RBC, hereafter) nonlinear model that allows us to describe, how the *fiscal limit*, the maximum level of debt that the government is able to service, depends on macroeconomic fundamentals. We also calculate the risk premia that emerge from agents' intertemporal choice taking into account the fact that the government might default on its outstanding liabilities.

We extend Bi (2011) and Bi and Leeper (2013) in important ways to make it more relevant in the context of Slovakia. The underlying growth in government transfers is calibrated to reflect the ageing population of Slovakia. We introduce a response to the economic cycle to both transfers and government spending. Moreover, we allow transfers to follow a regime–switching process that better reflects the political cycle in Slovakia. Finally, we draw total factor productivity – the only source of business cycle fluctuations in the model – from a distribution that approximates empirical cyclical conditions in Slovakia very well, and is heavy-tailed.

In this context, we find that the rate of growth of transfers in the economy, the degree to which policies respond to the economic cycle, and the distribution of cyclical conditions in the economy all affect the distribution of the fiscal limit significantly. We find that the distribution of the debt limit is considerably heavy-tailed, and an adverse combination of conditions and policies could generate high probabilities of default even at debt levels generally considered to be safe. Hence, our main policy conclusion is that the debt limit enshrined in the Stability and Growth Pact of 60 percent of GDP may not be "safe" for Slovakia.

Although in normal times the Maastricht debt limit is associated with only a 10% chance of default, this probability rises sharply to 40 percent when the country faces a significant drop in the productivity level. Furthermore, running a bad fiscal policy in bad times, i.e. allowing the age-related expenditures to rise faster relative to the no-policy-change scenario implies even a 50-60% chance that country would default on its liabilities. Therefore, governments would be well–advised to keep debt levels at a significantly lower level. To do so, it appears crucial to control the long-term growth of transfers. Hence, reforms to age-related spending should be a priority.

[1] Our study is the first step in the process of the safe debt level analysis. The conceptual framework of the CBR fiscal risk assessment is explained in detail in Odor (2014) (Chapter 4.2).

The rest of the paper is organized as follows. Section 2 presents the baseline model and its extensions. Sections 3 and 4 present our quantitative exercise with regards the distribution of the fiscal limit and the risk premium. Section 5 concludes.

2 The Model

2.1 The Intuition

The original model of (Bi (2011) and Bi and Leeper (2013)) considers a closed economy in which the government finances lump-sum transfers to homogeneous households and an exogenous level of purchases, produced along with consumption goods using a simple linear production function, by collecting distorting taxes levied on labour and issuing non-state- contingent debt. The government raises the time-varying tax rate when the debt level grows. Laffer curves arise endogenously from distorting taxes - if the tax rate is on the slippery side of the Laffer curve, then the government is unable to raise more tax revenue through higher tax rates. The lump-sum transfers follow a Markov regime-switching process, with one regime being stationary while the other explosive. If the government stays in the explosive-transfer regime for too long, the debt can rise to such a level that tax rate may eventually reach the peak of Laffer curve and the government will be unable to repay its debt in full amount. Even if the current tax rate is not there yet, a positive probability of eventually hitting the peak of Laffer curve in the future can prompt forward-looking households to demand a higher default risk premium on sovereign debt today.

Therefore, the concept of the *fiscal limit* is introduced, as the maximum level of debt that the government is able to service, which is defined as the sum of the discounted maximum fiscal primary surplus in all future periods. It is the point at which, for economic or political reasons, the government can no longer adjust taxes and spending (government consumption and transfers to households) to stabilize debt. An estimate of the tax rate at which the peak of the Laffer curve is also obtained. Given the persistence of exogenous disturbances, the fiscal limit depends on the current state of the economy (productivity level, regime of transfers, level of government purchase) and on random disturbances hitting the economy in the future. The fiscal limit is state-dependent and has a stochastic distribution.

To estimate country default risk premium one must solve the nonlinear model which uses the already calculated distribution of the fiscal limit. Even this simple model generates non-linearities that play a critical role in pricing sovereign debt. Due to the high non-linearities and the discontinuity one cannot employ log–linearisation to solve the model. Instead, the procedure is as follows. At each period, an effective fiscal limit is drawn from the state-dependent fiscal limit distribution. If the level of government debt hits the effective fiscal limit, then the government reneges on a fraction of its debt and the realized default rate follows an empirical distribution that is computed from historical data. Otherwise, the debt is repaid in full. Using the state-dependent distributions of fiscal limits and the empirical distribution of default rates, households can decide the quantity of government debts that they are willing to purchase and the price at which they are willing to pay. Furthermore, households make a decision about their level of consumption and labour supply, pay tax from their labour income and receive lump-sum transfers at level determined by the fiscal authority. The government collects tax revenues levied with rate reflecting current post–default debt from which it finances unproductive spending (government purchase) and transfers following the Markov–switching regime. The accumulated debt is priced by the forward–looking households considering the current primary surplus and their expectations of the next–period default rate and future changes in consumption. Then the default risk premium arises as the difference between this risky interest rate and the rate calculated under assumption of no default.

2.2 The Original Model

Here, we set out the model that builds on Bi (2011), Bi and Traum (2012), Bi and Leeper (2013) and Bi and Leeper (2010) formally. Within this paper we use the following notation:

Box 1: Notation

a_t	technology process
g_t	non–productive government purchase
c_t	household consumption
u_t	marginal utility of consumption
h_t	labour supply
z_t	transfers
r_t	transfers regime
b_t, b_t^d	public debt (pre–default, post–default)
Δ_t	actual default rate
δ_t	maximal default rate
b_t^*	fiscal limit
q_t	bond price
τ_t, $\tau_t^{\max}$	(peak of the Laffer curve) effective tax rate
Θ_t, $\Theta_t^{\max}$	(maximal) tax revenues
β	time–discount rate
ϕ	consumption–labour preference parameter
ς_t, $\varsigma_t^{\max}$	(maximal) primary surplus

Model Background: Our model is a closed economy with linear production function: output depending on the level of productivity a_t and household labour supply h_t is purchased by government g_t or consumed by households as expressed in the following aggregate resource constraint:

$$a_t h_t = y_t = c_t + g_t \,. \tag{1}$$

We assume that the deviation of the productivity from its steady state follows an AR(1) process

$$a_t = [a_{t-1}]^{\rho_a} [a]^{1-\rho_a} \exp\{\varepsilon_t^a\} \,, \qquad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2) \,. \tag{2}$$

The government finances their purchase g_t and lump-sum transfers to households z_t by issuing one–period bonds b_t with price q_t and collecting a levied tax τ_t on labour income. For each unit of the bond purchased in the beginning of the period, the government promises to pay the household one unit of consumption in the next period. The bond contract is not enforceable since at time t a partial default of fraction Δ_t on government liability issued in the beginning of that time period b_{t-1} is possible. Therefore, denoting the post–default debt liability b_t^d the government budget satisfies:

$$q_t b_t - b_t^d = z_t + g_t - a_t h_t \tau_t \,, \qquad b_t^d \equiv (1 - \Delta_t) b_{t-1} \,. \tag{3}$$

The default scheme at each period depends on the effective fiscal limit b_t^* drawn from a conditional distribution $\mathscr{B}^*(a_t, g_t, r_t)$. If government liability at the beginning of period t does not attain the effective fiscal limit, then no default occurs since it repays its debt in full amount.

Otherwise, a partial default takes place and the stochastic default rate follows an empirical distribution Ω[2]

$$\Delta_t = \begin{cases} 0, & b_{t-1} < b_t^*, \\ \delta_t, & b_{t-1} \geq b_t^*, \end{cases} \qquad b_t^* \sim \mathscr{B}^*(a_t, g_t, r_t), \quad \delta_t \sim \Omega. \tag{4}$$

Government purchases follow a simple AR(1) process

$$g_t = [g_{t-1}]^{\rho_g} [g]^{1-\rho_g} \exp\{\varepsilon_t^g\}, \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2). \tag{5}$$

Next, transfers are countercyclical with $\zeta^z < 0$ and always follow a Markov regime-switching process r_t driven by the constant[3] transition matrix

$$P = \begin{pmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{pmatrix}, \tag{6}$$

so are either stationary or they expand and grow exponentially with a growth rate $\mu > 1$. Therefore for a given regime of transfers it holds that

$$z_t = \begin{cases} z + \zeta^z(a_t - a), & r_t = 1, \\ \mu z_{t-1} + \zeta^z(a_t - a), & r_t = 2. \end{cases} \tag{7}$$

We assume that the government follows a simple Taylor–type rule that raises tax rate with adjustment parameter $\gamma > 0$ to retire the debt,

$$\tau_t = \tau + \gamma(b_t^d - b). \tag{8}$$

Therefore, for any $\gamma > 0$ an equilibrium exist with non necessarily bounded debt – to ensure that debt is bounded in the steady state γ must be sufficiently large.

At each time t a representative household chooses consumption c_t, labour supply h_t, and bond purchases b_t that would maximize

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U(c_{t+k}, h_{t+k}), \qquad U(c_t, h_t) = \log c_t + \phi \log(1 - h_t), \tag{9a}$$

subject to the intertemporal budget constraint with transfers z_t, tax rate τ_t and debt default rate Δ_t taken as given. $\mathbb{E}_t$ denotes the mathematical conditional expectation made based on the information available at time t, including the debt default information. The parameter $\beta \in (0,1)$ is the constant discount factor[4] and ϕ is the representative household leisure preference parameter[5]. The household utility function $U = U(c,h)$ is strictly concave and strictly increasing in leisure and consumption. Moreover, we assume that households consider the historical information captured in the empirical distribution Ω when pricing sovereign bonds.

$$(1 - \tau_t) a_t h_t - c_t + z_t = q_t b_t - b_t^d. \tag{9b}$$

[2]Concerning the empirical distribution Ω of the stochastic default rate δ_t we refer to Bi and Leeper (2010) and Bi (2011). They computed the distribution from the sovereign debt defaults and restructures observed in the emerging market economies during the period of 1983 to 2005 since few sovereign default has been observed in developed countries in the post-war era. The cumulative distribution function of the default rate distribution is illustrated on Figure C.1 in the Appendix.

[3]In this study we also illustrate an alternative approach and introduce a more structured state-dependent transition matrix that reflect the evolution of transfers and tax rate. For details see Appendix D.2.

[4]In this cashless economy, $1/\beta$ is the equilibrium risk-free interest rate.

[5]The parameter ϕ measures the household willingness to supply their labour services: households are less disposed to work if $\phi > 0$ is large. The exact value of ϕ is not calibrated but determined from the model steady state - see Equation (D.10) in the Appendix D.1.

We denote $u_c = \partial U/\partial c$ the marginal utility of consumption and $u_h = \partial U/\partial h$ the marginal disutility of labour:

$$u_c(c_t,h_t) \equiv \frac{\partial U(c_t,h_t)}{\partial c_t} = \frac{1}{c_t}, \qquad u_h(c_t,h_t) \equiv \frac{\partial U(c_t,h_t)}{\partial h_t} = -\frac{\phi}{1-h_t}.$$

Therefore, the optimal allocation of resources requires that the marginal rate of substitution between consumption and labour supply coincides with the after-tax wage and households price bonds taking into account their expectation about the next–period the probability and magnitude of sovereign default:

$$\phi \frac{c_t}{1-h_t} = -\frac{u_h(c_t,h_t)}{u_c(c_t,h_t)} = a_t(1-\tau_t), \tag{10a}$$

$$q_t = \beta \mathbb{E}_t\left[(1-\Delta_{t+1})\frac{u_c(c_{t+1},h_{t+1})}{u_c(c_t,h_t)}\right] = \beta \mathbb{E}_t\left[(1-\Delta_{t+1})\frac{c_t}{c_{t+1}}\right]. \tag{10b}$$

Finally, the optimal solution to the households maximization problem must also satisfy the subsequent transversality condition

$$\lim_{j\to\infty} \mathbb{E}_t\left\{\beta^{j+1}\frac{u_c(c_{t+j+1},h_{t+j+1})}{u_c(c_t,h_t)}(1-\Delta_{t+j+1})\,b_{t+j}\right\} = 0, \qquad \forall t \geq 0. \tag{10c}$$

The Laffer Curve: From the fiscal perspective, an increase in the proportional labour tax rate may or may not induce growth of tax revenues. This is the basis of the concept of the Laffer curve. Obviously, the point(s) on the curve where the tax revenues (measured as a function of tax rate) are at a maximum are particularly interesting. Within the baseline model where technology and government purchase follow standard autoregressive processes, there is a unique mapping between the state of economy characterised by the of technology and government purchases (a_t, g_t) and the tax rate $\tau_t^{\max}$ maximizing the collection of tax revenues $\Theta_t^{\max}$ given the state of the economy. Therefore, taking the current regime of transfers (political decision of the government), technology and government purchase as given, there is a unique maximum primary budget surplus and hence a *fiscal limit* defining the limit to the government's ability to service its debt.

2.3 Model Extensions

We modify the existing model of Bi (2011) and Bi and Leeper (2013) as follows. We allow transfers to follow a regime–switching process that better reflects developments in Slovakia. We also allow wasteful government purchases to respond to the cycle. Finally allow technology to follow a heavy–tailed empirical distribution rather than a normal one.

2.3.1 Extension 1: Regime–Switching Transfers Linked to Cycle

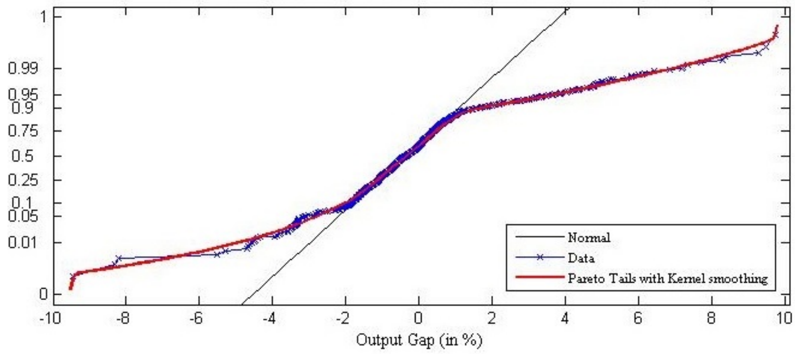
Within our model, transfers consist of all ageing–related government expenditures (Old-age, armed forces and disability pensions; healthcare, long–term care; education and unemployment benefits). Besides the projections of the fundamental demographic shifts expected in the next 50 years inducing increasing share of ageing–related government expenses on GDP even in the *no policy change* scenario (see Council for Budget Responsibility, (2014)), the government may adopt additional long–term measures that adjust (increase or cut) the ageing–related expenses. Therefore, in addition to allowing transfers to have countercyclical nature, we model them as either being in the *no policy change* regime (regime 1) of expansion or let them follow an alternative process associated with additional measures (temporarily) taken up by the government:

$$z_t = z_t(r_t, a_t) = \begin{cases} \mu_t^{(1)} z_{t-1} + \zeta_z (a_t - a) + \varepsilon_t^z, & r_t = 1, \\ \mu_t^{(2)} z_{t-1} + \zeta_z (a_t - a) + \varepsilon_t^z, & r_t = 2, \end{cases}, \tag{11}$$
$$\text{where} \quad \bar{\mu}^{(i)} > 1, \quad \forall t, i \in \{1,2\}, \quad \zeta_z < 0, \quad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2).$$

2.3.2 Extension 2: Business Cycle Distribution

To determine the distribution of the business cycle for Slovakia – a small open economy with short history, many structural breaks and changes in methodologies concerning the relevant data – we employ available estimates of the output gap. To overcome the uncertainty arising from the short time series and volatile data and increase the robustness of the output gap distribution estimation we consider all output gap time series for Slovakia published by several domestic and international[6] institutions as well as results obtained using standard filtering techniques[7] between 2000–2014[8]. Using this pragmatic approach we solve the lack–of–data issue and minimise the problems associated with small open economies and filtering techniques[9]. Inspecting the Slovak output gap data we find that extreme cases are not rare and the probability of keeping the output gap close to its mean decays rapidly.[10] Therefore, to model frequent structural breaks, we first use *Pareto tails* and the *kernel smoothing procedure* to estimate the distribution between these fat tails, as illustrated below in Figure 2.2. In the simulation of the fiscal limit, we draw random technology shock series employing this empirical distribution.

Figure 2.1 : Output gap data distribution



Comparison of the output gap data distribution (blue line with markers) to the normal distribution (black dashed line) and empirical Pareto-tailed kernel smoothing distribution (red thick line).

[6]Slovak Ministry of Finance, National Bank of Slovakia, European Commission, Bank for International Settlement.

[7]Hodrick–Prescott filter, multivariate Kalman filter, and Principal component analysis.

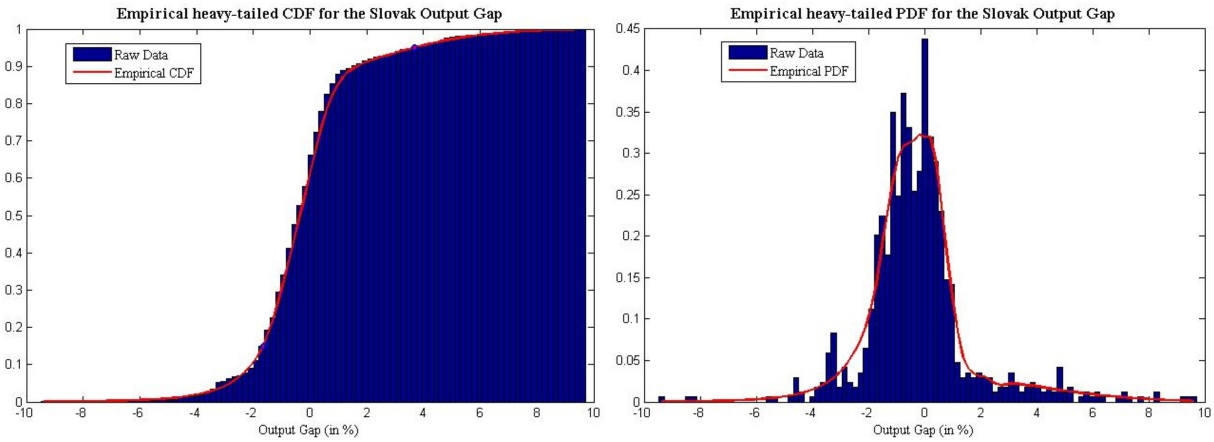
[8]Since these time series are usually on annual frequencies we interpolate them to obtain the approximations of quarterly output gap data.

[9]As shown in Odor and Jurasekova-Kucserova (2014) the current benchmark method in Europe based on the production function approach has in our view a lot of shortcomings in small and open economies – short-time series to estimate long-term trends with many structural breaks, high uncertainty around capital stock estimates, downplaying international capital and labour mobility, size of current account imbalances and banking sectors relative to GDP can be important in small and open economies, frequent supply side shocks, end-point problem of the HP-filter.

[10]Table B.3 in the Appendix B contains the detailed descriptive statistic of the collected output gap data.

Alternatively, we employ the *location-scaled t-distribution* to model the whole asymmetric heavy–tailed distribution of the Slovak business cycle (see Figure B.5 in the Appendix B).

Figure 2.2 : Empirical distribution of the Slovak business cycle



Empirical distribution of the Slovak business cycle as estimated from 2000-2014 data taken from Slovak Ministry of Finance, National Bank of Slovakia, European Commission, Bank for International Settlement and Hodrick–Prescott filter, multivariate Kalman filter, and Principal component analysis. Asymmetric fat tails (quantiles 0.15 and 0.95) are approximated by Pareto distribution whereas the Kernel smoothing procedure is employ to estimate the distribution between the tails.

2.3.3 Extension 3: Pro-cyclical Government Spending

Government purchases comprising fiscal expenditures independent of the demographic structure of the population usually evolve in an ad–hoc manner and based on Slovak fiscal data (see Figure B.1 and Table B.1 in the Appendix B.1) react the current business cycle. Thus we let them to evolve according to a standard autoregressive process,

$$g_t = \rho_g g_{t-1} + (1-\rho_g)g + \zeta_g(a_t - a) + \varepsilon_t^g\,, \qquad \varepsilon_t^g \sim \mathcal{N}(0,\sigma_g^2)\,. \tag{12}$$

Fiscal Limit and its Determination: To estimate the country's fiscal limit, defined as the sum of the expected discounted maximum primary surplus[11],

$$\begin{aligned} \mathscr{B}_t^* &= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \frac{u^{\max}(a_{t+k}, g_{t+k})}{u^{\max}(a_t, g_t)} \varsigma^{\max}(a_{t+k}, g_{t+k}, r_{t+k})\,, \\ \varsigma^{\max}(a_{t+k}, g_{t+k}, r_{t+k}) &= \Theta^{\max}(a_{t+k}, g_{t+k}) - g_{t+k} - z(r_{t+k}, a_{t+k})\,, \end{aligned} \tag{13a}$$

i.e the difference between the maximum tax revenues

$$\Theta_t^{\max} = (1+2\phi)\,a_t - \phi g_t - 2\sqrt{(1+\phi)\,\phi a_t\,(a_t - g_t)}\,, \tag{13b}$$

and expenditures (transfers and government purchases) in all future periods. A unique mapping between the state of economy (characterised by the technology and government purchase) and the unique rate $\tau_t^{\max}$

$$\tau_t^{\max} = 1 + \phi - \sqrt{(1+\phi)\,\phi\,(a_t - g_t)\,/a_t}\,, \tag{13c}$$

[11] A simple explanation of this idea is given in Appendix D.

mapping maximising the tax revenues exists[12]. Furthermore, $u^{\max} \equiv u^{\max}(a_t, g_t)$ from (13) is the marginal utility of consumption when the tax rate is at the peak of the Laffer curve, $\tau_t^{\max}$.

We also assume that household intertemporal utility is labour–leisure separable, $U(c_t, l_t) = \log c_t + \phi \log l_t$, where $l_t + h_t = 1$ and their budget constraint is $c_t + b_t q_t = b_{t-1}(1-\Delta_t) + z_t + (1-\tau_t) a_t h_t$. Combining the marginal utility of consumption, marginal utility of labour and the resource constraint $c_t + g_t = a_t h_t$, the relationships (17)–(16) are obtained. We can now express all our variables in terms of the state variables only - productivity and government purchase (see (16)–(17)).

The model equations are summarised in the following box.

Box 2: Fiscal Limit Equations

$$a_t = \rho_a a_{t-1} + (1-\rho_a) a + \varepsilon_t^a, \qquad \varepsilon_t^a \sim \mathscr{E}(\{\bar{y}^{gap}\}) \tag{14}$$

$$g_t = \rho_g g_{t-1} + (1-\rho_g) g + \varepsilon_t^g + \zeta_g (a_t - a), \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2), \tag{15}$$

$$c_t = \frac{(a_t - g_t)(1-\tau_t)}{1+\phi-\tau_t}, \tag{16}$$

$$h_t = \frac{a_t(1-\tau_t) + \phi g_t}{a_t(1+\phi-\tau_t)}, \tag{17}$$

$$z_t = \begin{cases} \mu_t^{(1)} z_{t-1} + \zeta_z (a_t - a) + \varepsilon_t^z, & r_t = 1, \\ \mu_t^{(2)} z_{t-1} + \zeta_z (a_t - a) + \varepsilon_t^z, & r_t = 2, \end{cases}, \qquad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2), \tag{18}$$

$$\tau_t^{\max} = 1 + \phi - \sqrt{(1+\phi)\phi(a_t - g_t)/a_t}, \tag{19}$$

$$\Theta_t^{\max} = (1+2\phi) a_t - \phi g_t - 2\sqrt{(1+\phi)\phi a_t (a_t - g_t)}, \tag{20}$$

$$\varsigma^{\max}(a_{t+k}, g_{t+k}, r_{t+k}) = \Theta^{\max}(a_{t+k}, g_{t+k}) - g_{t+k} - z(r_{t+k}, a_{t+k}), \tag{21}$$

$$\mathscr{B}_t^* = \sum_{k=0}^{\infty} \beta^k \frac{u^{\max}(a_{t+k}, g_{t+k})}{u^{\max}(a_t, g_t)} \varsigma^{\max}(a_{t+k}, g_{t+k}, r_{t+k}), \tag{22}$$

[12] For the derivation of the revenue maximising tax rate and the uniqueness of the mapping $(a_t, g_t) \mapsto \tau_t^{\max}$ see Appendix D.1

3 Analysis of the Fiscal Limit Distribution

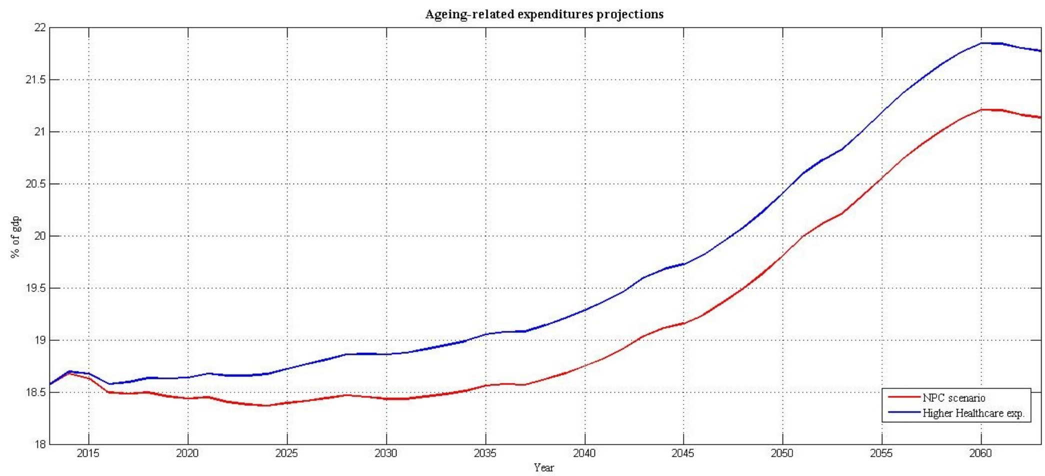
3.1 Model Calibration

In order to calibrate the model on annual frequency we use the 2013–2060 projections of Slovak data as published in Council for Budget Responsibility, (2014).

