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Closer to Finding Yeti

Rethinking the output gap methodology through the lens of fiscal policy stance plausibility

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Closer to Finding Yeti[1]

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ABSTRACT

This paper offers a synthesis of several approaches to measuring output gap in Slovakia and serves as an update of the original CBR work Finding Yeti after almost a decade. A "suite of models" approach is estimated and assessed to provide advantages over single models. Following the recommendation of the EU IFIs guide suggesting no one-size-fits-all approach for measuring output gap, our family of methods consist of two unobserved component models, principal component model, semi-structural model and Modified Hamilton filter. We propose a novelty approach to weighting the individual models capturing recent structural innovations in the economy to construct one central estimate of the output gap. Such a robust estimate is maximising its overall plausibility and applicability to prudent fiscal policy assessment.

Keywords: output gap, unobserved component, trend, cycle, plausibility, Bayesian analysis, estimation

JEL - codes: C11, C13, C32, E32, E62

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Content

List of figures and tables

1. Non-technical summary

Output gap estimates represent a fundamental toolkit supporting prudent policy decisions. Yet, as stated in the original CBR work Finding Yeti (Ódor and Kucserová (2014), hereafter FYWP), searching for this unobservable variable in a small open economy is an art rather than a science. By this paper, we are abandoning the strategy of simple averaging of six output gap estimates as outlined in the FYWP. The main "philosophical" change from the FYWP is that output gap estimates from international and domestic institutions with long historical expertise do not enter directly into the calculation of the CBR's output gap. In terms of technical differences from the FYWP, we propose various new models and the only methodology kept (but revised) from the FYWP is the principal component model. A novelty is the statistical weighting scheme, that anchors newly proposed models to output gaps produced by external institutions and finally leads to the CBR´s central output gap estimate. We closely pay attention to plausibility and consistency of signals and in terms of revisions, we assess stability and performance at key turning point.

Guided by the EU IFI recommendation, we first design "a suite of models", instead of relying on just a single approach. We start with a backward-looking unobserved component model (UCM A) and a model with a country-specific extension (UCM B). Thereafter, we design a forward-looking semi-structural model (SSM) estimated with Bayesian techniques. Owing to its semi-structural architecture, a strong story-telling feature facilitates deeper understanding of ongoing changes and substantive relationships between key economic variables. Bayesian technique enables us to encompass the certainty and significance of the model-drawn relations and their importance in observed economic development. By incorporating foreign economy variables, we address specific attributes of the Slovak economy - high openness and euro-zone membership[6]. Furthermore, from the group of simple statistical filters we apply only one-sided Modified Hamilton filter that reduces the end-point problem. As the last element of our toolkit, we revise the original CBR (from the FYWP) principal component model (PCA) with a new time-varying variable methodology, delivering more satisfactory results even during the Covid-19 pandemic.

From our point of view, the element of plausibility of real time output gap estimate is crucial, as it enters the calculation procedure when monitoring and assessing the underlying structural budget balance of public finances. For example, the output gap estimate might serve as a decisive factor for triggering an escape clause in a balanced budget rule (fiscal compact). That said, its importance dramatically increases when macroeconomic shocks complicate the trade-off between stabilisation and debt sustainability goals and, it is necessary to secure a properly and timely withdrawal of fiscal stimulus. Indeed, the consensus finding (e.g. Egert (2014), Auerbach (2009), Fatas and Mihov (2003)) is that correct cyclically-adjusted primary balance should provide a more reliable picture of the overall policy stance and guide the adjustment of policies and fine-tuning of automatic stabilizers that are anchored to past measures.

All newly developed models, as well as the original set of models from the FYWP, are tested against various criteria. We review stability of the estimates as a key for their reliability and accuracy, as well

6 Detailed analysis of SSM Bayesian estimation results and properties on model parameters are discussed in Box 1

as performance at turning points of business cycles. Stability tests shows that univariate statistical filters, both Hodrick-Prescott and Modified Hamilton filter, perform the worst together with original Multivariate Kalman filter (MVKF) from the FYWP. PCA (both the original from the FYWP and the newly developed model) and forward-looking SSM significantly beat other models in this property. However, PCA and MVKF from the FYWP give counterintuitive magnitude of the output gap in 2020, also if economic information from 2021 is considered. Revised PCA, SSM and newly developed UCMs provide more promising results in this respect. When assessing the ability of models to capture turning points, UCMs together with Modified Hamilton filter provide more satisfactory results than SSM and PCA model.