The baseline scenario of the long-term development of public finances, as defined by the Fiscal Responsibility Act, is developed by merging the medium-term scenario with long-term projections of the revenues and expenditures sensitive to population ageing and by incorporating other implicit and contingent liabilities. Therefore, transfers include social security payments and material social transfers (transfers in kind), i.e. all demography structure sensitive government payments. Under the baseline scenario between 2013–2060 a more than 13.8 percent increase in expenditures sensitive to population ageing (from 18.6 percent to more than 21 percent of GDP, see Figure 3.1) occurs which leads to average growth rate 1.0025. Alternatively, in the scenario with higher healthcare expenditures the share of transfers to GDP increases to more than 21.75 percent of GDP (average growth rate 1.0031).

Government purchase covers government final consumption of expenditures, subsidies, public wage bill and net capital transfers and in the steady-state covers 16.4 percent of the GDP. The average tax rate is defined as the ratio of the total tax revenue over the GDP, including social security and indirect and direct taxes and is consistent with 40 percent steady state debt–to–GDP and the annual discount rate 0.95. As a consequence, the steady state rate attains 39.14 percent which is considerably higher than the tax rate 31.68 percent observed in the data[13] (see Figure B.2 in the Appendix B.1).

Figure 3.1 : Projections of ageing related government expenditures



Projections of ageing related government expenditures: no-policy-change (baseline, red line) and higher healthcare expenditures (alternative, blue line) scenarios.

Source: *Council for Budget Responsibility, (2014)*

The leisure preference parameter ϕ set such that households spend 25 percent of their time by working (leisure is 75 percent of their time) and the Frisch elasticity of labour supply is 3.

[13] We use this fact and to be consistent with the data we prefer using the observed effective tax rate (31.68 percent) to the steady state rate (39.14) when estimating the tax rate adjustment parameter γ the OLS procedure to estimate the tax rate adjustment parameter γ.

Furthermore, we assume that productivity is unity in the steady state.

The coefficients affecting the model dynamics are obtained employing the Bayesian approach. Using the 2000–2014 time series for the transfers, government purchase and output[14], we estimate sensitivity of transfers and government purchase to current business cycle, $\zeta_z = -0.0159$, $\zeta_g = +0.0219$ and find out that both the technology and government purchase are rather persistent – $\rho_a = 0.7205$ and $\rho_g = 0.9229$ and standard deviations of fiscal shocks are moderate, $\sigma_g = 0.0233$ and $\sigma_z = 0.0277$.

For further details about historical fiscal data and Bayesian priors and posteriors see Tables B.1–B.2 in the Appendix B. Furthermore, as discussed deeply in Section 2.3.2 we assume that the business cycle in Slovakia follows a heavy–tailed empirical distribution with parameters and properties deeply described in Appendix B.3. The model calibration process is discussed in details in Appendix D.1.

3.2 Results

To understand the impact of various model parameters we proceed as follows. We start with a baseline case with only productivity and government purchase shocks while abstracting from varying the regimes for transfers. We then modify one parameter at a time, while keeping all other parameters the same as in the baseline case, to understand the quantitative impact of macroeconomic fundamentals upon the distribution of fiscal limits.

Specifically, in the baseline, *no–policy–change* scenario we assume that transfers reside in the first regime only ($p^{(1)} = 1$, $p^{(2)} = 0$). First, we ignore the effect of business cycle on fiscal variables. Next, we make fiscal expenditure items (either government purchase or transfers) sensitive to the business cycle. Then we study the case of permanently higher growth rate of transfers due to expected higher healthcare expenditures (see Figure 3.1), i.e. the risky scenario ($p^{(1)} = 0$, $p^{(2)} = 1$). Finally, reflecting four–year political cycle ($p^{(1)} = p^{(2)} = 0.75$) we let transfers to switch between two regimes with different growth rates of transfers. For details see the Table 3.1.

Table 3.1 : Simulation scenarios for the model calibrated on Slovak economy

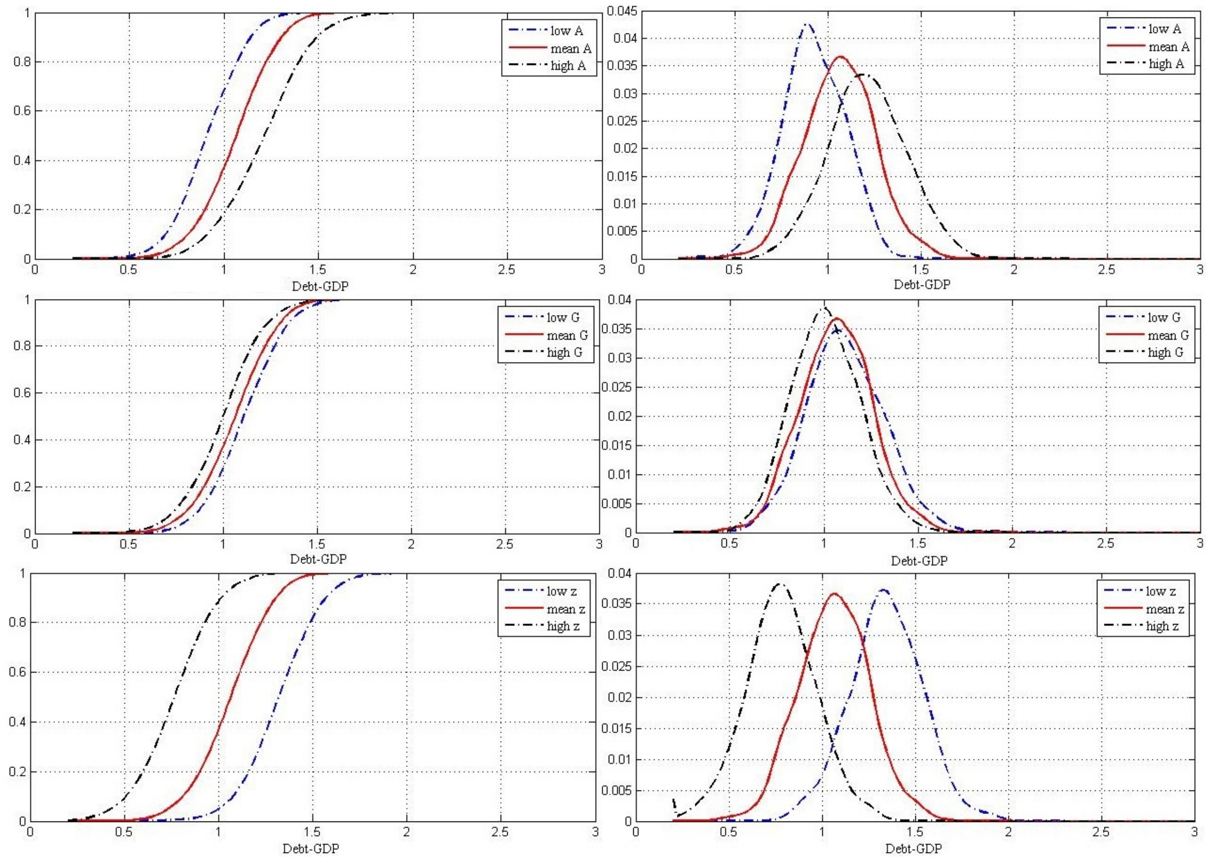
Scenario	$\bar{\mu}_1$	$\bar{\mu}_2$	ζ_g	ζ_z	$p^{(1)}/p^{(2)}$	ρ_a	ρ_g
no policy change (A.1)	1.0026	1.0032	0	0	1 / 0	0.7205	0.9229
– procyclical g.purchase (A.2)	1.0026	1.0032	0.0219	0	1 / 0	0.7205	0.9229
– countercyclical transfers (A.3)	1.0026	1.0032	0	-0.0159	1 / 0	0.7205	0.9229
risky scenario (A.4)	1.0026	1.0032	0	0	0 / 1	0.7205	0.9229
– two regimes of transfers (A.5)	1.0026	1.0032	0	0	0.75 / 0.75	0.7205	0.9229
– all features switched on (A.6)	1.0026	1.0032	0.0219	-0.0159	0.75 / 0.75	0.7205	0.9229

The state-dependent fiscal limit distributions are simulated employing the Markov Chain Monte Carlo method. The whole procedure detailed description and the associated code snippet are located in Appendix D.3).

The right panels in the Figure 3.2 (or Figure A.1 in the Appendix A) show the probability distribution functions (PDFs), while the left panels show the corresponding cumulative distribution functions (CDFs). Thus, the CDFs can be understood as the *probability of sovereign default at*

[14] We take the quarterly frequency time-series for real output per worker, real transfers and government purchase per worker between 2000-2014 from the National Bank of Slovakia Database.

Figure 3.2 : Fiscal Limits for the no-policy-change scenario



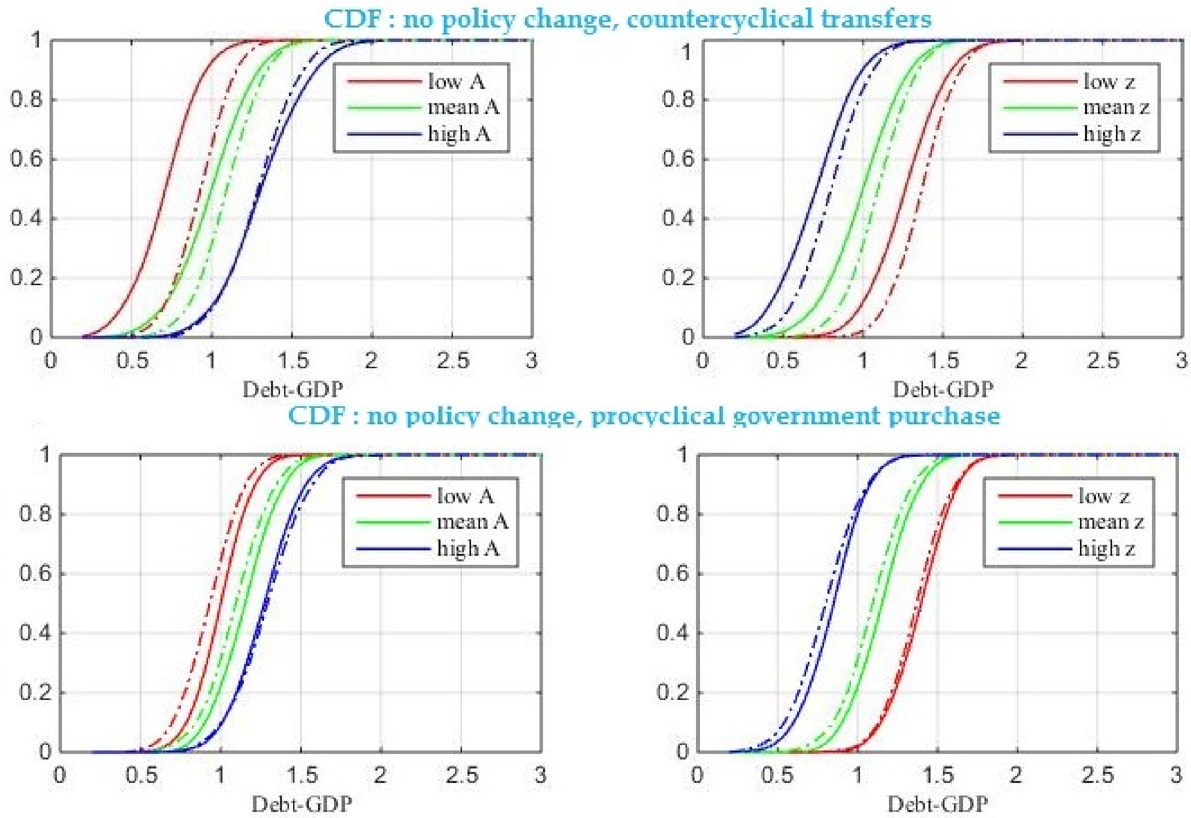
Distribution of the fiscal limits estimated for the no-policy-change scenario for various levels of technology, government purchase and transfers. The business cycle effect on fiscal variables is ignored. The left panels show the cumulative distribution functions while the right panels the corresponding probability distribution functions.

different debt levels. Furthermore, the top panels compare the state-dependent distributions at different productivity levels representing a sudden fall/increase in output by 7.64 percent of its steady-state level (which is $\pm 4\sigma_a$ as estimated from the empirical distribution), while keeping both the transfers and government purchases at their steady-state levels. The middle panels correspond to the comparison of the state-dependent distribution at various levels of the government purchase: the dashed lines illustrate the fiscal limit distribution in the extreme cases when government purchase is 9.32 percent (i.e $\pm 4\sigma_g$) below/above its steady-state level. Finally, the bottom panels in the Figure 3.2 depict the state-dependent distribution of the country fiscal limit for different levels of transfers (dashed lines illustrate the shifts in the distribution providing that transfers grow/decline by 11.08 percent, i.e. by $\pm 4\sigma_z$) under the assumption of technology and government purchase kept at their steady-state levels.

From Figure 3.2 it is evident that the impact of the productivity level on the fiscal limit distribution is significant while the effect of changes in the government purchase – though very volatile – is rather small. In case that the debt reaches 60 percent of output only 2 percent chance of default is associated with normal times. Furthermore, 7.64 percent reduction in the productivity level ($= -4\sigma_a$, depicted by dashed lines in the top two panels) shifts the distribution down by 8 percent of the steady-state output while the same drop in government purchase steady state level (equivalent to approx. 9.32 percent of the output) shifts the distribution by only 1 percent of the GDP. Furthermore, the increase in default probability is even more serious when the debt-to-GDP ratio approaches 100 percent of the output: it increases from 30 percent in the normal times to 65 percent in bad times. Likewise, the current level of transfers matters a lot – indeed, an 11.08 percent reduction in transfers causes a impressive fall in probability of country default when its debt reaches 100 percent of GDP, while the symmetric expansion raises dramatically the chance of default to 20 percent if the debt attains the Maastricht limit and hits 80 percent as debt approaches 100 percent of GDP. Moreover, notice that the profiles of non–symmetric density functions are essentially influenced by the initial state (technology, government purchase and transfers) – while probability distribution function is almost invariant in the level of transfers, it becomes more spiky and centred with decreasing level of technology. Finally, it must be emphasised that the distribution of the fiscal limit is rather heavy–tailed and dispersed even in the no policy change scenario where government decision about the spending policy is affected neither by the current phase of the business cycle nor by the raising growth rate of expenditures.

To fully understand the contribution of various model parameters to the fiscal limit distribution, we change or add only one parameter in each of the following experiments (see Table 3.1) while keeping other parameters the same as in the baseline case (dotted curves), and then plot the resulting distributions against the baseline case, as illustrated on Figure 3.3 or Figures A.1–A.6 in the Appendix A.

Figure 3.3 : Impact of macroeconomic fundamentals on fiscal limits distribution I.



Quantitative impact of macroeconomic fundamentals on fiscal limits distribution in case of Slovakia for various levels of technology and transfers. Upper panels illustrate the effect of counter-cyclical government transfers. Lower panels show the impact of pro-cyclical government purchase..

3.2.1 Pro-cyclical Government Purchases

The first scenario assumes procyclical government purchase with $\zeta_g = +0.0219$.[15] The impact of the sensitivity of the government purchase to the current phase of the business cycle on the distribution of the fiscal limit is relatively smaller than the influence of the transfers sensitivity (see Figure A.2 in Appendix A or plots on the bottom row of Figure 3.3). The reason is changes in the labour supply mitigate the impact. Therefore, the overall production and hence the tax revenues adjust and move accordingly the government purchase. Furthermore, the forward–looking households know that shocks to the level of government purchase are only temporary. Clearly, the distribution of the country fiscal limit becomes more concentrated and right–shifted, which is even more evident when the initial productivity level is low. This is because during bad times the government tends to reduce its expenses (public investment, infrastructure, subsidies, public wages, government consumption) to adapt to lower tax revenues and avoid excessive indebtedness.
On the other hand, in good times, increasing government purchase requires higher employment and hence, raising tax revenues help to fund these spending activities. Thus the shift and shape of the fiscal limit distribution is obvious. Therefore, while in case of bad times (fall in productivity by 7.64 percent) and 60 percent debt-to-GDP ratio the chance of country default drops by approx. 8 p.p., and even by 16 p.p. if the debt reaches 100 percent of GDP.

3.2.2 Counter-cyclical Transfers

If transfers are countercyclical as a result of discretionary countercyclical policy or large automatic stabilizer, $\zeta = -0.0159$, (middle figure in the upper row), the distribution becomes much more dispersed than in the baseline scenario where transfers do not react on current phase of the business cycle, and the distribution is extremely sensitive to the initial level of productivity (see Figure A.3 in Appendix A or the upper row plots on Figure 3.3).[16]

However, this can worsen the volatility of the fiscal limit since with low tax revenues and productivity in bad times the government has to supply more transfers to households which even more deepens country indebtedness. Truly, even thought in case of steady state technology level and 60 percent debt-to-GDP, the probability of default increase to 8 percent, an 7.64 percent fall in output leads to extreme increase in the probability of country default to nearly 35 percent. even if the current debt-to-GDP ratio is small (60 percent of the GDP).

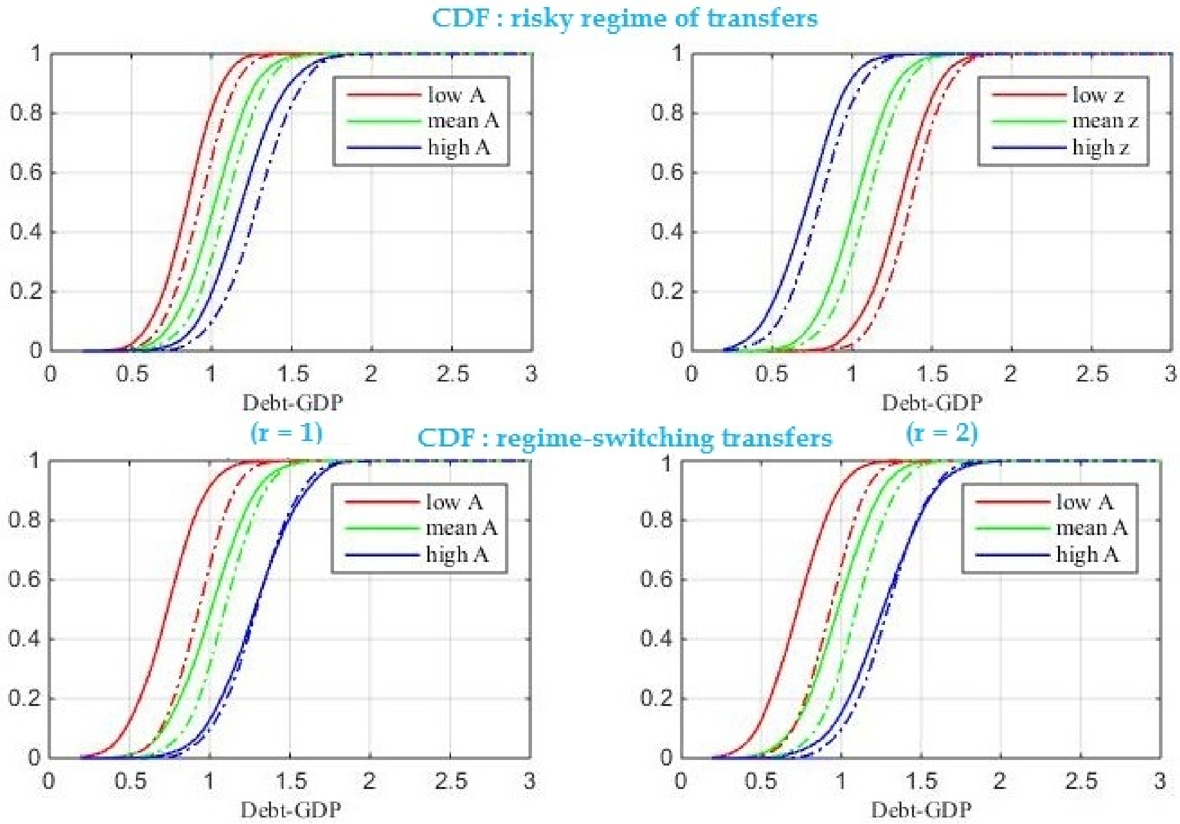
3.2.3 Transfers with Higher Rate of Growth

As another alternative, we consider the risky scenario (depicted by the red line on Figure 3.1) with highly increasing expenditures on healthcare implying increase in the transfers growth rate – the average rate of growth changes from 1.0026 in the no-policy-change scenario to 1.0032. More aggressive growth of transfers leads to higher chance of country default even with a relatively small debt as it projects considerably larger expected future expenditures – an approximately 8 p.p. increase in compare to the no policy change scenario (dashed lines) when debt attains 60 percent of GDP (see Figure A.4 in Appendix A or the upper plots on Figure 3.4).

[15] Equivalently, the elasticity of real detrended government purchase per worker w.r.t real detrended GDP per worker attains $0.0219y/g$.

[16] Equivalently, the elasticity of real detrended transfers per worker w.r.t real detrended GDP per worker attains $-0.0159y/z$.

Figure 3.4 : Impact of macroeconomic fundamentals on fiscal limits distribution II.



Quantitative impact of macroeconomic fundamentals on fiscal limits distribution in case of Slovakia for various levels of technology and transfers. Upper panels illustrate the effect of higher growth rate of transfers (risky regime). Lower panels show either the influence of regime-switching transfers providing that transfers are currently either in the NPC regime (left), or the risky regime (right).

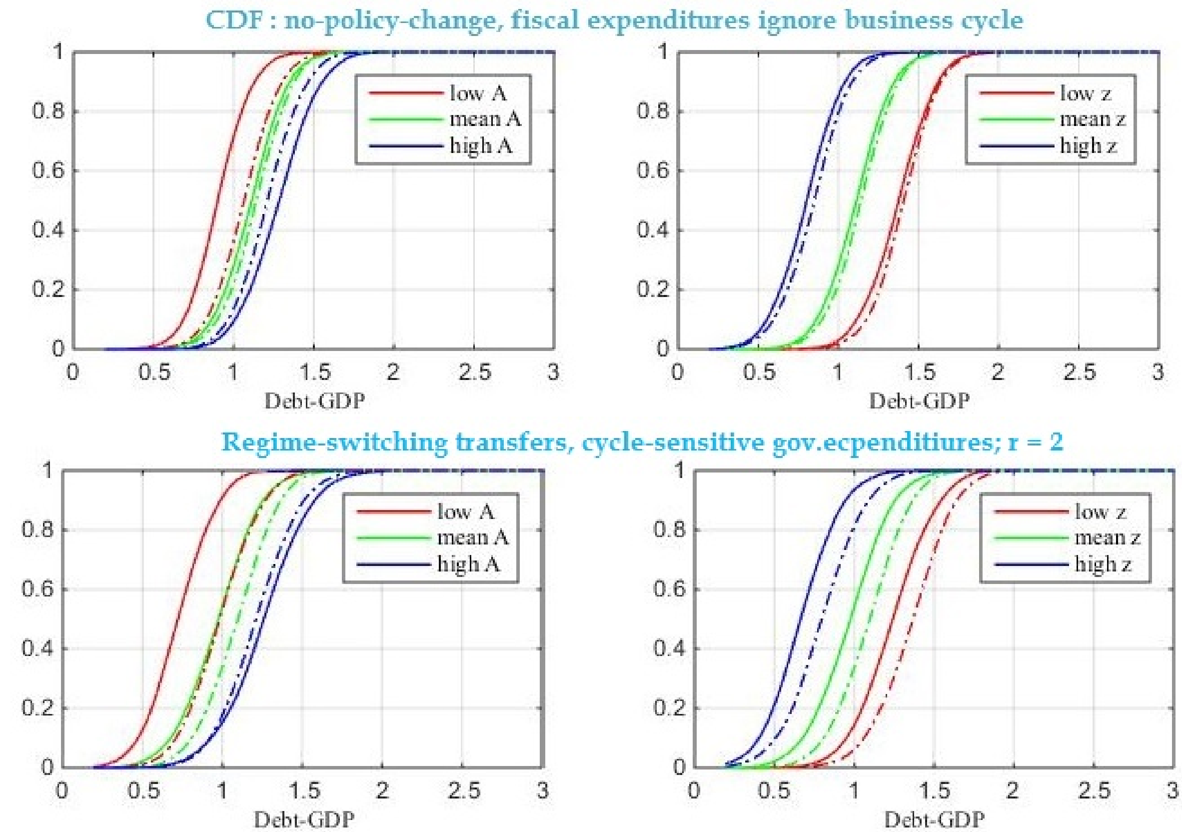
So, while 10 percent probability of default is associated with normal times and Maastricht debt limit, a sudden fall in productivity causes 16 percent chance of default on debt liabilities. Hence, running *bad policy in bad times* worsen significantly the fiscal outlook. Moreover, despite the low debt level, the situation becomes more dangerous if the current level of ageing–related expenses is high, since the probability of default augments steeply by 18 p.p. to almost 38 percent.

3.2.4 Regime–Switching Transfers

First, from lower plots on Figure 3.4it is evident that the left tail (the one closer to the lower bound of the distribution) is much fatter than in the baseline case with lower rate of growth. Although current transfers are in the no policy change regime with smaller growth rate a non–zero probability of highly explosive future transfers induces that future fiscal surplus might be essentially lower compared to the one in the baseline case. This restrains current the government's ability to service its future debt. The lower the probability of staying in the no policy change regime, the more dispersed and heavy–tailed the fiscal limit distribution is. This is even more evident when transfers are already in the highly explosive regime and the probability of keeping that behaviour is high .

Furthermore, since the regime of transfers reflecting the four–year political cycle is not stable, the initial choice of a particular regime in the Markov Chain Monte Carlo simulation does not play an important role in the fiscal limit distribution (Figure A.5). However, highly expansionary regime of transfers leads to a more dispersed distribution of the fiscal limit (see e.g. Figure A.5, A.6 in Appendix A or the lower right plot on Figure 3.3) and hence higher probability to default on debt liabilities even when the debt level is relatively small.

Figure 3.5 : **Comparison of the Fiscal Limit distribution for the heavy-tailed and normally distributed business cycle**



Fiscal Limit Distribution under two different assumptions on the business cycle distribution: dashed lines depict the distribution assuming that the technology shock is normally distributed ($\sigma_a = 0.0191$) whereas thick lines are associated with the heavy–tailed empirical distribution of the business cycle. The distributions are illustrated for various levels of technology and transfers. In both cases the distributions are estimated under the alternative scenario that considers transfers with natural social transfers, transition matrices reflecting the evolution of transfers and tax rates, and government expenditure items sensitive to business cycle. Red (blue) lines illustrate the fiscal limit distribution in the technology or transfers fall (increase) by 7.64 and 11.08 percent, respectively of GDP occurs.

3.2.5 Business Cycle Distribution

Next, it is important to discuss the effect of the heavy–tailed business cycle distribution on the Slovak fiscal limit. A larger probability of extreme negative situations in comparison with normally distributed shocks leads to higher chance of country default even it the debt is quite small, as demonstrated on Figure 3.5 (or Figures A.7–A.8 in the Appendix A for detailed information). Indeed, if business cycle has heavy tails and the economy suffers a 7.64 percent fall below its steady state, the default probability in the no policy change scenario and 60 percent debt-to-GDP is higher by 12 p.p. in compare to the country default probability obtained under the assumption of normally distributed technology shock – and this wedge doubles if transfers are in the explosive regime (and are regime-switching) and government expenditures (transfers, purchase) react to the current phase of the cycle. However, the default probabilities mostly coincide if country faces good times.

3.2.6 Shock Process

Finally, with higher persistence of technology process or business cycle volatility, the distributions become more heavy tailed and shift to the left with increasing shock volatility. This is obvious, since the effects on any shock in a highly persistent process last longer. The volatility implied by the prevailing random walk behaviour of technology process and its intrinsic heavy-tailed and left-skewed distribution are projected to more disperse and left-shifted distribution of the fiscal limit.

Box 3: Fiscal Limit : Summary

Thus, taking into account the different model extensions (explosive transfers even in the no policy change scenario, heavy-tailed left-skewed business cycle distribution, countercyclical transfers and procyclical government purchase), its calibration and the influence of model parameters on the fiscal limit distribution, it becomes clear that the well known 60 percent debt-to-GDP limit is not safe for Slovakia. Although in the equilibrium the chance of country default is 10 percent when the debt is 60 percent of GDP, it increases dramatically to approximately 40 percent in bad times (fall in the productivity by 7.64 percent). It also follows that a well–designed fiscal policy may change a lot and possibly offset the impact of negative externalities, since an 11 percent drop in the level of transfers reduces radically the current and expected future liabilities and thus causes a significant decline in the chance of default by almost 40 p.p. (30 p.p. in the NPC scenario) when the debt-to-GDP ratio reaches 100 percent.

4 The Default Risk Premium Model

The complete nonlinear model based on work of Bi and Leeper (2013) is constituted by the set of equations describing

- the households' optimal decisions about consumption and labour (17)–(16),
- the autoregressive shock processes for technology and government purchase (14)–(15),
- fiscal policy determining the level of transfers (18),
- the fiscal limit distribution (22) extended by the relationships introduced below :
  - country default scheme (25),
  - the Taylor–type rule for the tax rate (23) and the associated tax revenues (24),
  - the bond pricing rule (Euler equation) and the government budget constraint (the last two equations are combined into (26)).

Box 4: Nonlinear Model Equations

$$\tau_t = \tau + \gamma(b_{t-1}(1-\Delta_t) - b)\,, \tag{23}$$

$$\Theta_t = \tau_t \frac{a_t\,(1-\tau_t) + \phi g_t}{1+\phi-\tau_t}\,, \tag{24}$$

$$\Delta_t = \begin{cases} 0\,, & b_{t-1} < b_t^*\,, \quad b_t^* \sim \mathscr{B}^*(a_t, g_t, r_t) \\ \delta_t\,, & b_{t-1} \geq b_t^*\,, \quad \delta_t \sim \Omega\,, \end{cases} \tag{25}$$

$$\frac{(1-\Delta_t)b_{t-1} + g_t + z_t - \tau_t a_t h_t}{b_t} = \beta \mathbf{E}_t \left\{ [1-\Delta_{t+1}] \frac{c_t}{c_{t+1}} \right\}, \tag{26}$$

4.1 Method Description

In order to solve the nonlinear model numerically we employ the monotone mapping method introduced by Coleman (1991) and Davig (2004). Intuitively, the system of the optimal condition (14)–(18), (23)–(26) along with the fiscal limit distribution (19)–(22) is converted into the set of the first order difference equations which are solved iteratively. Given the fixed point in the state–space and the initial guess of the debt rule, the aim is to find the final debt rule at that point, $b_t = f^b(\psi_t)$ which is the end-of-period, pre–default debt, function of the current state.

Hence, in each step of the iteration we map the current state $\psi_t = (b_{t-1}, b_t^*, \delta_t, a_t, g_{t-1}, z_{t-1}, r_t)$ to obtain the updated guess of the debt rule $f^b(\psi_t)$ by solving the core equation of the model,

$$\frac{(1-\Delta_t)b_{t-1} + g_t(\psi_t) + z_t(\psi_t) - \tau_t a_t h_t(\psi_t)}{f_t^b} = \beta \mathbf{E}_t \left\{ [1-\Delta_{t+1}(\psi_{t+1})] \frac{c_t(\psi_t)}{c_{t+1}(\psi_{t+1})} \right\}, \tag{27}$$

with the expected state $\psi_{t+1} = (f^b(\psi), b_{t+1}^*, \delta_{t+1}, a_{t+1}, g_t, z_t, r_{t+1})$. The solution to the core equation above is determined numerically using the algorithm of Sims (1999)[17].

[17] Unfortunately, there is no guarantee that this algorithm is able to find the solution to (27) for every possible parametrization such that it lives within some reasonable boundaries. Therefore, the model is extended to include some built–in approximation techniques.

Pricing Rule and Default Risk Premium After obtaining the decision rules for each point in the discrete state–space, we employ the budget constraint to find the bond-pricing rule, $q_t = f^q(\psi_t)$ and the associated risky interest rate rule on government bond computed in terms of the current state, $R_t = f^R(\psi_t)$. On the other hand, the risk-free time–varying rate R_t^f is estimated using the similar approach with one difference – in this case we assume that the government never defaults. Therefore, we define the *default risk premium* r_t, as the difference between the risky and risk–free rates,

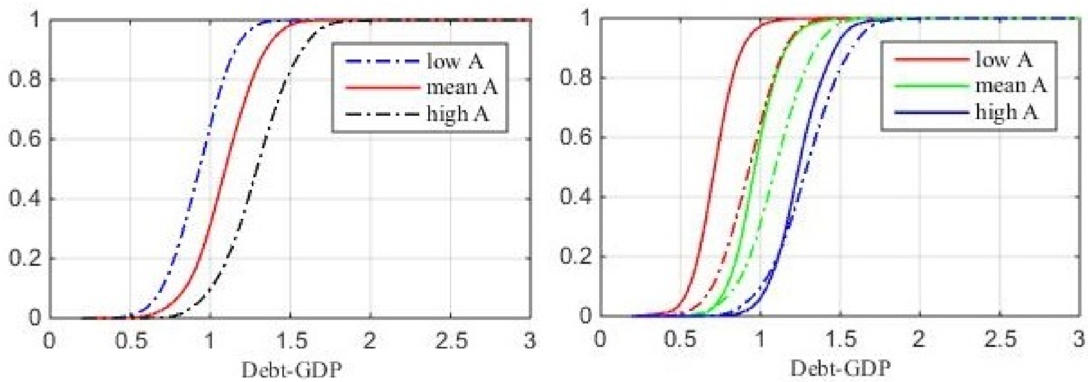
$$r_t = R_t - R_t^f = \frac{1}{q_t} - \frac{1}{q_t^{\Delta_t=0}}. \tag{28}$$

The technical details of the procedures for estimating the country default risk premium and the underlying debt rule are described in Appendix E.1 with the Matlab codes in Appendix E.2.

4.2 Model Calibration

The debt limit is drawn from the fiscal limit distribution derived following the procedure and model calibration established in Sections 2.3.2–3.1. The tax rate adjustment parameter is estimated[18] to 0.0724 representing the fact that the government raises tax rate by 1 point whenever the debt changes by 7.24 percent of the output.

Figure 4.1 : Underlying fiscal limit distributions



Distribution of the fiscal limit for various technology levels under the no-policy-change scenario (left) and regime-switching scenario with business cycle sensitive government expenditures (right).

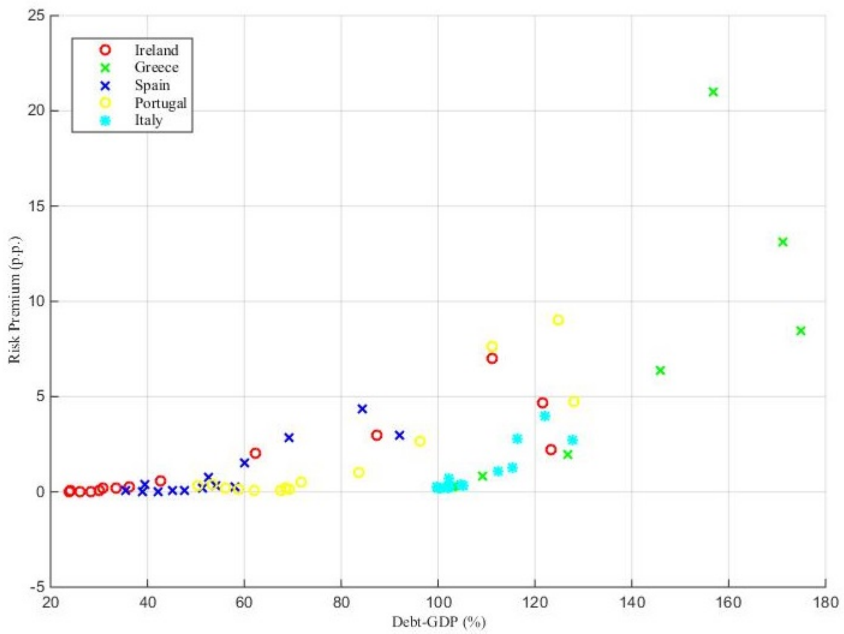
Figure 4.3 illustrates country's default risk premium charged by the investors above the default–free rate. The dependence of the interest rate and default risk premium on country debt-to-GDP ratio is obtained applying the method described in Section 4.1 (for details see the procedure in Appendix E.1) using the distribution of the fiscal limit for the baseline setting under no-policy-change scenario (see the left plot on Figure 4.1).

From Figure 4.3 (and Figure A.9 in Appendix A) it is evident that the net interest rate rises with the current debt in a non-linear way. This is in consistence with the reality: Figure 4.2 depicts the relationship between the sovereign debts and long–term risk premia for highly indebted Euro Area countries between 2000-2013. The evolution of the risk premium follows the distribution – being low and flat for the *safe* debt levels and sharply increases as debt approaches the fiscal

[18] Based on Eurostat data for Slovakia between 2000–2014 (see Figure B.2 in the Appendix B) the OLS procedure to estimate the tax rate adjustment parameter γ.

limit. The higher the current government liability, the more debt it has to issue for next period, and the more likely sovereign default will occur. Hence, the snowball effect makes the country's debt position even worse. The default risk premium attains its maximum even below the fiscal limit since forward–looking investors expected default already at lower levels of debt. [19]

Figure 4.2 : Sovereign debts and Risk Premia



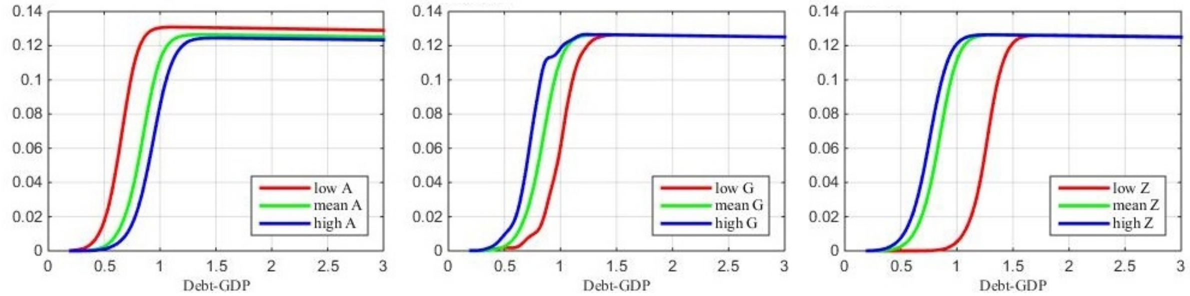
The impact of sovereign debt on risk premium for highly indebted Euro Area countries between 2000-2013. Country risk premium is calculated as the difference between the country's and German convergence criterion bond yields.

Source: *Eurostat*

Based on the fiscal limit distribution estimated for the no-policy-change scenario and government expenditures ignoring the business cycle (see the corresponding probability and cumulative distribution functions for this baseline setting on Figure 3.2 in Section 3 and Figure A.1 in Appendix A) we derive them for various levels of productivity, government purchase and transfers. On the left plot of Figure 4.3 green lines depict the dependence of the default risk premium on the current debt-to-GDP ratio assuming the steady state productivity; while the red lines (*bad times*) are associated with a sudden fall in productivity by 7.64 percent ($4\sigma_a$) and blue lines (*good times*) correspond to output higher by 7.64 percent. Likewise, the middle plot illustrates the dependence of the default risk premium on the current debt-to-GDP ratio assuming either the steady state (green line), or higher (blue line, increase by 9.32 percent from the steady state value), or lower (red line, decline by 9.32 percent from the steady state value) level of government purchase. Finally, the right plot on Figure 4.3 depicts the relationship between the default risk premium on the current debt-to-GDP ratio for various levels of transfers: high (11.08 percent above its steady state value), medium, and low (11.08 percent below its steady state level). The associated net net interest rate are illustrated on Figure A.9 in the Appendix A.

[19] If the debt hits the fiscal limit the default rate reaches its maximum (the default rate remains constant even if the debt increases above the fiscal limit) and the sovereign debt default occurs. Then, investors expecting lower tax rate levied by the government based on the post-default debt liability in the next period will increase their consumption, so the expected future marginal utility of consumption drops. Therefore, the interest rate slightly drops (or does not grow) whenever the debt surpasses the fiscal limit.

Figure 4.3 : Default risk premium for the no-policy-change scenario



Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers, under no-policy-change scenario with government expenditures (purchase, transfers) not reflecting the business cycle.

The productivity level has a substantial influence on the risk-free interest rate due to the intertemporal substitution effect (see Figure 4.3 above or Figure A.9 in Appendix A). The lower the productivity level, the higher the interest rate. In addition, a lower productivity level shifts down the state-dependent distribution and, therefore, raise the sovereign default probability at a given level of government debt. The effects of lower productivity and bad exogenous shocks on the default risk premium are much worse than the positive externalities inasmuch as investors price the debt taking into account the non-symmetric heavy tailed empiric distribution of the business cycle. During normal times, forward–looking investors charge no more than 1 p.p. default risk premium unless the debt remains below the Maastricht level associated with only a minor chance of country default (see the left plot on Figure 4.1). However, by taking into account that the default probability increases steeply (see Figure 4.1), to protect themselves against the default they ask for high premium even for the debt when the debt considerably lower than the country fiscal limit. Thus, assuming the steady state levels of technology and government expenditures, the risk premium culminates at 12.75 p.p. when the debt hits 110 percent of the production (the underlying fiscal limit is attained for 160 percent debt-to-GDP). Furthermore when country suffers a large sudden fall in the productivity, investors charge 4 p.p. premium if the debt-to-GDP is 60 percent and even 13 p.p. when it increase to 100 percent of the output.

The impact of different levels of government purchases on the interest rates is substantially smaller, which contrasts with the impact of various technology levels and transfers regime with different transfer regime or with different productivity level. This small impact of government purchase level on risk premium is due to short–lasting government purchase shocks (in compare to the long–term expectation of the transfers regime) and even though higher level of government purchases increase the debt, this is partially offset since the wealth effect of increased spending motivates households to work more. Higher production then leads to higher tax revenues helping to finance these purchases.

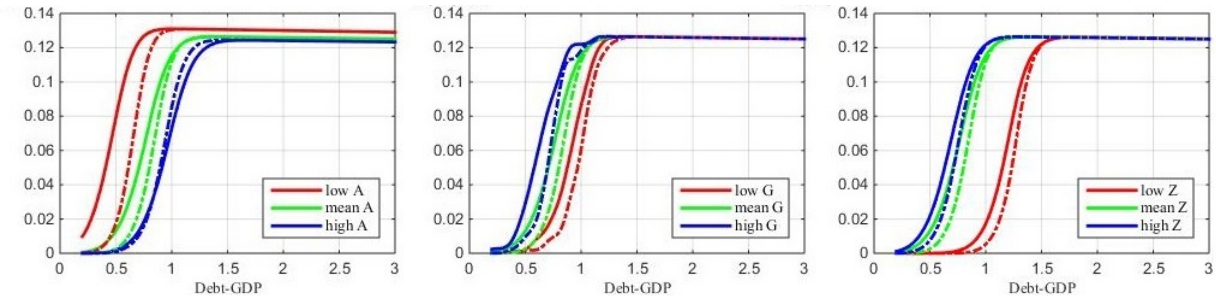
The current level of transfers significantly influences the interest rate and the associated default risk premium charged by lenders on government debt. High level of exponentially growing transfers (even in the no-policy-change scenario) and positive shocks have long–lasting effects and increase essentially expected future liabilities in a nonlinear way especially when the debt is high. On the other hand, transfers reduction may help a lot. Indeed, a large cut in the level of transfers reduces significantly the default risk premium, which becomes very low even for 100 percent debt-to-GDP (below 1 p.p.) and attains the high value only when it hits the fiscal limit.

4.3 Sensitivity Analysis of the Risk Premium

To understand how various model parameters affect the country default risk premium we again modify one parameter at a time, while keeping all other parameters the same as in the baseline case.

Specifically, in the baseline, *no–policy–change* scenario we assume that transfers reside in the first regime only ($p^{(1)} = 1$, $p^{(2)} = 0$). We ignore the effect of business cycle on fiscal variables. Next, we make fiscal expenditure items (either government purchase or transfers) sensitive to the business cycle. Then we study the case of permanently higher growth rate of transfers due to expected higher healthcare expenditures (see Figure 3.1), i.e. the risky scenario ($p^{(1)} = 0$, $p^{(2)} = 1$). Finally, reflecting four–year political cycle ($p^{(1)} = p^{(2)} = 0.75$) we let transfers to switch between two regimes with different growth rates of transfers. For details see the Table 3.1 in Section 3.

Figure 4.4 : Default risk premium for the no-policy-change scenario with countercyclical transfers



Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers, under no-policy-change scenario. Transfers are countercyclical and government purchase do not reflect the business cycle. Dashed lines correspond to the no-policy-change scenario with government expenditures (purchase, transfers) not reflecting the business cycle.

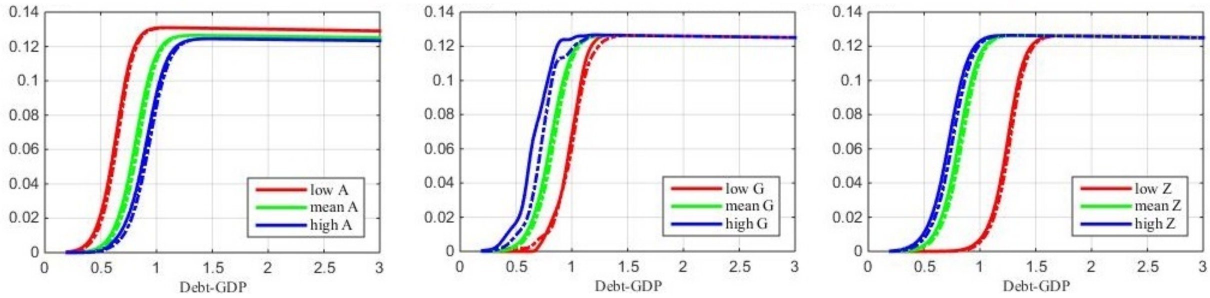
4.3.1 Government Expenditures and Business Cycle

The impact of countercyclical transfers on the default risk premium and the net interest rate is enormous (see Figure 4.4 above or Figure A.11 in the Appendix A). During bad times the government raises transfers and hence they explode much faster even in the no-policy-change scenario. These changes persist and contribute to debt increase acceleration. Therefore, forward–looking investors protect themselves (also in normal times taking into account the countercyclical fiscal policy as another source of indebtedness) against country default by asking much higher interest especially when the debt is quite small.

Assuming that the productivity is on its steady state level and the debt-to-GDP is 60 percent, the government is charged a 4 p.p. premium (the same as in the baseline scenario and bad times). Furthermore, when a sudden fall in productivity by 7.64 percent occurs, the 'safe' 60 percent debt-to-GDP is penalised by 12 p.p. premium (three times more than in case of transfers ignoring the business cycle).

On the other hand, the effect of procyclical government purchase on default risk premium is negligible with one exception (see Figure 4.5 above or Figure A.10 in the Appendix A): during bad times government cuts its purchase and hence improves the primary balance and lowers the debt. There are two reasons why the default risk premium is not significantly affected by

Figure 4.5 : Default risk premium for the no-policy-change scenario with procyclical government purchase



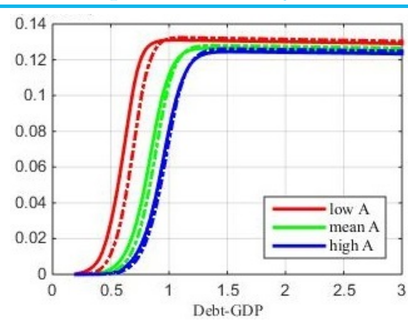
Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers, under no-policy-change scenario. Government purchases are pro-cyclical but transfers do not reflect the business cycle. Dashed lines correspond to the no-policy-change scenario with government expenditures (purchase, transfers) not reflecting the business cycle.

procyclical government purchase: low sensitivity on the fiscal limit distribution on the government purchase pro-cyclical behaviour, and the wealth effect lowering the labour supply and consumption, subsequently.

4.3.2 Business Cycle Distribution

Moreover, it is necessary to discuss the effect of the heavy–tailed business cycle distribution on the default risk premium. Simply, the default risk premium is higher if the business cycle is left-skewed heavy-tailed distributed in compare to normally distributed shocks (see Figure 4.6 in the Appendix A). The reason is obvious: investors take into account that extreme negative situations causing a sharp decline in government revenues and hence increasing the debt are not rare, so they ask for higher premium. Furthermore, the default probability is much higher than in case of normally distributed technology shocks.

Figure 4.6 : Comparison of the default risk premia for heavy-tailed and normally distributed business cycle



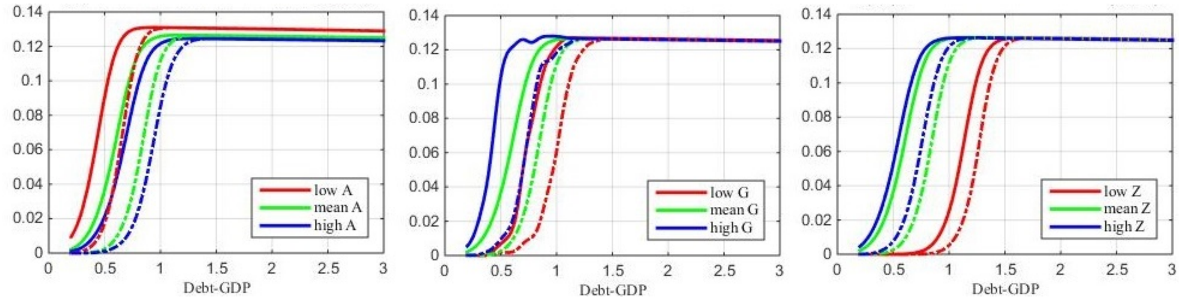
Default risk premium for Slovakia estimated for various levels of productivity under no-policy-change scenario. Notice that the business cycle is left-skewed heavy-tailed distributed. Dashed lines correspond to the no-policy-change scenario, but with normally distributed technology shocks.

If the productivity shock follow normal distribution, during normal times investors penalize country by only 0.15 p.p. (0.25 p.p.) risk premium when the debt hits 60 percent (100 percent) of the output and even during bad times the premium does not go above 2 p.p. providing that the debt is kept below 100 percent of the production.

4.3.3 Transfers with Higher Rate of Growth

The influence of the growth rate of transfers on the default risk premium becomes even more evident when their rate of growth is higher (as depicted on Figure 4.7, or Figure A.12 in the Appendix A), or the regime of transfers switches between the no-policy-change and the risky scenarios. If the current transfers are in the risky regime (characterized by higher rate of their growth), the interest rate starts to rise at a much lower level of debt than under no-policy-change like transfers.