Given the mixed conclusions of models' properties, a weighting procedure derived from estimates of external institutions is applied and secures credibility in obtaining one central output gap estimate. Routine use of models in the CBR estimation process will include automatic revisions, that we advise to update at an annual frequency[7]. More sophisticated revisions and updates of the whole modelling toolkit might be considered in the future as well, as "suite of models" do not have to accommodate all structural changes in economy and economic shocks that might potentially emerge. New models with more sophisticated economic background or estimation techniques will always be in the cards and must be carefully considered in the future as Closer to Finding Yeti is still a marathon rather than a sprint.

7 These updates include changes in models' parameters, shift of ten year moving window within which the central CBR output gap estimate is calculated and last but not least, weights of individual models are re-estimated annually as well.

2. Introduction

Almost a decade has passed since the pioneering work on the output gap methodology[8] by the Council for Budget Responsibility (CBR, 2014) was published, and another complete business cycle was observed. Longer time series history has opened options for practitioners to employ more sophisticated trend-cycle decompositions of gross domestic product, labour market variables or inflation. Moreover, Slovak country-specific issues are notable. As a converging economy, it experiences frequent structural changes and cannot rely on estimates from longer time series. These include particularly shocks to automotive industry and high concentration of EU funds absorption in some years. Together with global shocks such as Covid-19 pandemic, war in Ukraine and ongoing green transition it implies that modelling output gap and potential output requires paying close attention to various specific details and technicalities.

The main goal of this paper is to propose a robust and central CBR estimate of output gap. To fulfil this objective, we reconsider in-house estimation procedure of output gap and propose new models and approaches. The stress in this paper is put mainly on plausibility and consistency of signals, but at the same time stability and performance at key turning points are assessed. The CBR as a fiscal watchdog is mainly interested in the point estimate of output gap, as this value enters the calculation of the cyclically adjusted budget balance for monitoring compliance with fiscal rules and advising policy recommendations. Nonetheless, we pay close attention to uncertainties and model deviations from the point estimate. To fulfil our goals in all aspects, various univariate and multivariate models are proposed and tested. Afterwards, a weighting procedure of deriving a central estimate of the output gap from “a suite of models” is applied.[9] Nevertheless, we do not fully abandon checking the credibility of estimates’ range compared with other institutional estimates.

So far, the CBR has followed the methodology developed in the FYWP almost a decade ago. It means, that for the real-time assessment of the cyclical position of the economy, a simple average of three CBR’s methods and three output gap estimates produced by external institutions have been used. Up to now, the CBR’s toolkit has included the traditional Hodrick-Prescott (HP) filter, the Unobserved component model estimated with the Kalman filter (MVKF) and the principal components analysis (PCA) method for the historical and real-time estimates of the output gap. External output gap estimates have included the production function approach (PF), specifically from the National Bank of Slovakia (Reľovský and Široká, NBS 2009)[10], the Ministry of Finance (Priesol, MoF 2021) and the European Commission (Blondeau, Planas and Rossi, EC 2021).

In this paper, we abandon this strategy from the FYWP. Instead of that, we first propose to estimate the already mentioned “suite of models”[11] for nowcasting and backcasting of output gap estimate. In the second step, we calculate the mean of five output gap estimates from prominent domestic and

8 The FYWP.

9 In contrast, central banks might primarily seek for credible range estimates when the commitment to an inflation target is symmetric.

10 Production function is utilized in the forecasting procedure. For historical and nowcasting estimates, UCM model is the main tool employed by the NBS.

11 In contrast to Production function that is recommended to be used in the case of medium or long run projections.

foreign forecasting institutions. This institutional mean further serves as a benchmark for the calculation of deviations of each newly developed CBR model. Based on a calculated deviation from the benchmark, each model has assigned weight, and one central estimate is obtained. From our point of view, this procedure offers a transparent and competitive alternative to expert judgement practice, as it helps to reduce uncertainty around estimate.