Figure 4.7 : Default risk premium for the risky scenario



Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers, under risky scenario (transfers grow faster due to higher healthcare expenditures) with government expenditures not reflecting the business cycle. Dashed lines correspond to the no-policy-change scenario.

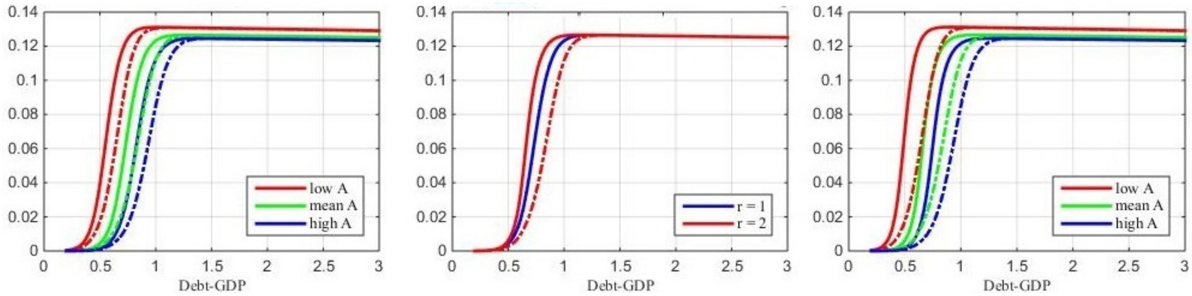
The explosive character of transfers causes that a significant increase in the risk premium is even more evident when country faces a strong negative productivity shock: though in normal times the risk premium increases by 5 p.p. in compare to the no-policy-change case, during bad times investors charge more than 12.5 p.p. premium when debt reaches 60 percent of the GDP (in compare to 1 p.p., and 4 p.p, respectively in the no-policy-change case) as can be deduced from Figure 4.7. Such behaviour is obvious, since the intertemporal substitution effect is emphasized by the strongly heavy-tailed fiscal limit distribution shifted down not only by the low productivity level but even more by rapidly growing transfers that generate exponentially higher expected future liabilities.
On the other hand, the risk premium remain very low even for high debt (less than 2 p.p. for the debt-to-GDP below 100 percent) providing that the level of transfers is low. (see Figure 4.7 above or Figure A.12 in the Appendix A).

4.3.4 Regime-switching transfers

If the growth rate of explosive transfers is allowed to switch randomly between small (no-policy-change case) and moderate (risky scenario) following a Markov Chain Monte Carlo process, the default risk premium is considerably higher than in the no-policy-change scenario.

Although current transfers in the no-policy-change regime grow at a smaller rate, forward–looking investors ask for additional premium since a non–zero probability of highly explosive future transfers restrains the current government's ability to service its future debt as future fiscal surplus might be essentially lower compared with those in the baseline case. The lower the probability of staying in the no policy change regime, the more risk premium is charged. This is even more evident when transfers are already in the risky regime and the probability of staying in that regime is high.

Figure 4.8 : Default risk premium for the regime-switching scenario



Default risk premium for Slovakia estimated for various levels of productivity under regime-switching scenario with government expenditures not reflecting the business cycle. On the left plot the risk premium is obtained assuming that transfers are currently in the no-policy-change regime (lower rate of growth), while the right one is estimated if we start with the risky regime of transfers (higher rate of growth). The middle plot illustrates the difference between these two premiums. Dashed lines correspond to the no-policy-change scenario.

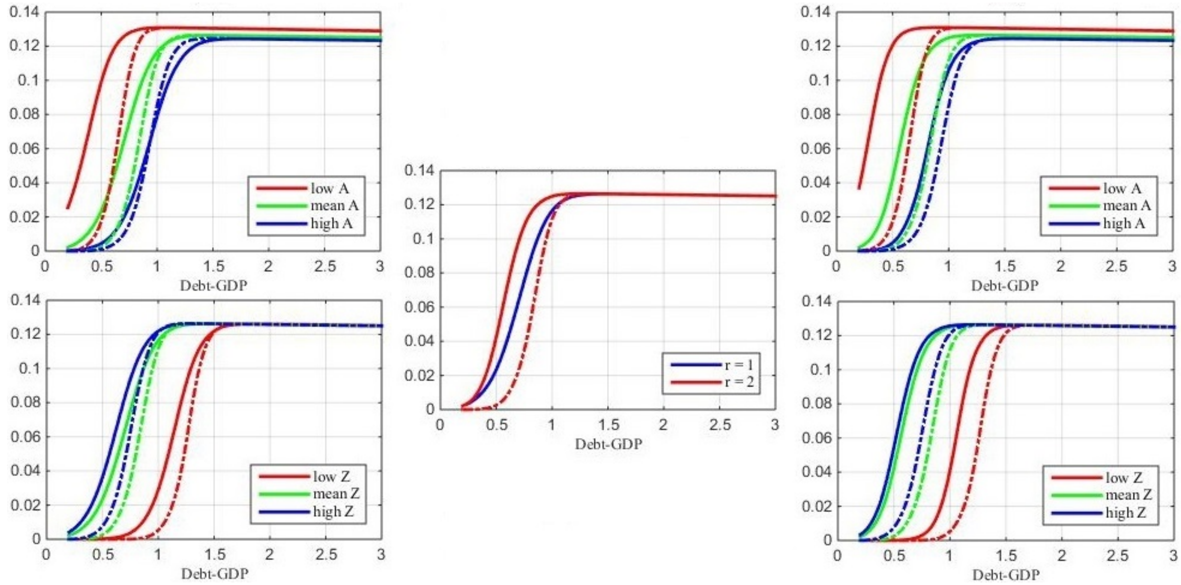
Therefore regardless the state of economy (levels of productivity, transfers and government purchase) a considerably higher interest is paid when the debt is below the fiscal limit. Moreover, due to the strongly (left) heavy-tailed business cycle distribution, whenever a large sudden fall occurs the risk premium persist above the one associated with steady state productivity even when the debt is above the fiscal limit. However, highly expansionary regime of transfers leads to higher interest rate and the associated risk premium even when the debt is rather small (60 percent): although being in the no-policy-change regime of transfers is associated with 2 p.p. premium, quickly growing transfers are charged with 3.5 p.p. default premium in case of steady state productivity. Obviously, during bad times the increase in risk premium is significantly higher especially when transfers are in the risky regime.

4.3.5 Regime-Switching Policy with Cyclically-Sensitive Expenditures

From Figure 4.9 it is evident that in case of the heavy–tailed business cycle, procyclical stationary government purchase and countercyclical transfers with random-switching rate of growth, investors ask for a risk premium whenever the country is indebted. The premium is quite dispersed, rises sharply and culminates for debt levels significantly lower than the underlying fiscal limit depicted on Figure 4.1.

First, observe that the default risk premium is very sensitive to the current regime of transfers: if they grow according to the no-policy-change scenario (lower growth rate), the Maastricht debt is penalised by 4 p.p. premium (four times more than in the no-policy-change case with only one regime of transfers and government expenditures ignoring the business cycle) and even 8 p.p. when transfers currently follow the risky scenario of growth (see the middle plot on Figure 4.9). Next, the business cycle phase is very important as transfers (always explosive) are countercyclical. Therefore, when a large sudden fall in productivity occurs, the risk premium rises sharply to 12 p.p. (13 p.p.) when transfers grow slowly (quickly) as illustrated on upper plots on Figure 4.9. Furthermore, although high level of transfers is charged by an additional 1.5-2 p.p. risk premium above the one associated with the steady state level of transfers, a significant reduction of transfers can help a lot with debt financing as for 60 percent debt-to-GDP it cuts the interest payments to almost the default-free costs.

Figure 4.9 : Default risk premium for regime-switching scenario & cycle-sensitive fiscal expenses



Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers, with regime-switching countercyclical transfers (higher vs. lower rate of growth) and stationary procyclical government purchases. Left panels correspond to risk premia estimated for transfers currently in the no-policy-change regime, while the right panels are associated with risky regime of transfers. Dashed lines correspond to the no-policy-change scenario with default setting (government expenditures ignore the business cycle).

Box 5: Default Risk Premium : Summary

Taking into account the model features (explosive transfers even in the no policy change scenario, heavy-tailed left-skewed business cycle distribution, countercyclical transfers and procyclical government purchases), its calibration and the influence of model parameters on the fiscal limit distribution, it is evident that the well known 60 percent debt-to-GDP limit associated with 10-20 percent chance to default may bring 4-8 p.p. default premium charged above the default–free rate during normal times. The heavy–tailed character of the distribution represents another source of uncertainty that investors can directly project into higher default risk premium. A sudden large fall of productivity by 7.64 percent implies 12-13 p.p. premium. On the other hand, a well–designed fiscal policy may change a lot. A drop by 11 percent in the level of transfers reduces radically current and expected future liabilities and thus causes a significant decline in the default risk premium which falls to less than 0.5 p.p. for the Maastricht debt level.

5 Concluding Remarks

We built a simple general equilibrium framework that is able to capture important aspects of the Slovak economic and fiscal policy environment such as heavy-tailed distribution of cyclical conditions, significant growth in demography-related spending, pro- and counter-cyclicality of government spending and transfers, respectively.

We used this simple model to provide an estimate of the fiscal limit - the maximum serviceable level of public debt and a model-based estimate of the associated risk premium demanded by households aware of the fact that the government might default on its obligations. We have shown that both are very sensitive to the rate of growth of transfers. Controlling age-related spending is thus a key task if the government wants to avoid financing difficulties. We have also demonstrated that the nature of economic conditions makes the distribution of the fiscal limit heavy-tailed, and – as a consequence – apparently safe levels (and legislated limits) of public debt might not be safe in reality.

The model we use can be developed further. In particular, there is currently no feedback loop between the fiscal limit and the risk premium. Introducing such an interaction would likely alter the shape of both the fiscal limit distribution as well as the levels of fiscal premia in a way that would make the current estimates look optimistic. A richer model could also provide a better account of the consequences of the cyclical nature of spending items. It may also be interesting to formally account for the openness of the economy. We leave these extensions on our future research agenda. However, the key policy messages arising from this paper should continue to hold if not with a greater force.

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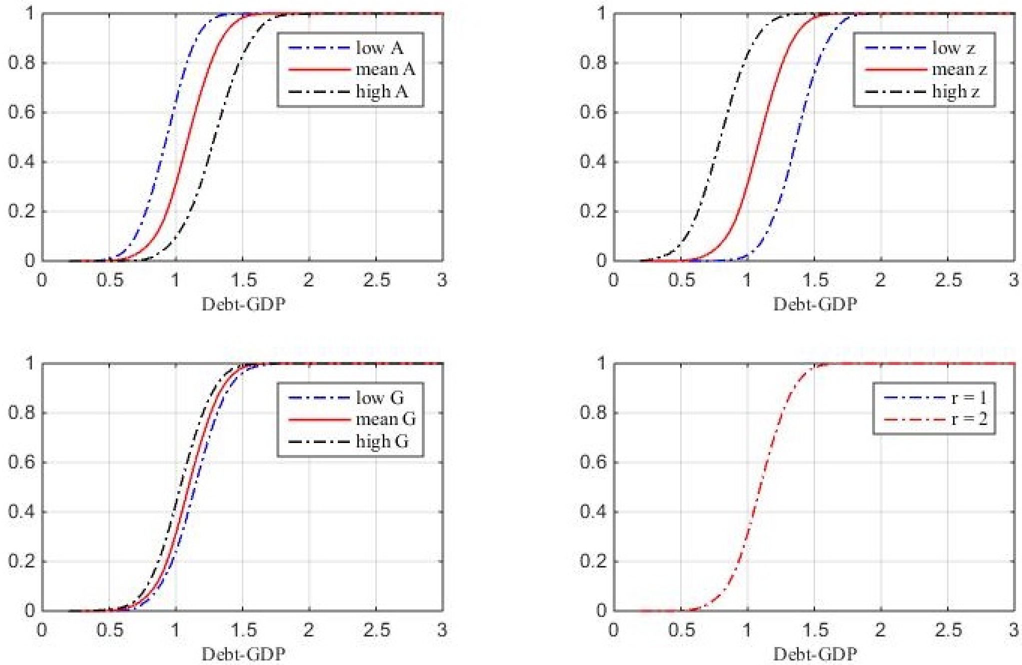
Appendix A Graphical Results

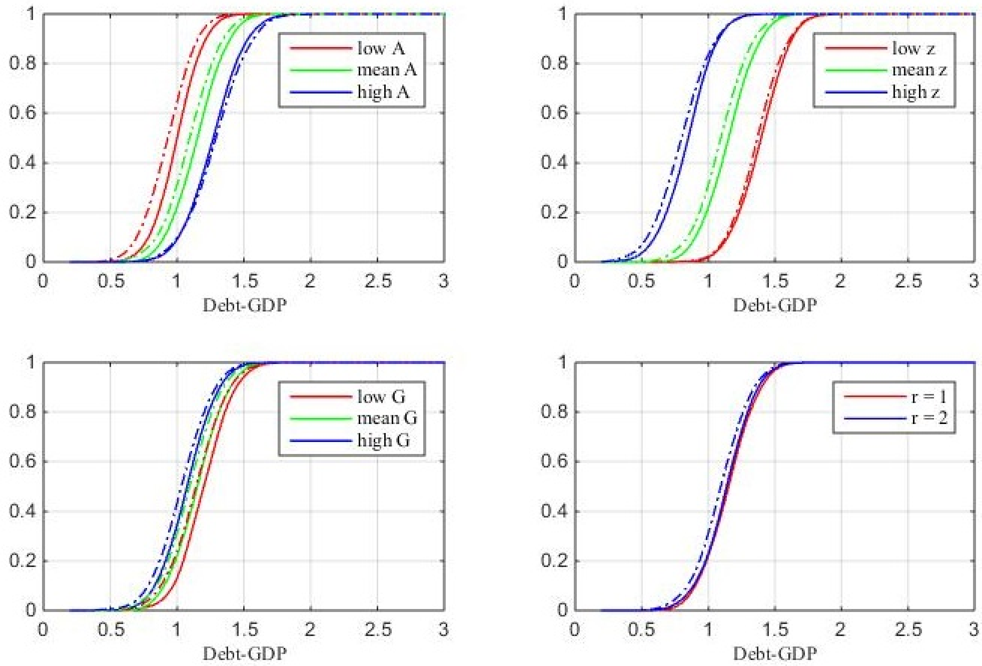
Appendix A.1 Fiscal Limit Distribution

Figure A.1 : Fiscal Limit Distribution : no policy change scenario & fiscal variables ignoring business cycle.



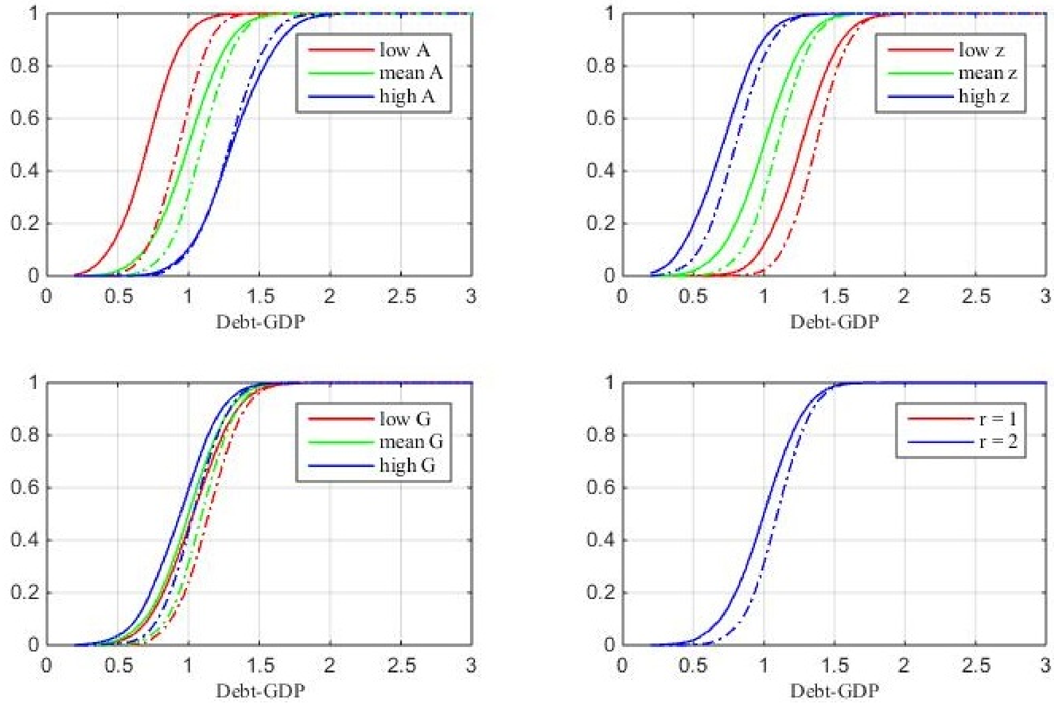
Distribution of the fiscal limits estimated for the no-policy-change scenario for various levels of technology (upper left), government purchase (lower left) and transfers (upper right). The business cycle effect on fiscal variables is ignored. Dashed blue lines represent the distribution assuming that either productivity, or gov.purchase, or transfers are above their equilibrium levels while dashed black lines illustrate the fiscal limit distribution assuming that they are below equilibrium.

Figure A.2 : Fiscal Limit Distribution : no policy change scenario & pro-cyclical government purchase



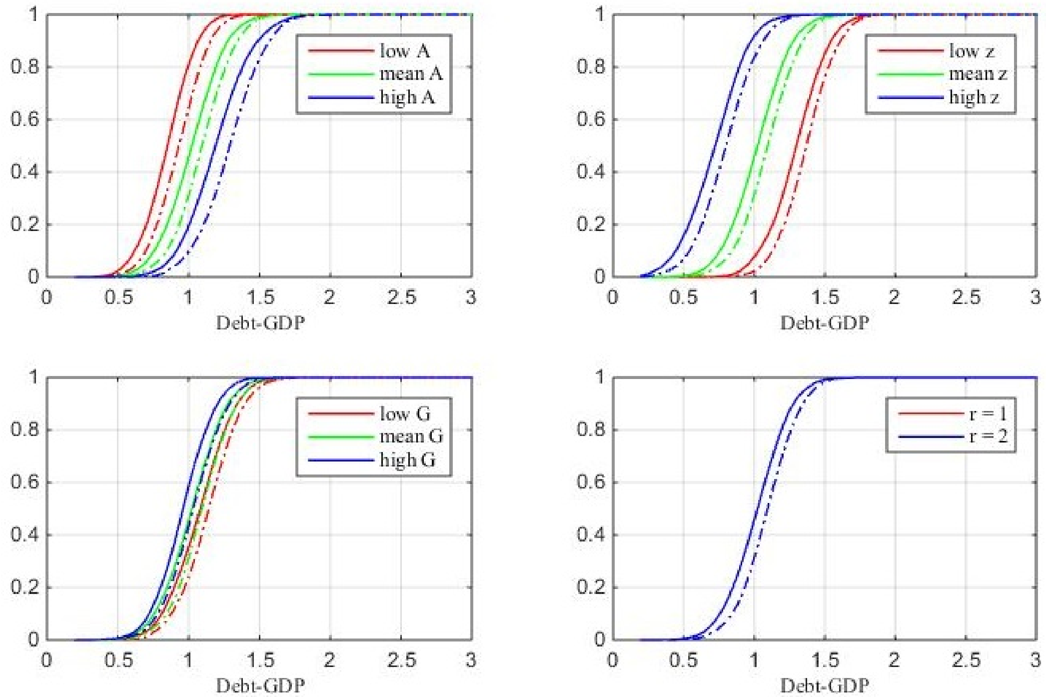
Fiscal Limit Distribution for Slovakia: no policy change scenario with pro-cyclical government purchase. Transfers ignore the business cycle. Dashed lines correspond to the baseline NPC scenario with all features switched off.

Figure A.3 : Fiscal Limit Distribution : no policy change scenario & counter-cyclical transfers



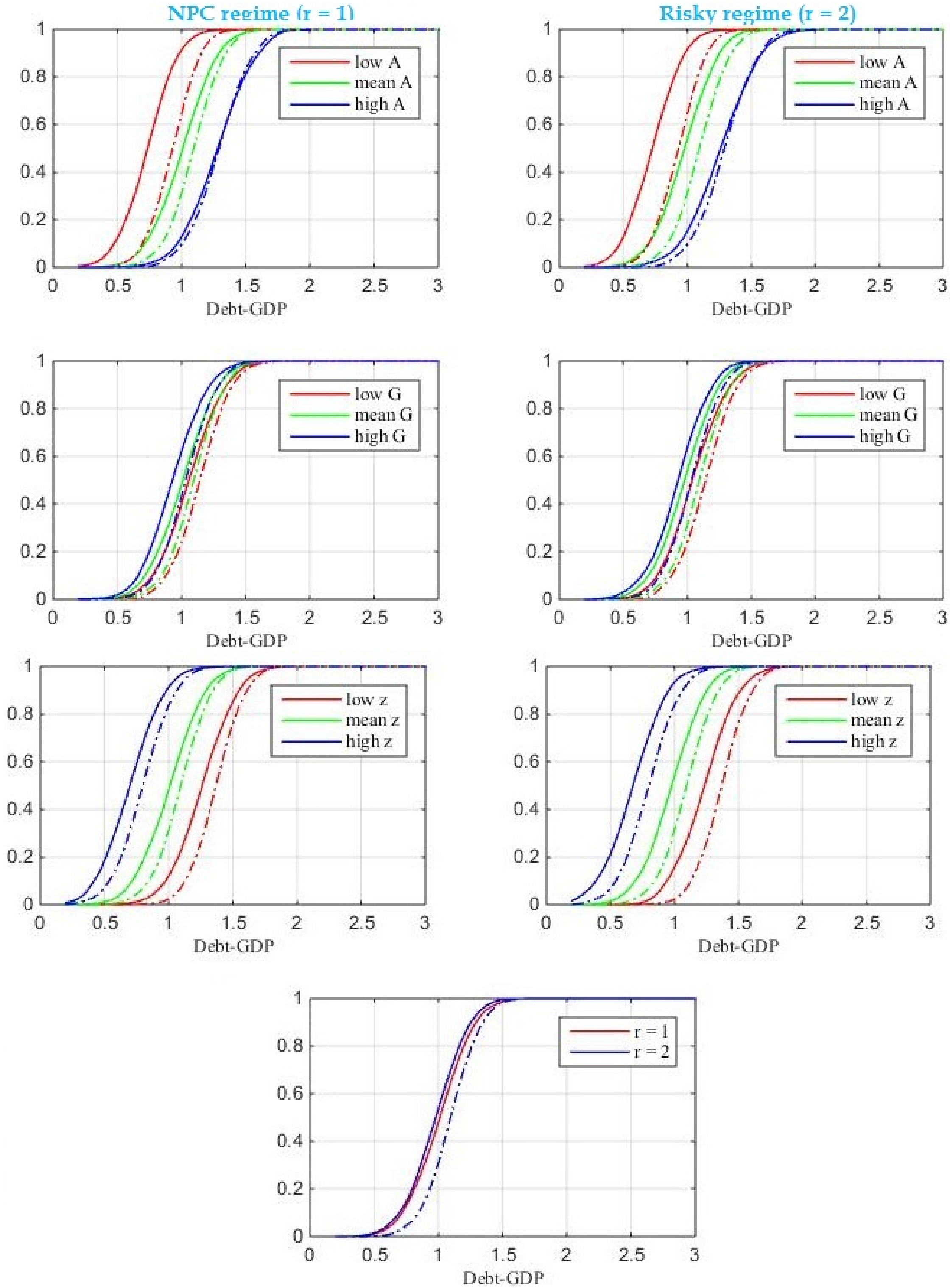
Fiscal Limit Distribution for Slovakia: no policy change scenario with countercyclical transfers. Government purchase ignores the business cycle. Dashed lines correspond to the baseline NPC scenario with all features switched off.

Figure A.4 : Fiscal Limit Distribution : risky scenario (higher growth rate of transfers)



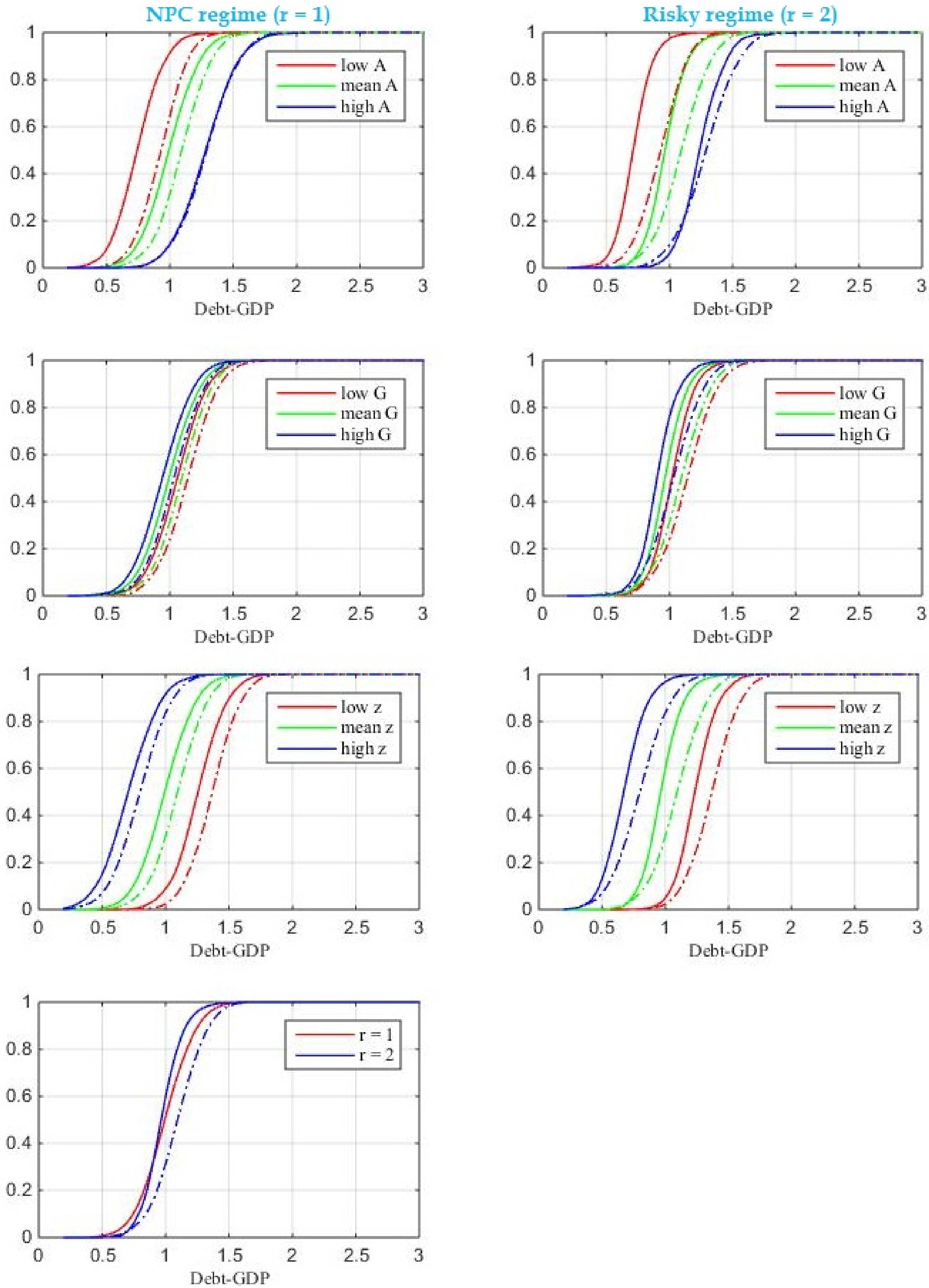
Fiscal Limit Distribution for Slovakia: alternative scenario with higher healthcare expenditures. Fiscal expenses ignore the business cycle. Dashed lines correspond to the baseline NPC scenario with all features switched off.

Figure A.5 : Fiscal Limit Distribution : regime-switching transfers



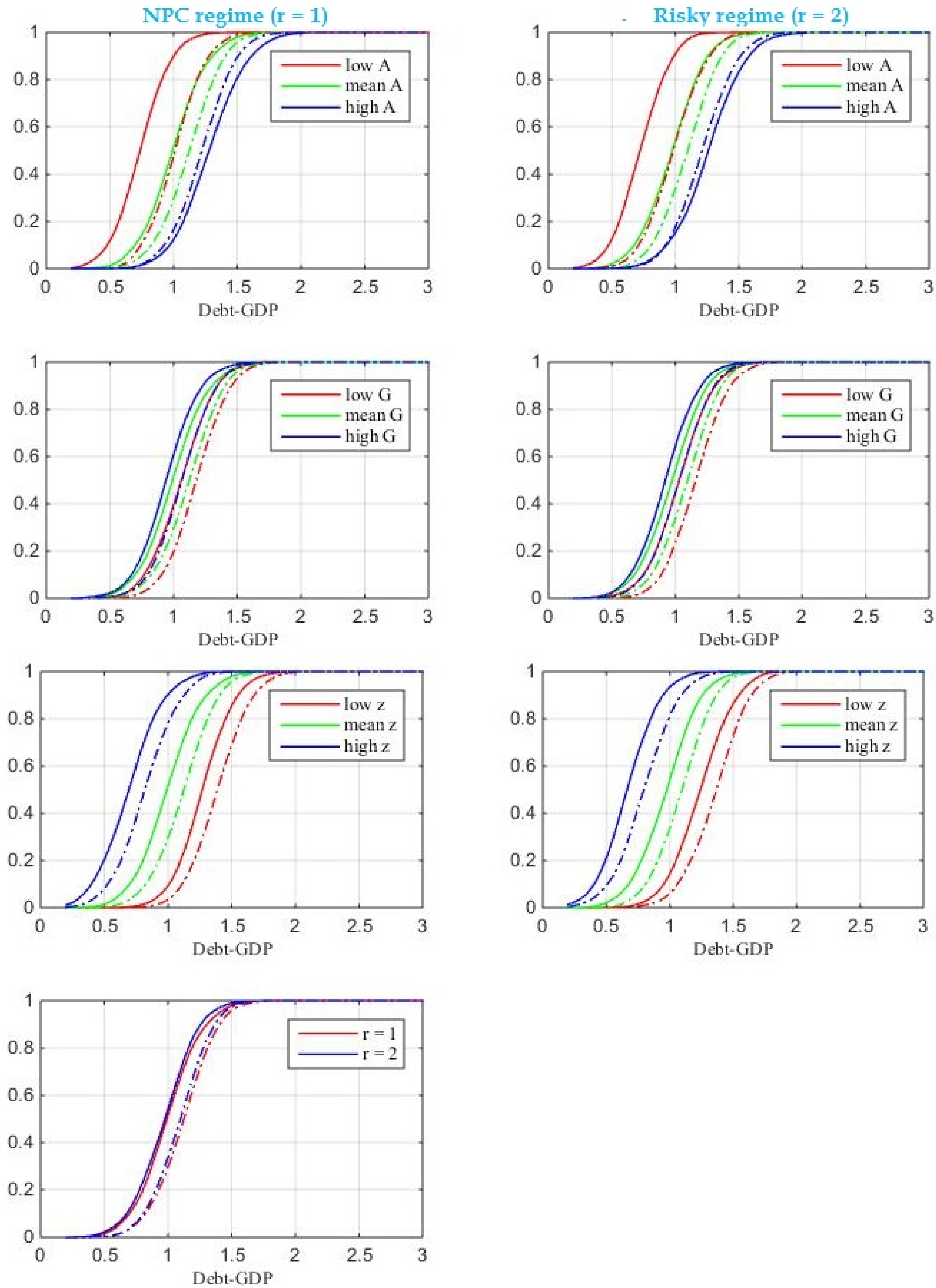
Fiscal Limit Distribution for Slovakia: transfers switch between no policy change scenario (lower rate of growth) and risky scenario (higher rate of growth) accordingly to the transition matrix. Fiscal expenses ignore the business cycle. Left panels illustrate the fiscal limit distribution providing that transfers are initially in the NPC regime while the right panels depict the distribution assuming the initially risky regime of transfers. Dashed lines correspond to the baseline NPC scenario with all features switched off.

Figure A.6 : Fiscal Limit Distribution : regime-switching transfers, all features switched on



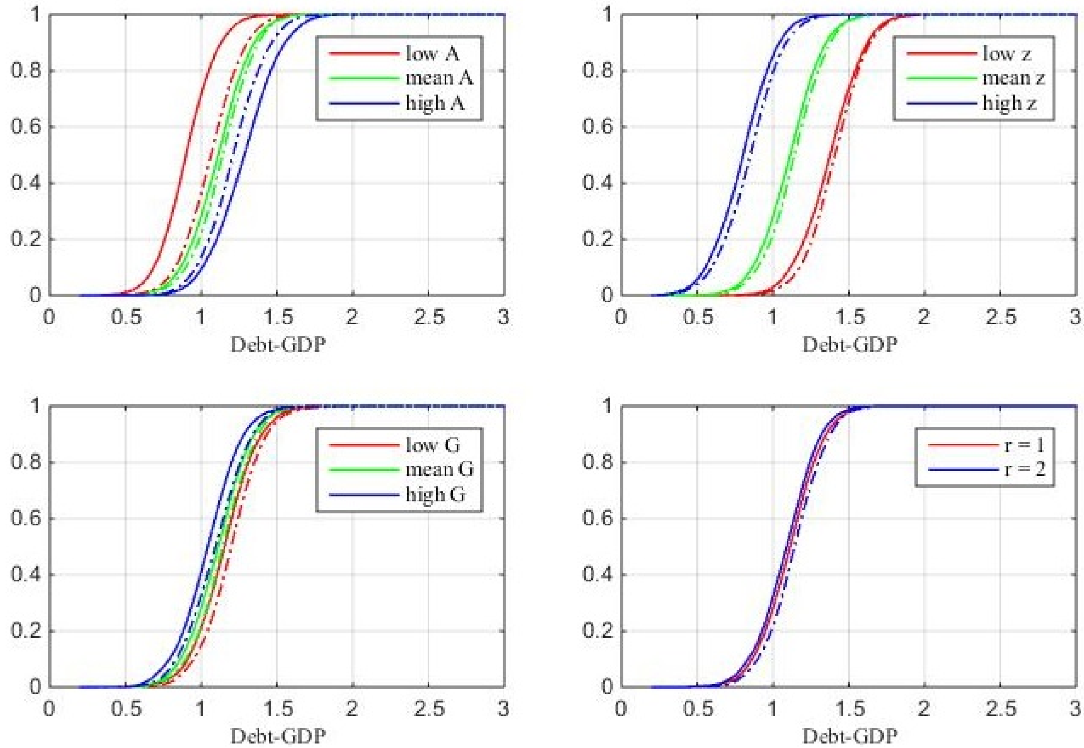
Fiscal Limit Distribution for Slovakia:transfers switch between no policy change scenario (lower rate of growth) and risky scenario (higher rate of growth) accordingly to the transition matrix. Furthermore, transfers are countercyclical and government purchase pro-cyclical. Left panels illustrate the fiscal limit distribution providing that transfers are initially in the NPC regime while the right panels depict the distribution assuming the initially risky regime of transfers. Dashed lines correspond to the baseline NPC scenario with all features switched off.

Figure A.7 : Fiscal Limit Distribution : heavy-tailed vs. normally distributed business cycle



Fiscal Limit Distribution for Slovakia: comparison of heavy-tailed distributed (bold line) and normally distributed business cycle (dashed line) for the regime switching scenario (NPC vs. risky regime of transfers switching accordingly to the transition matrix) with countercyclical transfers and pro-cyclical government purchase.

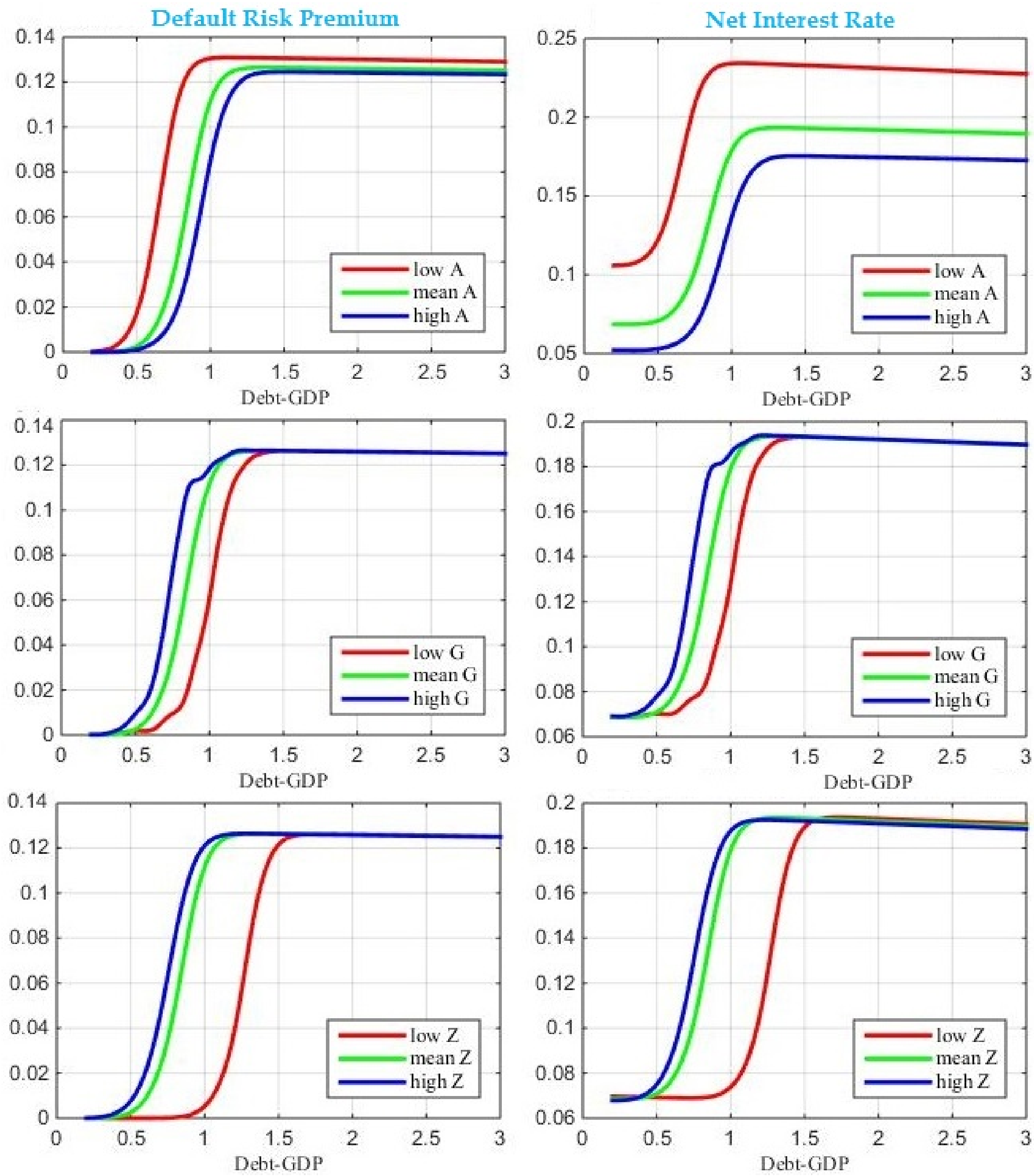
Figure A.8 : Fiscal Limit Distribution : heavy-tailed vs. normally distributed business cycle



Fiscal Limit Distribution for Slovakia: comparison of heavy-tailed distributed (bold line) and normally distributed business cycle (dashed line) for the baseline NPC scenario (all features switched off).

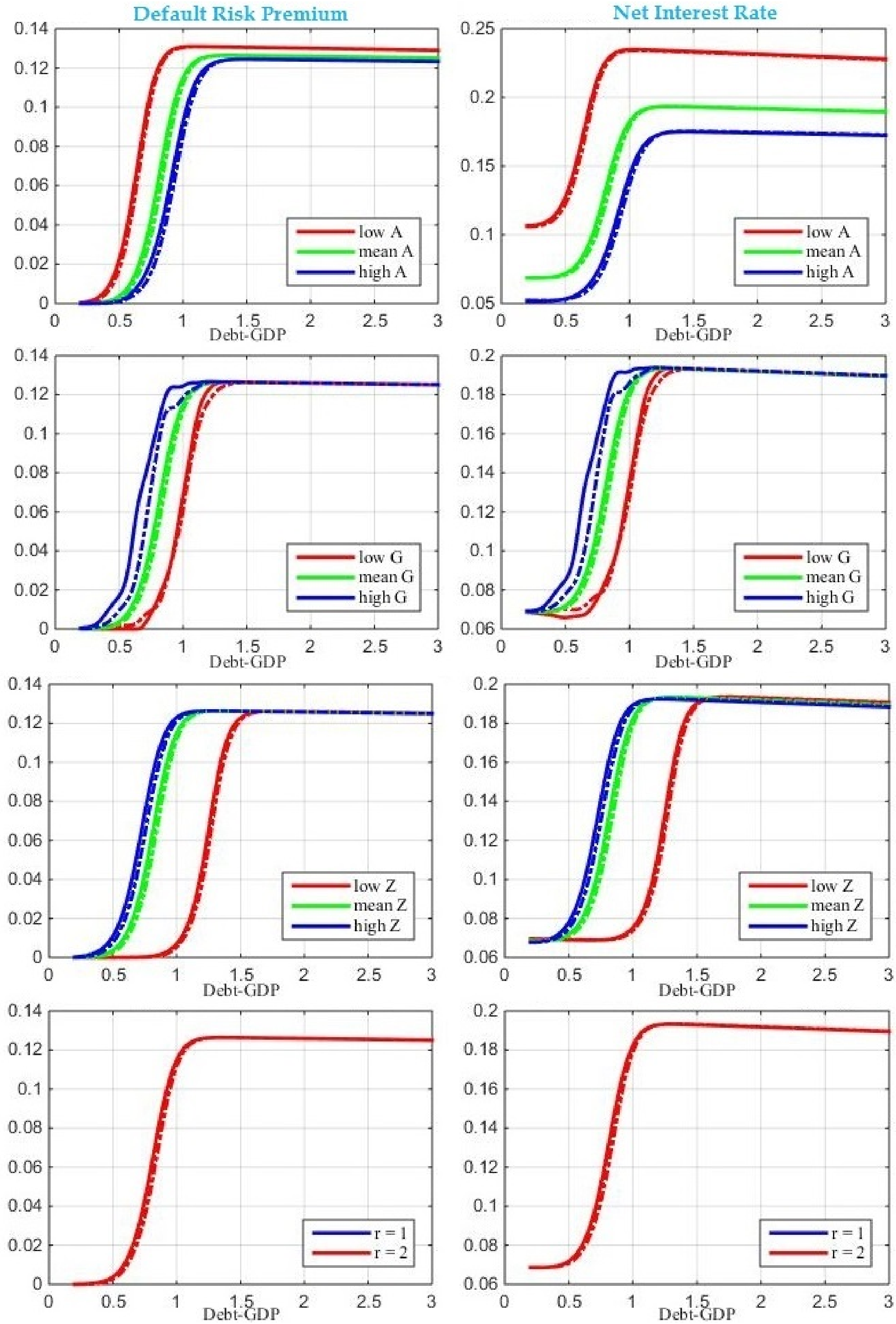
Appendix A.2 Default Risk Premium

Figure A.9 : Default Risk Premium : no-policy-change, baseline setting



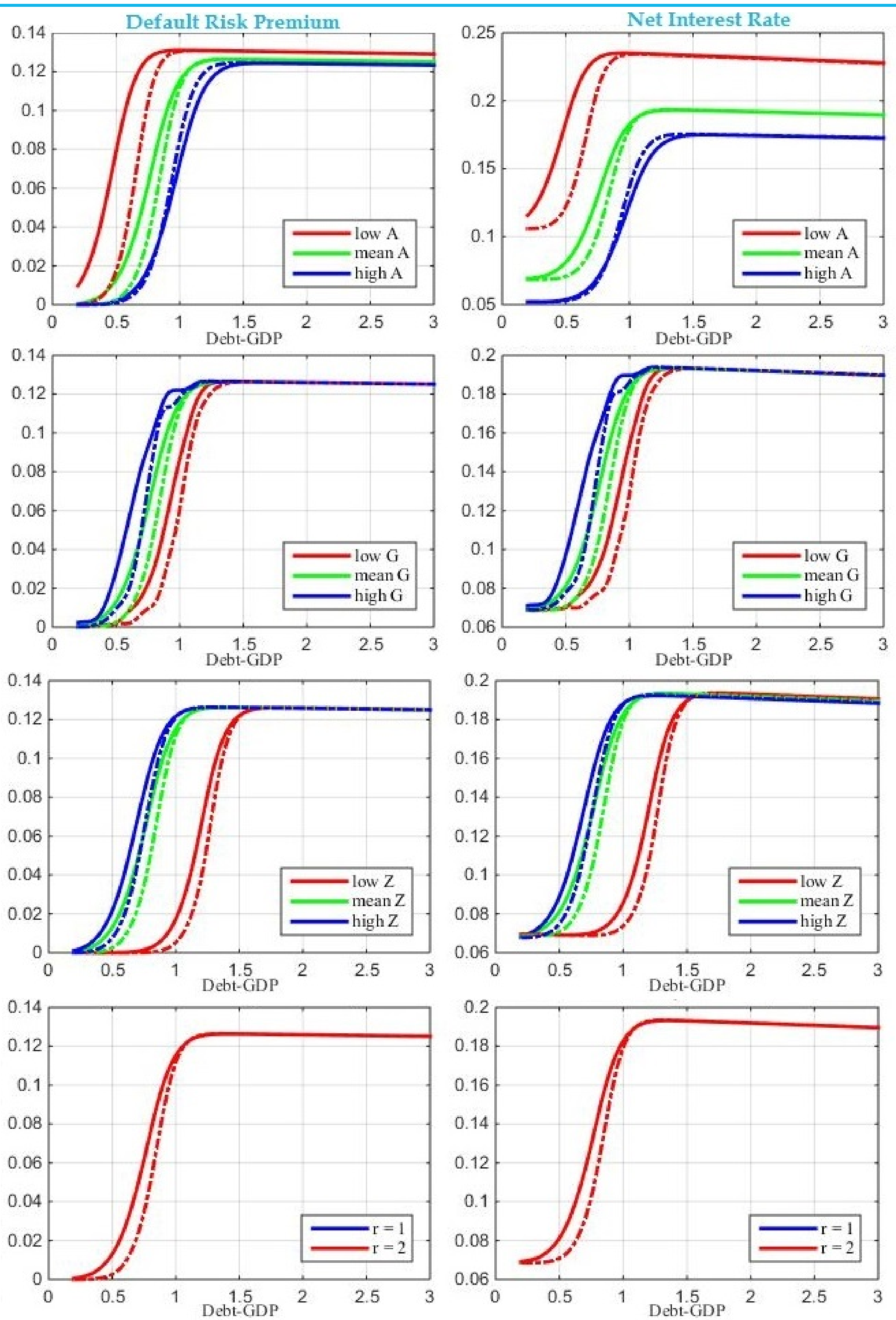
Default risk premium (left) and the corresponding net interest rate (right) for the no-policy-change scenario with default setting where fiscal expenditures (transfers, government purchase) ignore the business cycle following the heavy-tailed empirical distribution. Upper plots describe risk premium and interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers.

Figure A.10 : Default Risk Premium : no-policy-change, pro-cyclical government purchase



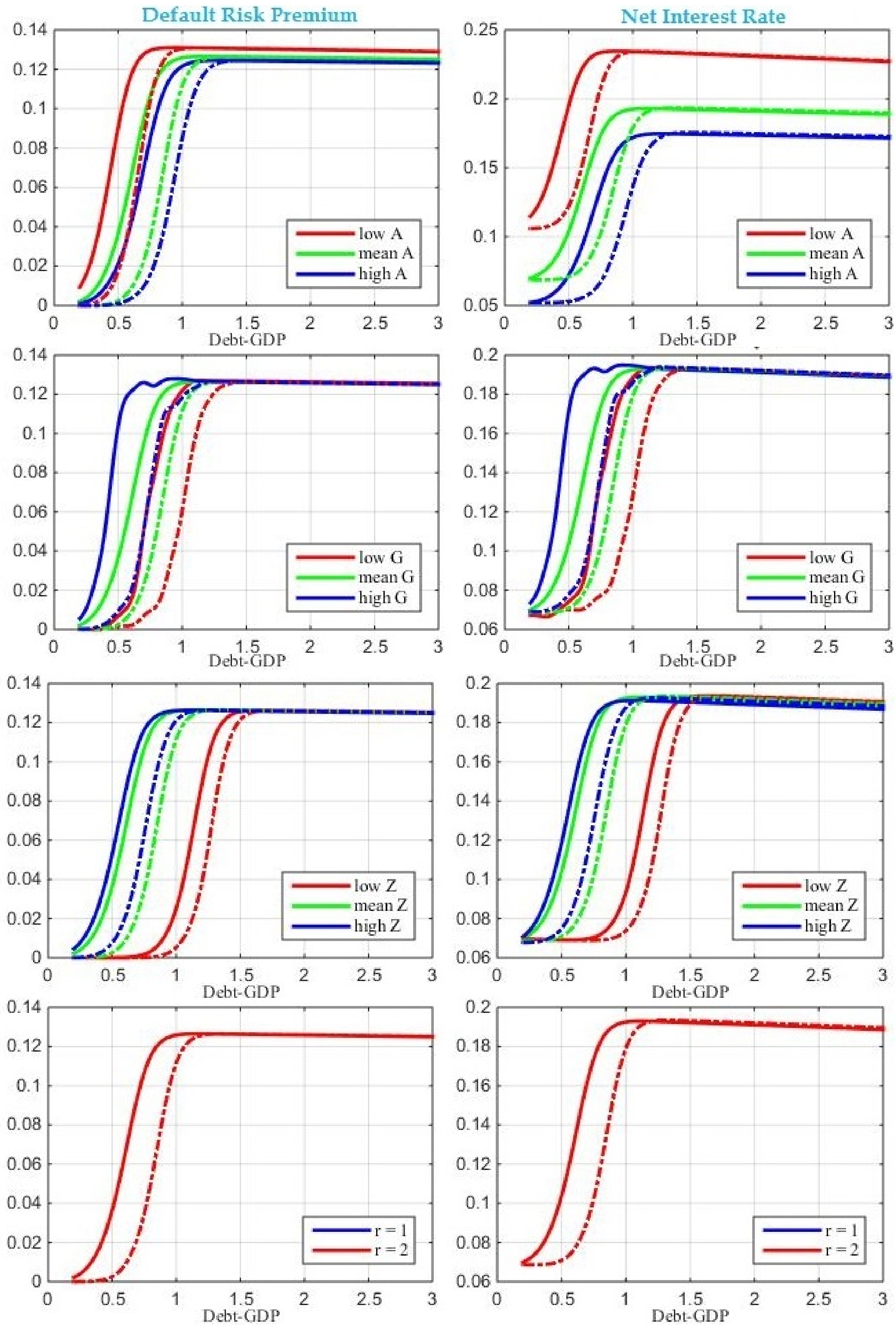
Default risk premium (left) and the corresponding net interest rate (right) for the risky scenario (permanently higher growth rate of transfers due to faster growing healthcare expenditures) with procyclical government purchase and transfers ignoring the business cycle following the heavy-tailed empirical distribution (thick lines). Upper plots describe risk premium and interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario default premium and interest rates.