The rest of the paper is organized as follows. Section 3 briefly summarizes the latest developments in the output gap estimation methodologies in relevant empirical literature. Section 4 is dedicated to modelling techniques and provides a detailed description of all newly proposed models. Results and properties of the methods are discussed in Section 5, including a technical roadmap to extract the central output gap estimate of the CBR. Section 6 concludes the article and proposes further work.

3. Overview of literature and models used by other institutions

Our main reference point is the EU IFIs guide for testing output gaps (2022) in which authors developed a framework to assess and compare standard approaches – univariate filters, multivariate filters and a production function. All methods are inspected in 6 categories – stability, plausibility, performance at turning points, consistency of signals, variability of potential output growth and avoiding procyclicality of potential output growth. Their findings are strongly encouraging towards a multivariate approach[12], as it satisfies the most properties in the analysed areas. On the other hand, it cannot beat a "suite of models" approach, which is shown to be superior vis-a-vis the individual output gap estimation methods. The suite of models is advocated also by Casey (2019, IFAC[13]), Office for Budget Responsibility (2011, OBR), Stock and Watson (1999) and ultimately also by Ódor and Kucserová (2014, CBR). In addition, most of these authors highlight the concept of "expert judgment" as perhaps unavoidable element in the process.

For instance, the role of judgement is explained in detail by the UK's Office for Budget Responsibility. Murray (2014, OBR) indicates, that there is probably no single method sufficient for all different periods of business cycle to be adequately assessed. Therefore, he proposed nine different approaches to be compared during business cycle[14]. More recently, OBR (2022) even more stresses the importance of expert judgment over mechanically derived estimates, especially during the periods like the coronavirus pandemic.

The European Commission (EC) reflects its contribution to the topic in the paper written by Blondeau, Planas and Rossi (2021). Authors exhaustively describe the process (and also software) of building the production function type of output gap estimate, based on the Commonly Agreed Methodology by the European Union. The production function type of output gap is also used by the MoF (Priesol, 2021), as a product from a backward-looking structural econometric model.

The contribution of the EU IFIs' (2022) assessment is the finding that the production function has the most procyclical estimates of potential output growth compared to other models. This assessment was conducted on 10-year average, and we will later use this value for statistical weighting scheme.

Regarding central banks contributions, we start with the National Bank of Slovakia. Ostapenko (2022, NBS) carried out an extensive analysis on Slovak data with the newly adopted models such as Bayesian vector autoregression, Modified Hamilton filter and a Dynamic factor model, for the purpose of widening the NBS forecasting and analytical toolkit. She finds, that the newly developed models perform well in terms of stability of output gap and they are similar in magnitudes to the official NBS estimate derived from the large scale UCM model in the spirit of Tóth (2021, ECB)[15], calibrated for Slovak data. Ostapenko also tests information about the financial cycle in connection to output gap. Her study identifies no link between financial variables and the Slovak output gap confirming earlier

12 State-space system estimated with Kalman filter.

13 Irish Fiscal Advisory Council

14 Various univariate and multivariate filters (various small scale UCMs), principal component model as well as a production function.

15 The author links an unobserved component model of 6 variables with Cobb-Douglas production function.

results of the CBR (2014). This is a reason why we also do not examine financial variables when developing UCM, although we do include one financial indicator in our PCA output gap estimate, as it helps to make the resulting PCA output gap more plausible. Last but not least, important message by Ostapenko (2022) is the application of expert judgement in the official NBS output gap estimates even in the situation of rich institutional toolkit.

After careful review of all already mentioned models and considering the most recent available information on the output gap estimation, we decided to shape our baseline model in the spirit of Melolinna and Tóth's (2016, BoE[16]) small-scale Unobserved component model (UCM)[17]. The authors decompose key macroeconomic observable variables (GDP, inflation and unemployment) into trends and unobservable cyclical components. Subsequently, they enrich the model with relevant variables of interest. We adequately follow and modify their strategy, particularly we do not consider financial cycle as a relevant explanatory variable for Slovakia.

Instead, we follow the strategy of Beneš et al. (2010, IMF), incorporating capacity utilization as an additional explanatory variable. Authors showed for various countries, that a small macroeconomic model (UCM) performs better against a random walk model in terms of forecasting inflation and revisions of current estimates of output gap are less sensitive in comparison to HP filter.