Figure A.11 : Default Risk Premium : no-policy-change, countercyclical transfers



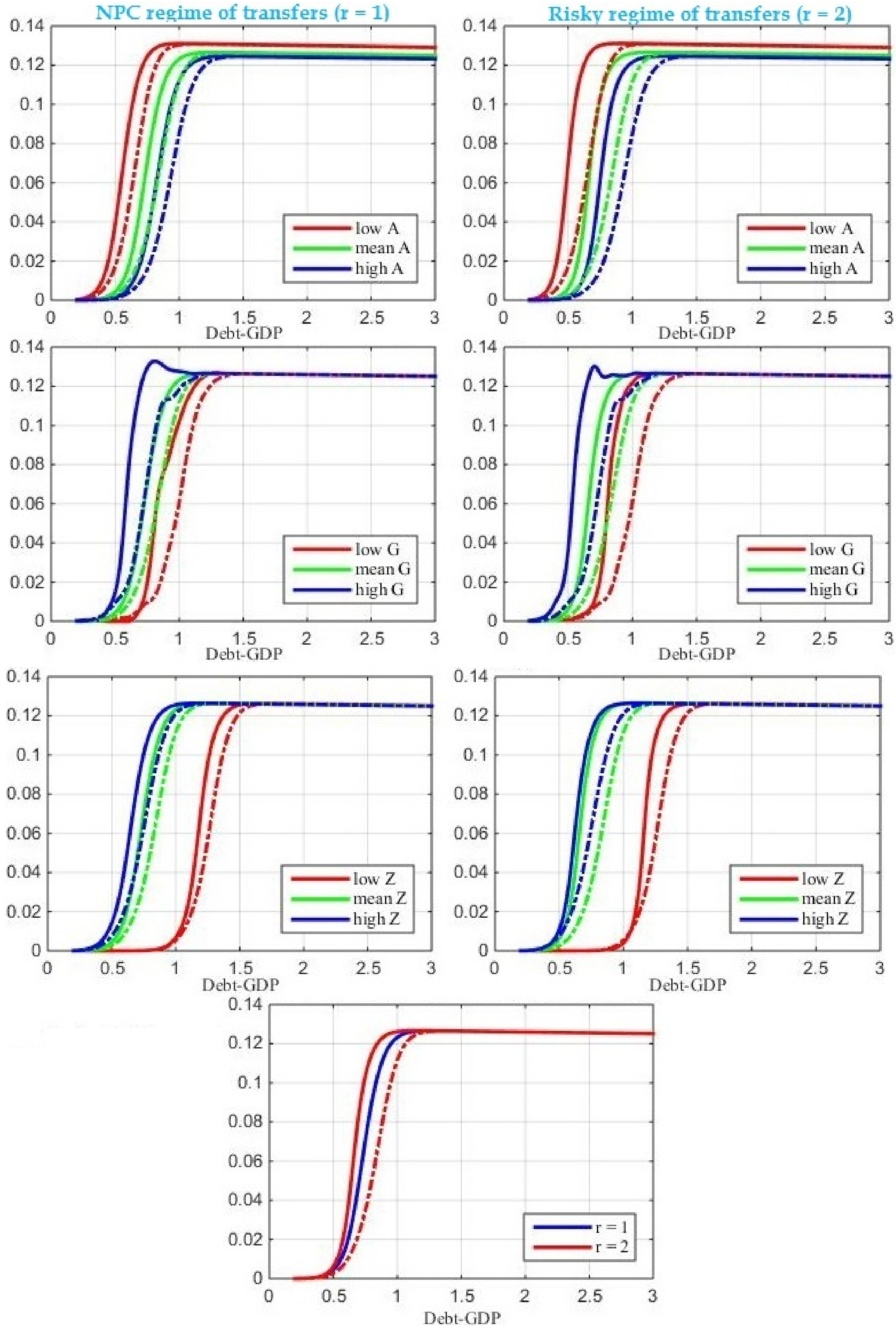
Default risk premium (left) and the corresponding net interest rate (right) for the risky scenario (permanently higher growth rate of transfers due to faster growing healthcare expenditures) with countercyclical transfers and government purchase ignoring the business cycle following the heavy-tailed empirical distribution (thick lines). Upper plots describe risk premium and interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario default premium and interest rates.

Figure A.12 : Default Risk Premium : risky scenario, baseline setting



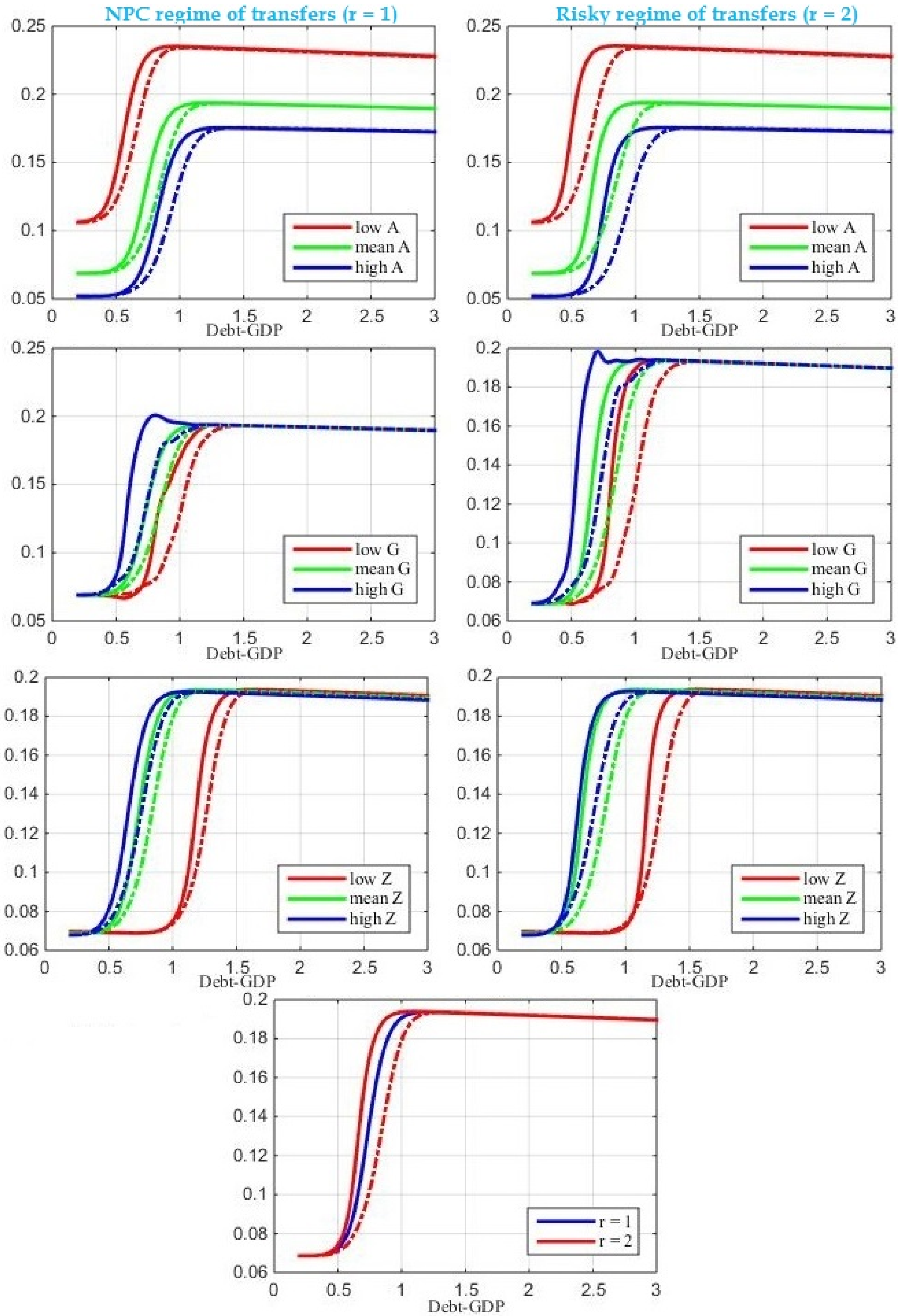
Default risk premium (left) and the corresponding net interest rate (right) for the risky scenario (permanently higher growth rate of transfers due to faster growing healthcare expenditures) with default setting where fiscal expenditures (transfers, government purchase) ignore the business cycle following the heavy-tailed empirical distribution (thick lines). Upper plots describe risk premium and interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario default premia and interest rates.

Figure A.13 : Default Risk Premium : regime-switching transfers, baseline setting.



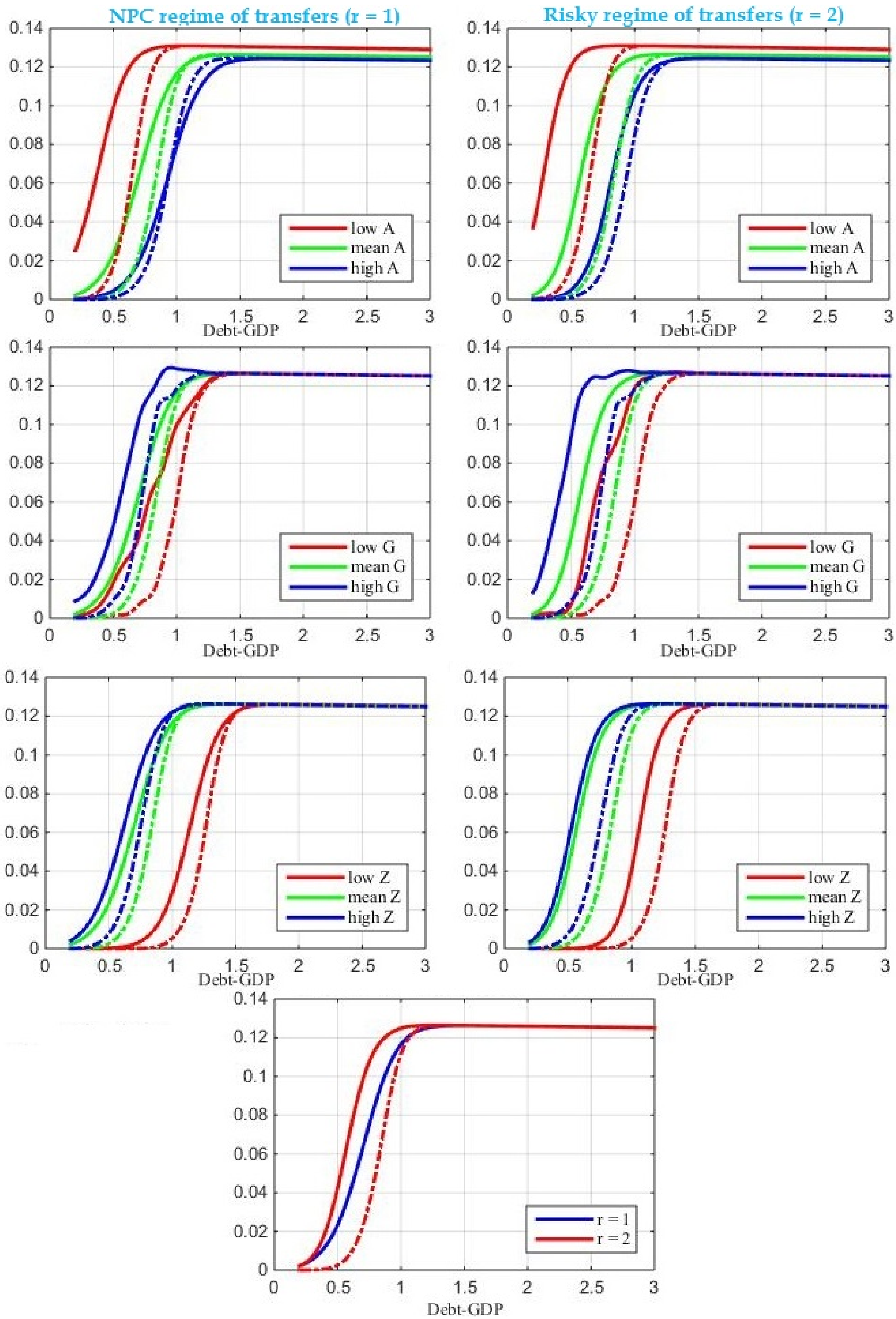
Default risk premium for the regime-switching scenario with default setting where fiscal expenditures (transfers, government purchase) ignore the business cycle following the heavy-tailed empirical distribution (thick lines). Upper plots describe risk premium for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario default premia.

Figure A.14 : Net Interest Rate : regime-switching transfers, baseline setting



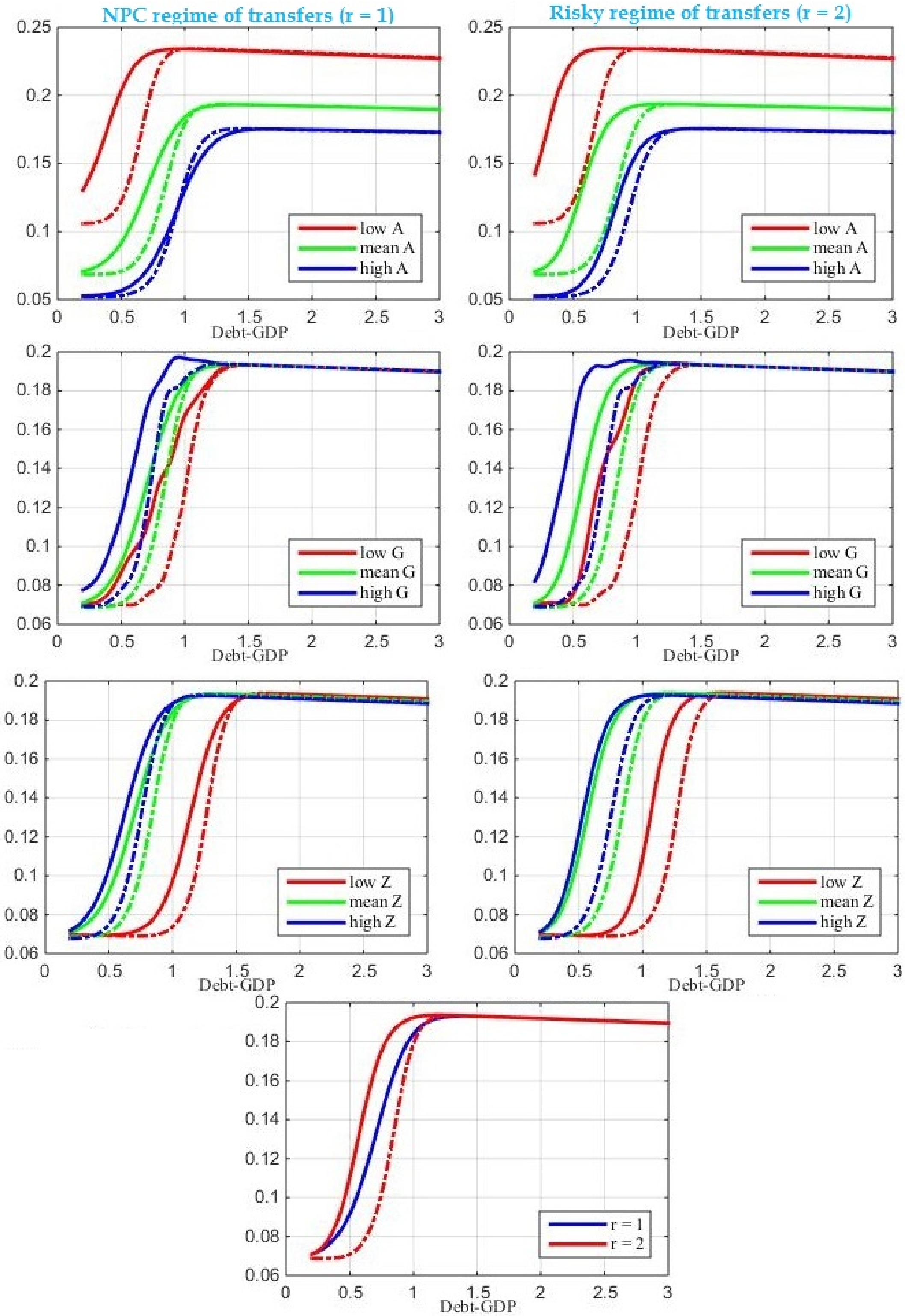
Net interest rate for the regime-switching scenario with default setting where fiscal expenditures (transfers, government purchase) ignore the business cycle following the heavy-tailed empirical distribution (thick lines). Upper plots describe net interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario net interest rate.

Figure A.15 : Default Risk Premium : regime-switching transfers, all features on



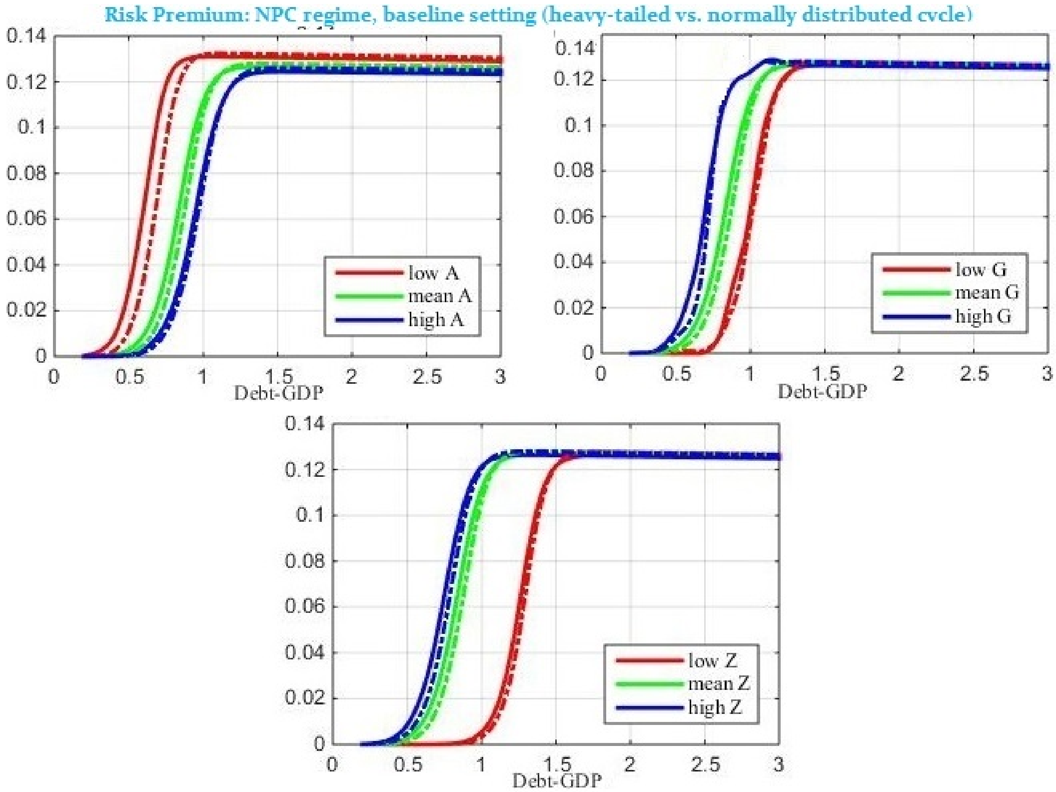
Default risk premium for the regime-switching scenario (with randomly varying between two rates of growth of transfers) for countercyclical transfers. Government purchase are pro-cyclical and business cycle follows the heavy-tailed empirical distribution (thick lines). Upper plots describe the default risk premium for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario net interest rate.

Figure A.16 : Net Interest Rate : regime-switching transfers, all features on



Net interest rate for the regime-switching scenario (with randomly varying between two rates of growth of transfers) for countercyclical transfers. Government purchase are pro-cyclical and business cycle follows the heavy-tailed empirical distribution (thick lines). Upper plots describe net interest rate for different technology levels, middle plots illustrate them for various government purchase levels and the bottom plots depict them for more levels of transfers. Dashed lines correspond to the no-policy-change scenario net interest rate.

Figure A.17 : Default Risk Premium : heavy-tailed vs. normally distributed business cycle



Default risk premium for Slovakia estimated for various levels of productivity, government purchase and transfers under no-policy-change (left) scenario. Notice that the business cycle is left-skewed heavy-tailed distributed. Dashed lines correspond to the no-policy-change scenario, but with normally distributed technology shocks.

Appendix B Slovak Data

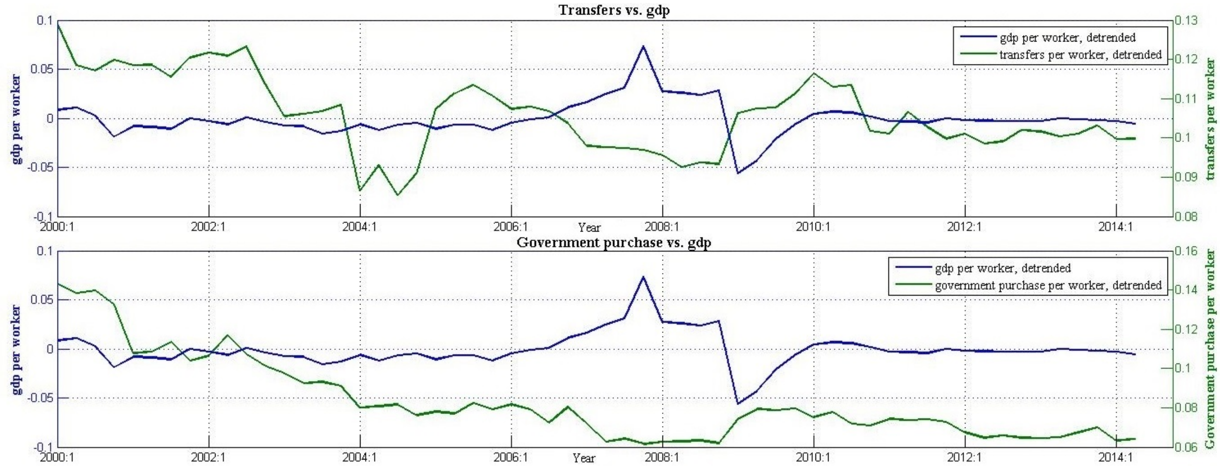
Appendix B.1 Fiscal Data

Table B.1 : Fiscal data for Slovak economy (in % of GDP)

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Real GDP growth rate	4.6	4.8	5.1	6.7	8.3	10.5	5.8	-4.9	4.4	3.0	1.8	0.9
Intermediate Consumption	5.5	5.6	5.1	4.7	5.6	4.9	4.6	5.8	5.5	5.4	5.2	5.2
Gross Capital Formation	4.2	3.2	3.1	3.6	3.7	3.3	3.5	3.8	3.6	3.7	3	3.1
Subsidies	1.5	1.6	1.8	1.2	1.2	1.1	1.6	1.6	1.3	0.8	1	1
Compensation of Employees	9.1	8.9	8.1	7.3	7.2	6.6	6.8	7.7	7.7	7.4	7.3	7.6
Social benefits (excl. *)	13.5	11.7	12.4	12.5	12.1	11.8	11.5	14	14.3	13.8	14	14
Social transfers in kind (*)	2.8	3.3	3	4.3	4.3	4.4	4.6	5.1	5.2	4.8	4.9	5

Source: *Eurostat, NBS database (2002-2013)*

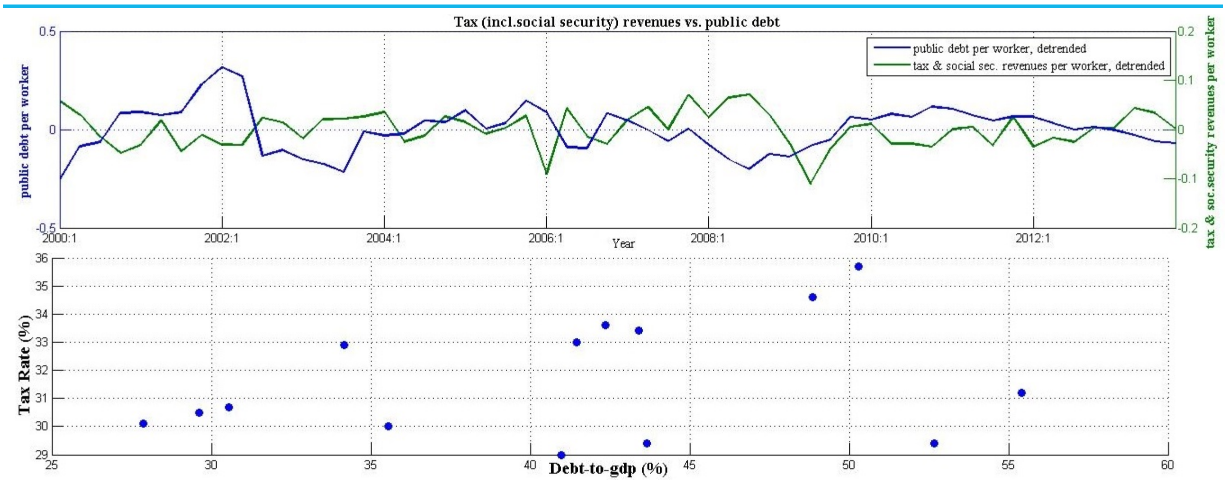
Figure B.1 : Transfers and government purchase vs. GDP per worker between 2000-2014



Transfers (upper panel) and government purchase (lower panel) vs. GDP (real, detrended) per worker between 2000-2014

Source: *NBS Database, Eurostat*

Figure B.2 : Tax, social security revenues and effective tax rate vs. public debt



Tax and Social Security Revenues vs. public debt (real, detrended) , per worker (upper panel) and Effective tax rate vs. Public debt-to-GDP for Slovakia (lower panel) Scatter plot, data between 2000-2014

Source: *NBS Database, Eurostat*

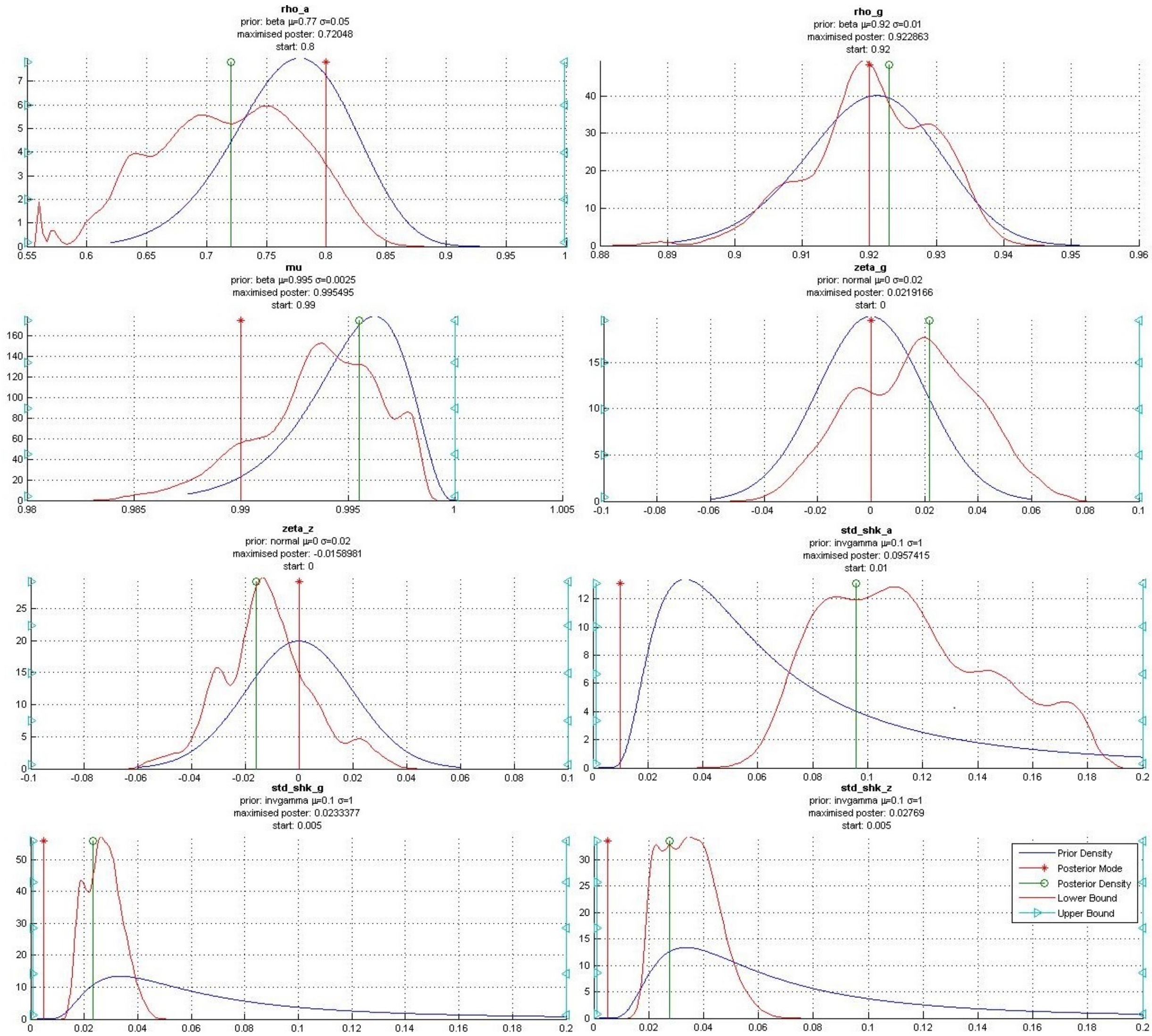
Appendix B.2 Model Estimation

Table B.2 : Model Priors and Posteriors

Parameter		Prior Type	Prior Mean	Prior St.Dev.	Posterior Mean	Posterior St.Dev
Technology persistence	ρ_a	β	0.80	0.05	0.7205	0.0587
Government purchase persistence	ρ_g	β	0.92	0.01	0.9229	0.0090
Government purchase sensitivity to b.cycle	ζ_g	$\mathcal{N}$	0	0.02	+0.0219	0.0221
Transfers sensitivity to business cycle	ζ_z	$\mathcal{N}$	0	0.02	-0.0159	0.0166
Government purchase shock : Standard deviation	σ_g	Γ^{-1}	0.1	1	0.0233	0.0063
Transfers shock : Standard deviation	σ_z	Γ^{-1}	0.1	1	0.0277	0.0095

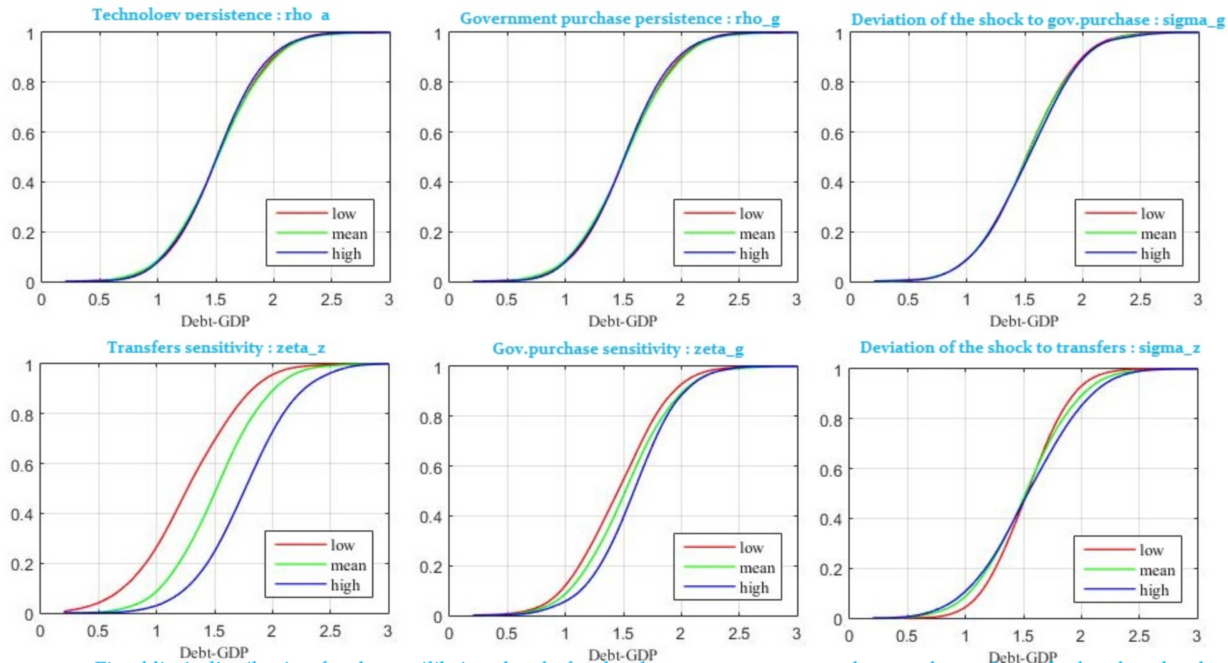
Source: *Eurostat, NBS database*

Figure B.3 : Prior and posterior distribution of model parameters



Source: *NBS Database (1997–2014)*

Figure B.4 : Sensitivity of the Fiscal Limit distribution to the posterior distribution of parameters



Fiscal limit distribution for the equilibrium level of technology, government purchase and transfers calculated under the assumption of low (red; posterior mean minus 2 posterior deviations), mean (green; posterior mean) or high (blue; posterior mean plus 2 posterior deviations) value of the parameters estimated using Bayesian approach.

Appendix B.3 Business Cycle in Slovakia

Table B.3 : Descriptive Statistics for Slovak output gap data

	Range	Mean	Standard Deviation	Interquartile Range	α-Quantiles (0, 0.05, 0.15, 0.85, 0.95, 1)	Skewness	Kurtosis
Annual	17.8244	-0.1986	2.0241	1.5194	-9.4710 -3.1601 -1.6065 0.9315 3.7674 8.3534	0.8914	8.3644
Quarterly	19.1882	-0.2010	1.9514	1.5133	-9.4710 -3.1574 -1.6061 0.8865 3.6702 8.3534	1.0878	8.2908

Source: *Slovak MinFin, NBS, EC, BIS (2000-2014)*

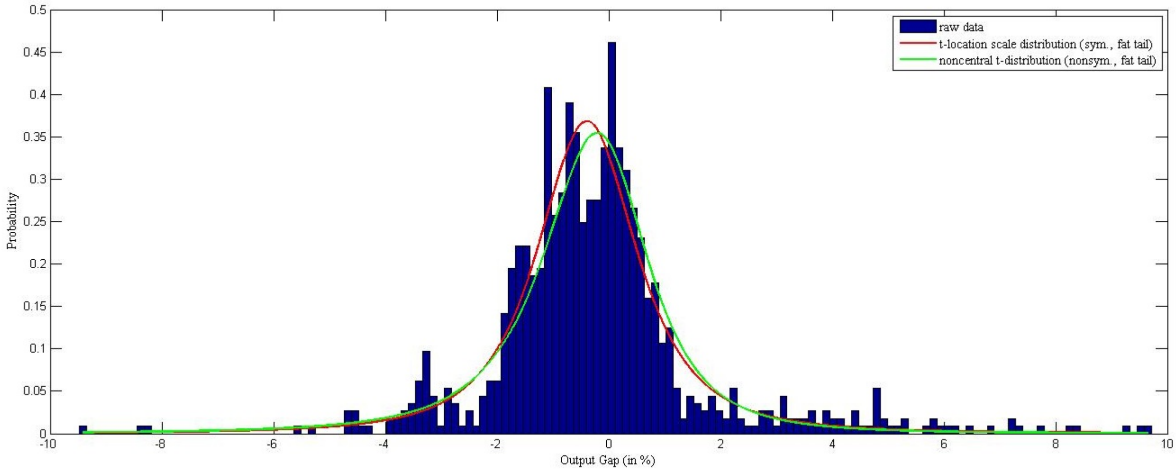
Table B.4 : Characteristics of the empirical heavy-tailed distribution estimate fitting the Slovak output gap data.

	Lower Tail		Upper Tail		Interior	Neg. Log-Likelihood
	Range	Distribution	Range	Distribution		
Annual	$x < -1.6787$ $\alpha < 0.15$	Generalised Pareto (0.1375,1.1532)	$x > 1.0552$ $\alpha > 0.85$	Generalised Pareto (0.1060, 2.8302)	Interp. kernel smooth cdf	403.6468
Quarterly	$x < -1.6464$ $\alpha < 0.15$	Generalised Pareto (0.1475,0.9856)	$x > 2.5024$ $\alpha > 0.925$	Generalised Pareto (0.1336, 3.0415)	Interp. kernel smooth cdf	1.6195e+03

Table B.5 : Characteristics of t-location scale distribution estimate fitting the Slovak output gap data.

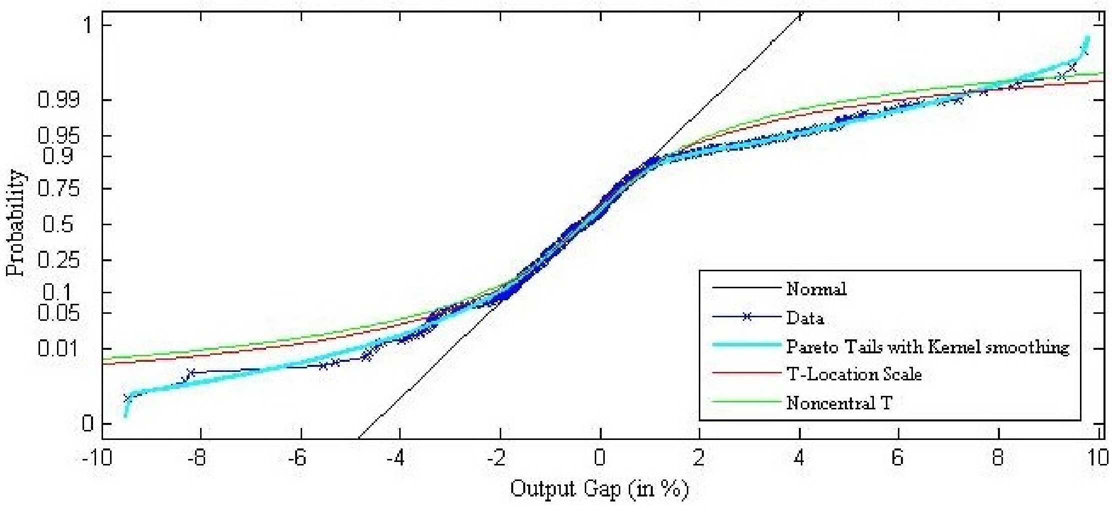
	Distribution	Parameters			Descriptive Statistics		Neg. Log-likelihood
		Location	Scale	Deg. of freedom	Mean	Std. Deviation	
Annual	t-location scale	-0.3927	0.9682	2.0158	-0.3927	10.9384	423.1749
Quarterly	t-location scale	-0.3894	0.9634	2.0727	-0.3894	5.1464	1.6770e+03

Figure B.5 : Probability distribution function of the Slovak business cycle



Estimation of the Slovak business cycle PDF based on data from 2000-2014. We use the t-location scale distribution (with parameters obtained from the maximal likelihood procedure) to fit the data properly.

Figure B.6 : Non-normality of output gap data

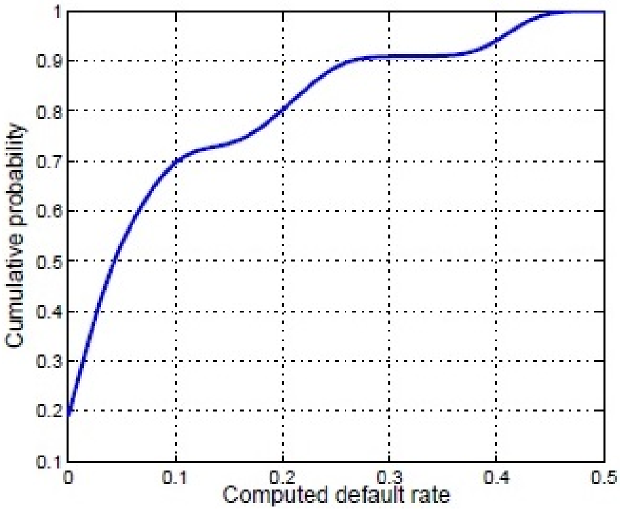


Q-Q plot for the comparison of the output gap data distribution (blue line with markers) to the normal distribution (black dashed line) empirical Pareto-tailed kernel smoothing distribution (thick cyan line) and location-scale T-distribution (thin red line).

Appendix C Stochastic Default Rate

At time t the stochastic default rate δ follows the empirical distribution Ω computed by Bi and Leeper (2010) and Bi (2011) from the sovereign debt defaults and restructures observed in the emerging market economies during the period of 1983 to 2005.

Figure C.1 : Stochastic Default Rate Distribution



Cumulative distribution function of the stochastic default rate defining the empirical distribution Ω computed from the sovereign debt defaults and restructures observed in the emerging market economies during the period of 1983 to 2005.

Source: *Bi and Leeper (2010), Bi (2011)*

Appendix D Simulation Scheme for Fiscal Limits

The concept of the fiscal limit is based on the subsequent simple idea. Denote the primary surplus $\varsigma_t = \tau_t a_t h_t - z_t - g_t$ and iterate the government budget constraint[20]:

$$b_t^d = q_t b_t + \varsigma_t = q_t \mathbb{E}_t (q_{t+1} b_{t+1} + \varsigma_{t+1}) + \varsigma_t = \ldots = \mathbb{E}_t \sum_{k=0}^{T} \left[\prod_{j=0}^{k-1} q_{t+j} \right] \varsigma_{t+k} + \mathbb{E}_t \prod_{j=0}^{T} q_{t+j} b_{t+T} \,.$$

Observe that due to trasversality conditio[21] (10c) and the Euler equation (10b) the second term in the equation above tends to zero as $T \to \infty$. Therefore, to achieve the largest possible post-default debt b_t^d we need to maximize the present value of the sum of the current and all future primary surpluses.

Appendix D.1 Model Calibration and Maximal Tax Revenues

The household choices about their level of consumption and labour supply only depend on the income tax rate τ_t and the exogenous state variables , technology a_t and government purchase g_t.
Therefore, referring to (9a) assume the utility function is $u(c,h) = \log c + \phi \log(1-h)$.

Optimal Tax Rate: The household first-order conditions (see (10a)) can be written as

$$c_t = \frac{(a_t - g_t)(1-\tau_t)}{1+\phi-\tau_t} \,, \tag{D.1}$$

[20] At this very first model approach, to model the default risk premium we need at *any* reasonable (though very simplified) approximation of the fiscal limit distribution. Thus to simplify the estimation of the fiscal limit distribution we derive it under the assumption of no future defaults – therefore b_{t+k}^d coincides with b_{t+k-1}. Providing that the non-zero future default probability is taken into consideration, the iterated budget constraint takes the form of

$$b_t^d = \mathbb{E}_t \sum_{k=0}^{T} \left[\prod_{j=1}^{k} \frac{q_{t+j-1}}{1-\Delta_{t+j}} \right] \varsigma_{t+k} + \mathbb{E}_t \prod_{j=1}^{T} \frac{q_{t+j-1}}{1-\Delta_{t+j}} q_{t+T} b_{t+T} \,.$$

[21] The transversality condition becomes even stronger requirement in this simplified case

$$h_t = \frac{a_t(1-\tau_t)+\phi g_t}{a_t(1+\phi-\tau_t)}. \tag{D.2}$$

Then, the first derivative of the tax revenue $\Theta_t = \tau_t a_t h_t$ with respect to the tax rate τ_t,

$$\frac{\partial \Theta_t}{\partial \tau_t} = \frac{a_t\tau_t^2 - 2a_t(1+\phi)\tau_t + (1+\phi)(a_t+\phi g_t)}{(1+\phi-\tau_t)^2}, \tag{D.3}$$

have two distinct roots[22]

$$0 < \tau_t^{(-)} \equiv 1+\phi - \sqrt{\phi(1+\phi)\frac{a_t-g_t}{a_t}} < 1 < \tau_t^{(+)} \equiv 1+\phi + \sqrt{\phi(1+\phi)\frac{a_t-g_t}{a_t}}. \tag{D.4}$$

Thus, since $\partial\Theta_t/\partial\tau_t < 0$ iff $\tau_t \in (\tau_t^{(-)}, \tau_t^{(+)})$ one can straightforwardly deduce that

$$\tau_t^{\max} \equiv \tau_t^{(-)} = 1+\phi - \sqrt{\phi(1+\phi)\frac{a_t-g_t}{a_t}} \tag{D.5}$$

is the unique tax revenues maximiser and

$$\Theta_t^{\max} \equiv \Theta_t^{\max}(a_t, g_t) = (1+2\phi)a_t - \phi g_t - 2\sqrt{\phi(1+\phi)a_t(a_t-g_t)}. \tag{D.6}$$

Inasmuch as

$$\frac{\partial \Theta_t^{\max}}{\partial g_t} = -\phi + \sqrt{\frac{\phi(1+\phi a_t)}{a_t-g_t}} > 0, \qquad \phi > 0, a_t > g_t > 0,$$

the maximal tax revenues $\Theta_t^{\max}$ increases with the level of government purchase, g_t. Next, as

$$\frac{\partial \Theta_t^{\max}}{\partial a_t} = 1+2\phi - \phi\zeta_g - (2a_t - g_t - \zeta_g a_t)\sqrt{\frac{\phi(1+\phi)}{a_t(a_t-g_t)}} \in (0,1)$$

for any ζ_g small enough (any any negative), $\Theta_t^{\max}$ increases with the technology, a_t and moreover, fluctuations in exogenous productivity are projected into changes in the maximal tax revenues with lower magnitude[23].

Finally, combining (D.1)–(D.2) with (D.5) leads to the following optimal levels of consumption and labour supply depending on current technology and government purchase assuming that the tax rate is set such that it maximises the tax revenues:

$$h_t = 1 - \sqrt{\frac{\phi}{1+\phi}\left(1-\frac{g_t}{a_t}\right)}, \tag{D.7}$$

$$c_t = -\sqrt{\phi\frac{a_t(a_t-g_t)}{1+\phi}} + (a_t - g_t). \tag{D.8}$$

Evidently, labour supply declines with technology but increases with government purchase, even cycle–sensitive. The opposite behaviour is typical for consumption – it grows with technology, but decreases with government purchase.

Calibration: In order to calibrate the model properly and determine the coefficient ϕ we assume that in the steady state, $h = 0.25$, and $a = 1$, so $y = 0.25$. Next, g/y, z/y, b/y and β (on annual frequency) are given, so the steady state tax rate satisfies

$$\tau = \frac{b}{y}(1-\beta) + \left(\frac{z}{y} + \frac{g}{y}\right) \tag{D.9}$$

[22] Since $a_t > g_t > 0$ and $\phi > 0$, it is evident that $0 < \tau_t^{(-)} < 1 < \tau_t^{(+)}$.

[23] This attribute of the maximal tax revenues is not in consistence with our observation of Slovak data.

Then, plugging (D.9) into (D.1) evaluated in the steady–state leads to the following:

$$\phi = (1-\tau)\left(\frac{a}{y}-1\right)\left[1-\frac{g}{y}\right]^{-1} = 3(1-\tau)\left[1-\frac{g}{y}\right]^{-1}. \tag{D.10}$$

Appendix D.2 State-Dependent Transition Matrix

A possible extension of the model that makes it more realistic can be obtained when we do not insist on the constancy of the transition matrix P.

Thus, we let it to reflect the evolution of the transfers and optimal tax rate. Government running the no-policy-change scenario with lower rate of growth of transfers is rewarded by higher level of trust (and hence, higher chance to have power in the next time period) when transfers raise, and looses sympathies with transfers cuts. The marginal popularity drops with increasing transfers. On the other side, the fiscal authority preferring rapidly increasing transfers (the risky scenario) looses chance to be re–elected when tax rate increases and the marginal unpopularity even raise with higher tax rate. This idea can be expressed as follows:

$$\begin{aligned} P_t &= \begin{bmatrix} p_t^{(1)} & 1-p_t^{(1)} \\ 1-p_t^{(2)} & p_t^{(2)} \end{bmatrix}, \\ p_t^{(1)} &= q^{(1,1)} + q^{(1,2)} \arctan\left[\frac{\Delta p_{t-1}^{(1)}}{p^{(1)}} + \alpha_{p^{(1)}}\frac{\Delta z_t}{z}\right], \\ p_t^{(1)} &= q^{(2,1)} + q^{(2,2)} \arctan\left[\frac{\Delta p_{t-1}^{(2)}}{p^{(2)}} - \alpha_{p^{(2)}}\frac{\Delta \tau_t}{\tau}\right]. \end{aligned} \tag{D.11}$$

Above, $P = [p^{(1)}, 1-p^{(1)}; 1-p^{(2)}, p^{(2)}]$ represents the originally used constant transition matrix, $\Delta p_t^{(i)} = p_t^{(i)} - p_t^{(i-1)}$ reflects the evolution of state–dependent transition matrix main diagonal coefficients, constant parameters $\alpha_{p^{(1)}}, \alpha_{p^{(2)}} \in [0,1]$ control the level of consideration of changes in detrended transfers, $\Delta z_t = z_t - \mu_1(t) z_{t-1}$ and tax rate, $\Delta \tau_t = \tau_t - \tau_{t-1}$ and

$$q^{(i,1)} = \frac{1}{2}\left(\bar{p}^{(i)} + \underline{p}^{(i)}\right), \qquad q^{(i,2)} = \frac{1}{\pi}\left(\bar{p}^{(i)} - \underline{p}^{(i)}\right), \qquad i \in \{1,2\}.$$

Furthermore notice that the marginal increase in popularity is decreasing, which is very natural and consistent with reality in Slovakia: a fiscal policy which is intended is to lower the distributivity of the allocated wealth becomes even less popular when the transfers do not grow accordingly to households expectations and when they are raising it increases the popularity of the government only a bit.

Appendix D.3 Procedure Description

Used Equations: (14)–(22)

Aim: Obtain the conditional distribution of the fiscal limit $\mathscr{B}$ given the exogenous processes $\{a_{t+k}\}_{k=0}^{\infty}$, $\{g_{t+k}\}_{k=0}^{\infty}$, $\{z_{t+k}\}_{k=0}^{\infty}$, $\{r_{t+k}\}_{k=0}^{\infty}$ and employing Markov Chain Monte Carlo simulation.

To obtain the distribution of the fiscal limit we proceed as follows:

1. Discretize state space (a_t, g_t, z_t, r_t).
2. For each simulation i randomly draw normally distributed shocks $\left\{\varepsilon_{t+k}^{g}\right\}_{k=1}^{T}$, $\left\{\varepsilon_{t+k}^{z}\right\}_{k=1}^{T}$, empirically distributed shock $\left\{\varepsilon_{t+k}^{a}\right\}_{k=1}^{T}$ (following kernel smoothing Pareto–tailed empirical distribution) and uniformly distributed $\left\{\varepsilon_{t+k}^{r} \in [0,1]\right\}_{k=1}^{T}$ for $T = 200$ conditional on initial state $(a_t, g_t, z_t, r_t, f_t, d_t)$.

3. Within each simulation i for each time $k = 1, \dots, T$
   (a) Find a_{t+k} given a_{t+k-1}, ε^a_{t+k} and using (14).
   (b) Find g_{t+k} given g_{t+k-1}, ε^g_{t+k}, a_{t+k} in terms of (15).
   (c) Use MCMC procedure find the current regime of transfers r_{t+k} given the previous state r_{t+k-1}, constant transition matrix P and shock ε^r_{t+k}.
   Technically, we take the state associated with the first positive element of a past state row vector of the cumulative matrix P affected by the shock ε^r_t, i.e.
   $$r_{t+k} = \min\left[\text{find}\left(P(r_{t+k-1},:) - \varepsilon^r_{t+k} > 0\right)\right].$$
   (d) Find z_{t+k} using (18) and current regime r_{t+k} from the previous step.
   (e) Employ (20) to get the maximal tax revenue $\Theta^{\max}_{t+k}$. Use $\Theta^{\max}_{t+k}$, z_{t+k} and g_{t+k} to get the corresponding maximal primary surplus(21) ς_{t+k} at time $t+k$.

   Finally, combine $\varsigma_{t+k}, k = 1, \dots, T$ to obtain the discounted sum of maximum fiscal surplus (22) for the current simulation $B^*_i(t)$.
4. Repeat Step 2 – Step 3 10^6 times and obtain the conditional distribution of $\mathscr{B}(a_t, g_t, r_t)$ using the simulated $B^*_i(t)$ for $i = 1, \dots, 10^6$.
5. Repeat Step 2 – Step 4 for all possible exogenous states (a_t, g_t, r_t, z_t) within the discretized state space.
6. Bound the empirical distribution such that $0.2y \leq \mathscr{B}_t \leq 3.0y$ and re-estimate its pdf and cdf on this compact support.

Appendix D.4 Matlab Code

```
B_star = zeros(N, shk_a_num, shk_g_num, shk_z_num, r_num);
for idx_a = 1 : shk_a_num
    shk_a = shk_a_grid(idx_a);
    a_0 = a_ss  + shk_a;
    for idx_g = 1 : shk_g_num
        shk_g = shk_g_grid(idx_g);
        g_0 = g_ss + shk_g;
        tax_0 = 1 + phi - sqrt((1 + phi) * phi * (a_0 - g_0) / a_0);
        c_0 = (a_0 - g_0) * (1 - tax_0) / (1 + phi - tax_0);
        for idx_z = 1 : shk_z_num
            shk_z = z_ss * shk_z_grid(idx_z);
            z_0 = z_ss + shk_z;
            for idx_r = 1 : r_num
                r_0 = r_grid(idx_r);
                for k = 1 : N
                    shk_a = random(shk_a_obj, T, 1);
                    shk_g = randn(T, 1) * std_shk_g;
                    shk_z = randn(T, 1) * std_shk_z;
                    shk_r = rand(T, 1);
                    pb = zeros(T, 1);
                    a = a_0;
                    g = g_0;
                    r = r_0;
                    z = z_0;
                    for t = 1 : T
                      a = rho_a * a + (1 - rho_a) * a_ss + shk_a(t);
```

```
                    g = rho_g * g + (1 - rho_g) * g_ss + shk_g(t) ...
                        + zeta_g * (a - a_ss);
                    r = min(find(cumP(r, :) - shk_r(t)>0)); %#ok<*MXFND,*PFBNS>
                    if r == 1
                       z = mu_1 * z + zeta_z * (a - a_ss) + shk_z(t);
                    else
                       z = mu_2 * z + zeta_z * (a - a_ss) + shk_z(t);
                    end
                    u = sqrt((1 + phi) * phi * (a - g) / a);
                    v = u - phi;
                    pb(t) = (betta^(t-1) * u / ((a - g) * v)) * ...
                        ((1 - 2 * v) * a - z - (1 + phi) * g);
                end
                B_star(k,idx_a,idx_g, idx_z, idx_r) = c_0 * sum(pb);
            end
        end
    end
  end
end

B_low  = 0.2 * y_ss;
B_high = 3.0 * y_ss;
x_grid = B_low : (B_high-B_low)/(x_num-1) : B_high;
dim = [shk_a_num, shk_g_num, shk_z_num, r_num];
cdf_x = repmat(0,[x_num, dim]);
pdf_x = cdf_x;
m = prod(dim);
for k = 1 : m
    B_star_vector = B_star(:, [k]);
    cdf_x_vect = ksdensity(B_star_vector,x_grid,'function','cdf');
    cdf_x_vect_prev = [0; cdf_x_vect(1 : end - 1)];
    pdf_x(:, [k]) = cdf_x_vect - cdf_x_vect_prev;
    cdf_x(:, [k]) = cdf_x_vect;
end
```

Appendix E Decision Rule and Risk Premium

Aim: Solve the nonlinear model formed by equations (14)–(18), (23), (25)–(26). Due to the presence of fiscal limits in cannot be solved using the standard approach through log–linearisation.
The core equation (26) is solved iteratively on the state–space grid using Sim's algorithm that maps the current 7–dimensional state $(a_t, g_{t-1}, b_{t-1}, z_{t-1}, r_t, b_t^*, \delta_t)$ into the end–of–period unknown (pre–default) debt b_t (scalar). Then using the government budget constraint we obtain the pricing rule q_t and the associate interest rate $i_t = -log(q_t)$.

Appendix E.1 Procedure Description

At the beginning of each period the realization of productivity, government purchase and the regime of transfers determine the state dependent distribution of the fiscal limit $\mathscr{B}$ calculated employing the procedure from the previous section. If the debt at the beginning of the period b_{t-1} (obtained in the last period) exceeds the effective fiscal limit b_t^* drawn from the fiscal limit distribution $\mathscr{B}$ then the stochastic default rate $\delta_t \in \Omega$ takes place, otherwise the debt is paid back in full amount. The post–default debt level determines the effective tax rate, and so it affects labour supply and consumption. Furthermore, based on the current state $(a_t, g_{t-1}, r_t, z_{t-1}, b_t^d)$ households decide about the bond price q_t and amount of bonds to purchase.

1. Discretize state space $(b_{t-1}, a_t, g_{t-1}, z_{t-1}, r_t)$ where b_{t-1} is considered for the initial guess of the debt decision rule f_0^b and assume that the distributions (CDF's) of the fiscal limit $\mathscr{B}(a_t, g_t, r_t)$ and the default rate Ω are known (already determined).
2. At each grid point $(b_{t-1}, a_t, g_{t-1}, z_t, r_t)$ solve the core equation (26) non–linear equation to obtain the debt rule b_t.
   The debt rule is estimated iteratively as follows:
   (a) Set the initial guess of the debt rule f_0^b.
   (b) To obtain the updated rule f_i^b from f_{i-1}^b solve

$$\frac{b_t^d + z_t(\psi_t) + g_t(\psi_t) - \tau_t(\psi_t) a_t h_t(\psi_t)}{f_i^b(\psi_t)} = \beta \mathbf{E}_t \left[\frac{c(\psi_t)}{\psi_{t+1}} (1 - \Delta(\psi_{t+1})) \right]. \tag{E.1}$$

   Above, $\psi_t = (b_t^d, a_t, g_{t-1}, z_{t-1}, r_t)$ and the left-hand side variables g_t, z_t, τ_t, h_t and c_t are evaluated using ψ_t and model equations (15), (18), (23), (17) and (16), respectively. Next, the right-hand side integral is evaluated over the time–shifted grid $[a_{t+1}, g_t, r_{t+1}, z_t, \delta_{t+1}, b_{t+1}^*]$ with normally distributed shocks ε_{t+1}^g, ε_{t+1}^z and heavy-tailed empirically distributed shock ε_{t+1}^a using numerical quadrature and interpolating over fiscal limit distribution as follows:

$$\begin{aligned}
\mathbf{E}_t \left[\frac{1-\Delta_{t+1}}{c_{t+1}} \right] &= \int_{\varepsilon_{t+1}^a} \int_{\varepsilon_{t+1}^g} \int_{\{R_{t+1}\}} \int_{b_{t+1}^*} \int_{\delta_{t+1}} \frac{1-\Delta_{t+1}}{c_{t+1}} \\
&= \int_{\varepsilon_{t+1}^a} \int_{\varepsilon_{t+1}^g} \left\{ \left[\sum_{r=1}^{2} P_{r,1} \left(1 - \Phi(b_t \geq b_{t+1}^* | r_t = r)\right) \right] \frac{1}{c_{t+1}} |_{\text{no default}} \right\} \\
&+ \int_{\varepsilon_{t+1}^a} \int_{\varepsilon_{t+1}^g} \left\{ \left[\sum_{r=1}^{2} P_{r,1} \Phi(b_t \geq b_{t+1}^* | r_t = r) \right] \int_{\delta_{t+1}} \frac{1-\delta_{t+1}}{c_{t+1}} |_{\text{default}} \right]
\end{aligned} \tag{E.2}$$

   Technically,
   i. Given a grid point (a_t, g_t, r_t, z_t) and following (14), (15) and (18) you evaluate a_{t+1}, g_{t+1}, and z_{t+1} for all possible ε_{t+1}^a, ε_{t+1}^g, ε_{t+1}^z and states $r_{t+1} = \{1, 2\}$.
   ii. For both transfers regime interpolate over the fiscal limit grid for each grid point

$$(f_i^b, a_{t+1}, g_{t+1}, z_{t+1})$$

   (all a_{t+1}, g_{t+1}, z_{t+1} vary) to obtain the probabilities of country default, $\Phi(b_t \geq b_{t+1}^* | r_t = r)$, for $r \in \{1, 2\}$.
   iii. Calculate $\tau_{t+1} = \tau_{t+1}(f_i^b)$ for the case of no default and the associated consumption $c_{t+1} = c_{t+1}(a_{t+1}, g_{t+1}, \tau_{t+1})$.
   iv. Discretize $\{\delta_{t+1}\}$ and under the assumption of country default for each grid point calculate b_{t+1}^d, and the associated tax rate, consumption and evaluate $(1-\delta_{t+1})/c_{t+1}$. Compute the expected $(1-\delta_{t+1})/c_{t+1}$ over the whole δ_{t+1} space.
   v. Combine results from the previous two steps to get $(1-\Delta(\psi_{t+1}))/c(\psi_{t+1})$ evaluated for a specific $(a_{t+1}, g_{t+1}, z_{t+1})$ given (a_t, g_t, r_t, z_t) and $\mathscr{B}_{t+1}$, Ω.
   vi. Using numerical quadrature estimate the right–hand side of the integral (E.2) over the grid.

   (c) Repeat the whole procedure for the LHS and RHS with Sim's algorithm unless the convergence is achieved. The rule for which $|LHS - RHS| < 10^{-6}$ is f_i^b.
3. Check the convergence between the successive iteration of the debt decision rule. If $|f_i^b - f_{i-1}^b| < 10^{-6}$, f_i^b is the decision rule b_t. Otherwise, if the number of iteration does not surpass the limit $k^{\max}$ go to Step 2b, else the model does not converges at this grid point.

4. For a given $(a_t, g_{t-1}, r_t, z_{t-1})$ interpolate the solution over all starting points b_{t-1} to define the the decision rule for those grid points where the iteration method does not converges or the obtained debt rule does not live within the interval $[0,1]$.

Once the debt rule $b_t = f^b(\psi_t)$ is determined at each point in the discretized model state–space the estimation of the default risk premium profile is trivial indeed. We proceed as follows.

1. Discretize state space $(b_{t-1}, a_t, g_{t-1}, z_{t-1}, r_t)$.
2. At each grid point $(b_{t-1}, a_t, g_{t-1}, z_{t-1}, r_t)$
   (a) For the current state $\psi_t = (b_{t-1}, a_t, g_{t-1}, z_{t-1}, r_t)$ find the level of transfers z_t, government purchase g_t, tax rate τ_t and consumption c_t to find bond price rule $q_t = f^q(\psi_t, b_t)$ where $b_t = f^b(\psi_t)$ is the already estimated debt rule:
   $$q_t(\psi_t) = \frac{b_{t-1} + g_t(\psi_t) + z_t(\psi_t) - a_t \tau_t(\psi_t)\left[c_t(\psi_t) + g_t\right]}{b_t(\psi_t)} \tag{E.3}$$
   (b) Solve the core equation (26) non–linear equation under the assumption of no default. The risk–free debt rule is estimated iteratively as follows:
      i. Set the initial guess of the debt rule f_0^f.
      ii. To obtain the updated rule f_i^f from f_{i-1}^f solve
      $$\frac{b_{t-1} + z_t(\psi_t) + g_t(\psi_t) - \tau_t(\psi_t) a_t h_t(\psi_t)}{f^f(\psi_t)} = \beta \mathbf{E}_t\left[\frac{c(\psi_t)}{c(\psi_{t+1}})\right]. \tag{E.4}$$
      Above, $\psi_t = (b_{t-1}, a_t, g_{t-1}, z_{t-1}, r_t)$ and the left-hand side variables z_t, g_t, τ_t, h_t and c_t are evaluated using ψ_t and model equations (18), (15), (23), (17) and (16). Next, the right-hand side integral is evaluated over $[a_{t+1}, g_{t+1}, r_{t+1}, z_t, b^*_{t+1}]$ with normally distributed shocks using numerical quadrature and interpolating over fiscal limit distribution as follows:
      $$\mathbf{E}_t\left[\frac{1}{c_{t+1}}\right] = \int_{\varepsilon^a_{t+1}}\int_{\varepsilon^g_{t+1}}\int_{r_{t+1}}\int_{b^*_{t+1}} \frac{1}{c_{t+1}}_{\text{no default}} = \int_{\varepsilon^a_{t+1}}\int_{\varepsilon^g_{t+1}}\left\{\frac{1}{c_{t+1}}\Big|_{\text{no default}}\right\} \tag{E.5}$$
      Technically,
         A. Given a grid point (a_t, g_t, r_t, z_t) and following (14), (15) evaluate a_{t+1}, and g_{t+1} for all possible ε^a_{t+1}, ε^g_{t+1}.
         B. Calculate $\tau_{t+1} = \tau_{t+1}(f^f(\psi_t), a_{t+1})$ for the case of no default, the associated consumption $c_{t+1} = c_{t+1}(a_{t+1}, g_{t+1}, \tau_{t+1})$ and $1/c_{t+1}$.
         C. Using numerical quadrature estimate the right–hand side of the integral (E.5) over the grid.
      iii. Repeat the whole procedure for the LHS and RHS with Sim's algorithm unless the convergence is achieved. The rule for which $|LHS - RHS| < 10^{-6}$ is considered for f_i^f.
      iv. Check the convergence between the successive iteration of the debt decision rule. If $|f_i^f - f_{i-1}^f| < 1e-6$, f_i^f is the risk–free debt rule b_t^f. Otherwise, if the number of iteration does not surpass the limit $k^{\max}$ go to Step 2(b)ii, else the model does not converges at this grid point.
      v. For a given $(a_t, g_{t-1}, r_t, z_{t-1})$ interpolate the solution over all starting points b_{t-1} to define the decision rule for those grid points where the iteration method does not converges or the obtained debt rule does not live within $[0,1]$.
   (c) Calculate the risk–free bond price for the obtained risk–free debt rule $b_t^f = t^f(\psi_t)$
   $$q_t^f(\psi_t) = \frac{b_{t-1} + g_t(\psi_t) + z_t(\psi_t) - a_t \tau_t(\psi_t)\left[c_t(\psi_t) + g_t\right]}{b_t^f(\psi_t)}. \tag{E.6}$$

(d) Finally combining (E.3) and (E.6) evaluate the default risk premium for ψ_t:

$$r_t(\psi_t) = \frac{1}{q_t(\psi_t)} - \frac{1}{q_t^f(\psi_t)}. \tag{E.7}$$

Appendix E.2 Matlab Code

Debt Rule and Default Interest Rate

```
upper_limit = 1;
lower_limit = 0;
q_grid = zeros(b_num, a_num, g_num, z_num, r_num);

for idx_b = 1 : b_num
    for idx_a = 1 : a_num
        for idx_g = 1 : g_num
            for idx_z = 1 : z_num
                for idx_r = 1 : r_num
                    k = 0;
                    ttax = tax_ss + gamma * (b_grid(idx_b) - b_ss);
                    gg = g_grid(idx_g) * rho_g + g_ss * (1-rho_g) ...
                        + (a_grid(idx_a)  -  a_ss) * zeta_g;
                    cc = (a_grid(idx_a) - gg) * (1 - ttax) / (1 + phi - ttax);
                    if idx_r == 1
                        zz = mu_1 * z_grid(idx_z) ...
                            + zeta_z * (a_grid(idx_a) - a_ss);
                    else
                        zz = mu_2 * z_grid(idx_z) ...
                            + zeta_z * (a_grid(idx_a) - a_ss);
                    end
                    b_val = b_grid(idx_b) + gg + zz - ttax * (cc + gg);
                    b_rule = b_val / betta;
                    rc = -1;
                    b_diff = Inf;
                    while  k <= k_max && (rc ~= 0 || b_diff >= tol_conv)
                        ind_state = [b_grid(idx_b), a_grid(idx_a), ...
                             g_grid(idx_g), z_grid(idx_z), idx_r];
                        [x,rc] = csolve_nonlin(@EulerEqn, ...
                            b_rule, ind_state);
                        b_diff = abs(b_rule - x(1));
                        b_rule = x(1);
                        k = k + 1;
                    end
                    if k > k_max || b_rule < lower_limit ...
                         || b_rule > upper_limit
                        q_grid(idx_b, idx_a, idx_g, idx_z, idx_r) = NaN;
                    else
                        q_grid(idx_b, idx_a, idx_g, idx_z, idx_r) = ...
                            b_val / b_rule;
                    end
                end
            end
        end
    end
```

```
end

function [ diff ] = EulerEqn( b, varargin ) %#ok<*DEFNU>
    rule = b;
    args = cell2mat(varargin);
    b_d = args(1);
    a = args(2);
    g = rho_g * args(3) + (1 - rho_g) * g_ss + zeta_g * (a - a_ss);
    r = args(5);
    if r == 1
        z = mu_1 * args(4) +  zeta_z * (a - a_ss);
    else
        z = mu_2 * args(4) +  zeta_z * (a - a_ss);
    end
    e_rhs_a  = zeros(shk_a_num, 1);
    e_rhs_ag = zeros(shk_g_num, 1);
    e_rhs_agz = zeros(shk_z_num, 1);
    tmp_d_1 = zeros(num_delta, 1);
    tmp_d_2 = zeros(num_delta, 1);
    tax = tax_ss + gamma * (b_d - b_ss);
    c = (a - g) * (1 - tax) / (1 + phi - tax);
    lhs = (b_d + g + z - tax * (c + g)) / rule;
    e_tax_n = tax_ss + gamma * (rule - b_ss);
    e_inv_c_tmp = (1 + phi - e_tax_n) / (1 - e_tax_n);
    for i_a = 1 : shk_a_num
        e_a = rho_a * a + (1 - rho_a) * a_ss + shk_a_grid(i_a);
        for i_g = 1 : shk_g_num
            e_g = rho_g * g + (1 - rho_g) * g_ss + zeta_g * (e_a - a_ss) ...
                + shk_g_grid(i_g);
            for i_z = 1 : shk_z_num
                e_z_1 = mu_1 * z + zeta_z * (e_a - a_ss) + shk_z_grid(i_z);
                e_z_2 = mu_2 * z + zeta_z * (e_a - a_ss) + shk_z_grid(i_z);
                Phi_1 = InterpolateGrid(4,...
                    rule,  x_num,   x_grid, e_a,   a_x_num, a_x_grid, ...
                    e_g, g_x_num, g_x_grid, e_z_1, z_x_num, z_x_grid, ...
                    cdf_x(:, :, :, :, 1));
                Phi_2 = InterpolateGrid(4,...
                    rule,  x_num,   x_grid, e_a,   a_x_num, a_x_grid, ...
                    e_g, g_x_num, g_x_grid, e_z_2, z_x_num, z_x_grid, ...
                    cdf_x(:, :, :, :, 2));
                e_inv_c_n = e_inv_c_tmp / (e_a - e_g);
                for i_d = 1 : num_delta
                    e_tax_d = tax_ss ...
                        + gamma * ((1 - delta_grid(i_d)) * rule - b_ss);
                    e_c_d = (e_a - e_g) * (1 - e_tax_d) / (1 + phi - e_tax_d);
                    tmp_d(i_d) = (1 - delta_grid(i_d)) ...
                        * pdf_delta(i_d) / e_c_d;
                end
                e_inv_c_d = sum(tmp_d_1);
                e_rhs_agz(i_z) = betta * c  * pdf_shk_z(i_z) * ( ...
                    e_inv_c_d * (Phi_1 * P(r, 1)  ...
                        + Phi_2 * P(r, 2) ) ...
                    + e_inv_c_n * ((1 - Phi_1) * P(r, 1) ...
                        + (1 - Phi_2) * P(r, 2) ));
```

```
                end
                e_rhs_ag(i_g) = 0.5 * (2 * sum(e_rhs_agz) - e_rhs_agz(1) ...
                    - e_rhs_agz(shk_z_num)) * shk_z_step * pdf_shk_g(i_g);
            end
            e_rhs_a(i_a) = 0.5 * (2 * sum(e_rhs_ag) - e_rhs_ag(1) ...
                -  e_rhs_ag(shk_g_num)) * shk_g_step * pdf_shk_a(i_a);
        end
        e_rhs = 0.5 * (2 * sum(e_rhs_a) - e_rhs_a(1) - e_rhs_a(shk_a_num)) ...
            * shk_a_step;
        diff = lhs - e_rhs;
end
```

Risk Premium

```
prem_grid = zeros(b_num, a_num, g_num, z_num, r_num);

for idx_b = 1 : b_num
    for idx_a = 1 : a_num
        for idx_g = 1 : g_num
            for idx_z = 1 : z_num
                for idx_r = 1 : r_num
                    k = 0;
                    ttax = tax_ss + gamma * (b_grid(idx_b) - b_ss);
                    gg = g_grid(idx_g) * rho_g + g_ss * (1-rho_g) ...
                        + (a_grid(idx_a)  -  a_ss) * zeta_g;
                    cc = (a_grid(idx_a) - gg) * (1 - ttax) / (1 + phi - ttax);
                    if idx_r == 1
                        zz = mu_1 * z_grid(idx_z) + zeta_z * (a_grid(idx_a) - a_ss);
                    else
                        zz = mu_2 * z_grid(idx_z) + zeta_z * (a_grid(idx_a) - a_ss);
                    end
                    b_val = b_grid(idx_b) + gg + zz - ttax * (cc + gg);
                    b_rule = b_val / betta;
                    rc = -1;
                    b_diff = Inf;
                     while  k <= k_max && (rc ~= 0 || b_diff >= tol_conv)
                            ind_state = [b_grid(idx_b), a_grid(idx_a) , ...
                                 g_grid(idx_g) , z_grid(idx_z) , idx_r];
                            [x,rc] = hFnc(...
                                hEulerFncNoDefault, ...
                                b_rule, ...
                                ind_state);
                            b_diff = abs(b_rule - x(1));
                            b_rule = x(1);
                            k = k + 1;
                        end
                        if k > k_max || b_rule < lower_limit || b_rule > upper_limit
                            prem_grid(idx_b, idx_a, idx_g, idx_z, idx_r) = NaN;
                        else
                            q_grid(idx_b, idx_a, idx_g, idx_z, idx_r) = b_val / b_rule;
                        end
                end
            end
        end
    end
end
```

```
function [ diff ] = EulerEqn_noDef( b, varargin ) %#ok<*DEFNU>
    rule = b;
    args = cell2mat(varargin);
    b_lag = args(1);
    a = args(2);
    z = args(4);
    r = args(5);
    g = rho_g * args(3) + (1 - rho_g) * g_ss + zeta_g * (a - a_ss);
    if r == 1
        z = mu_1 * z + zeta_z * (a - a_ss);
    else
        z = mu_2 * z + zeta_z * (a - a_ss);
    end
    e_rhs_a  = zeros(shk_a_num, 1);
    e_rhs_ag = zeros(shk_g_num, 1);
    tax = tax_ss + gamma * (b_d - b_ss);
    c = (a - g) * (1 - tax) / (1 + phi - tax);
    lhs = (b_d + g + z - tax * (c + g)) / rule;
    e_tax = tax_ss + gamma * (rule - b_ss);
    e_inv_c_tmp = (1 + phi - e_tax) / (1 - e_tax);
    for i_a = 1 : shk_a_num
        e_a = rho_a * a + (1 - rho_a) * a_ss + shk_a_grid(i_a);
        for i_g = 1 : shk_g_num
            e_g = rho_g * g + (1 - rho_g) * g_ss + zeta_g * (e_a - a_ss) ...
                + shk_g_grid(i_g);
            e_rhs_ag(i_g) = pdf_shk_g(i_g) / (e_a - e_g);
        end
        e_rhs_a(i_a) = 0.5 * (2 * sum(e_rhs_ag) - e_rhs_ag(1) - e_rhs_ag(shk_g_num)) ...
            * shk_g_step * pdf_shk_a(i_a);
    end
    e_rhs = e_inv_c_tmp * betta * c * 0.5 * (2 * sum(e_rhs_a) - e_rhs_a(1) ...
        - e_rhs_a(shk_a_num)) * shk_a_step;
    diff = lhs - e_rhs;
end
```

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