Supporting our decision about the UCM development strategy, Barbarino et al. (2021, FED[18]) compare different modifications of a small scale UCM, similar to our model, and find in terms of various metrics, no evidence that any model specification is clearly superior to others. On the other hand, the authors suggest including Okun's law that improves real-time stability by alleviating the end-point problem. Authors also document the superiority of UCMs over univariate models in terms of stability.

One possible alternative method to the UCM is represented by a family of small-scale semi-structural macroeconomic models introduced by Alichi (2015) and Blagrave et al. (2015). Although their structure is very similar to the UCM, they employ forward-looking Phillips curve instead of the purely backward-looking method preferred by UCM models. They highlight the crucial role of including capacity utilization as a latent variable in explanation of business cycle development and in reliable identification of potential output. Furthermore, they emphasize the importance of incorporating labour force participation in the model building blocks. As the participation rate has been steadily increasing over last two decades (with exception of the Covid-19 pandemic shock), this feature could be relevant in proper estimation of the output gap and identification of its drivers.

When addressing forecasting and policy analysis objectives, central banks often utilize various-scale quarterly projections macroeconomic models (e.g. Armour et al. (2002), Carabenciov et al. (2008), De Resende et al. (2022), Benk et al. (2006), Beneš et al. (2017), Juillard et al. (2008)). New-Keynesian

[16] Bank of England

[17] We discard the ambition to develop large-scale UCM by Toth (2021) on the back of two reasons. First, it is operated by NBS and its implementation to the CBR routine would present a duplicity of analytical work at national level, while some degree of co-fluence is expected among relatively small analytical bodies. Second, our preference is to adopt models that can be easily operated in quarterly routine of CBR, or when flash estimates of cyclically adjusted balance are needed.

[18] Board of Governors of the Federal Reserve System

building principles and semi-structural form enables them to incorporate numerous features of domestic economy, capture specifics of open economies or different monetary policy rules at any level of detail. Backed by micro-foundation, consistency with economic theory, strong explanation ability and Bayesian estimation techniques (see e.g. Berger (2011) and Planas et al.(2008)), they provide both stable and plausible dynamics and sensible forecasts of the output gap.

To complement the multivariate modelling toolkit, we add a univariate filter to the portfolio of our models. Traditional Hodrick-Prescott (1997) filter has been replaced by the Hamilton (2018) filter in the empirical literature in recent years. We follow a modification by Quast and Wolters (2020). Applied to Slovak data by Ostapenko (2022), it was found that together with one-sided HP filter (and BVAR) it outperforms other methods in predicting core inflation. Additionally, it also beats models with labour market indicators in predicting wage inflation, which yield benefits in times of elevated price-wage spiral. Ultimately, important expected contribution resides in reducing the end-point problem in contrast to HP filter.

4. Modelling techniques

4.1. Backward looking approach – (UCM A, B)

In line with the literature review and best institutional practice, we start with the small backward looking semi-structural model (UCM A), inspired by Melolinna and Tóth (2016). It is a linear unobserved component model designed in a state space form. Employing Kalman filter, three key observable variables are jointly decomposed into the trend ($\bar{x}$) and the unobserved ($\hat{x}$) component (1. 2) - (1. 4). Taking into account that Slovakia is highly industrialized economy and new export capacities explain a decisive part of gross domestic product growth over past two decades, we add capacity utilization as an additional measurement equation (1. 5) to be decomposed in the alternative model specification (UCM B) as proposed by Beneš et al. (2010) and Kátay et al. (2020):

(Model) $$x_t = \bar{x}_t + \hat{x}_t \qquad x = \{y, u, \pi, \kappa\} \tag{1. 1}$$

(Output - real GDP) $$y_t = \bar{y}_t + \hat{y}_t \tag{1. 2}$$

(Unemployment rate) $$u_t = \bar{u}_t + \hat{u}_t \tag{1. 3}$$

(Inflation) $$\pi_t = \bar{\pi}_t + \hat{\pi}_t \tag{1. 4}$$

(Capacity utilization) $$\kappa_t = \bar{\kappa}_t + \hat{\kappa}_t \tag{1. 5}$$

We treat potential output as an I(1) process. Therefore, the random walk model (1. 6) with drift (g) is specified, which itself follows a random walk (1. 7). This specification allows to capture dynamic adjustments in the growth rate of trend output, for example due to changing productivity or demography (Barbarino, 2020), present also in the Slovak economy. The corresponding output gap equation (1. 8) is governed by an AR(2) process:

(Potential output) $$\bar{y}_t = \bar{y}_{t-1} + g_{t-1} + \varepsilon_t^{\bar{y}} \tag{1. 6}$$

$$g_t = g_{t-1} + \varepsilon_t^{g} \tag{1. 7}$$

(Output gap) $$\hat{y}_t = \alpha_1 \hat{y}_{t-1} - \alpha_2 \hat{y}_{t-2} + \varepsilon_t^{\hat{y}} \tag{1. 8}$$

Following well established macroeconomic relationships, labour marker structure is represented by the Okun´s law (1962). It links unemployment gap to lagged output gap to and its own lag (1. 9). The trend component – an approximation of long run equilibrium unemployment rate, is set as a local linear trend model (1. 10), where innovations are captured also by its "growth" rate (1. 11):

(Okun's law) $$\hat{u}_t = \gamma_1 \hat{u}_{t-1} - \gamma_2 \hat{y}_{t-1} + \varepsilon_t^{\hat{u}} \tag{1. 9}$$

(NAIRU) $$\bar{u}_t = \bar{u}_{t-1} + \overline{gu}_{t-1} + \varepsilon_t^{\bar{u}} \tag{1. 10}$$

$$\overline{gu}_t = \rho_1 \overline{gu}_{t-1} + \varepsilon_t^{\overline{gu}} \tag{1. 11}$$

For modelling inflation, we follow the consensus of recent studies, where cyclical inflation is modelled as an AR(1) process, while it is also correlated with lagged output gap (1. 12). Trend inflation is considered being a random walk without drift (1. 13):

(Phillips curve) $$\hat{\pi}_t = \beta_1 \hat{\pi}_{t-1} + \beta_2 \hat{y}_{t-1} + \varepsilon_t^{\hat{\pi}} \quad (1.\ 12)$$

(Trend inflation) $$\bar{\pi}_t = \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}} \quad (1.\ 13)$$

The model UC B incorporates industrial capacity utilization. In this case, the idea of extending the model is very similar to the Phillips curve (1. 14), except for the assumption of output gap to have lagged form:

(Capacity utilization gap) $$\hat{\kappa}_t = \theta_1 \hat{\kappa}_{t-1} + \theta_2 \hat{y}_t + \varepsilon_t^{\hat{\kappa}} \quad (1.\ 14)$$

(Potential capacity utilization) $$\bar{\kappa}_t = \bar{\kappa}_{t-1} + \varepsilon_t^{\kappa} \quad (1.\ 15)$$

The estimation strategy for both UC models is similar. Parameters are estimated by maximum likelihood method that is used for Kalman filter as well as smoother. For the purpose of this paper, we utilized the derived values from the smoother.

4.2. Forward looking approach - (SSM)

The theoretically richer method combines New-Keynesian principles with the semi-structural trend-cycle formulation from the UCM in a very parsimonious way, while allowing for an interaction of key economy features. Hence, the UCM is extended in three ways.

Firstly, following Alichi (2015) and Blagrave et al (2015) to describe the price dynamics we employ a Phillips curve whose forward-looking nature enables us to incorporate market inflation expectations. Secondly, inspired by Armour et al. (2002) and Benk et al. (2006) in order to emphasize that Slovakia is a small open economy we enrich our model by a foreign economy block. International trade dynamics affect not only domestic output gap fluctuations via changes in foreign demand, but also domestic price level as production sector in Slovakia relies heavily on imports. Thirdly, increasing labour market participation rate[19] can partially explain gradual decline of unemployment rate. Consistently with UCM B we do not embrace financial variables in the model. Future extension plans do consider this option.

The state-space model is designed in a reduced trend-cycle form. The stochastic process describing the economy output is driven by three equations, and subject to three types of shocks. The level of potential output ($\bar{y}_t$) dynamics (2. *2*) follows a random walk with a drift term, the potential growth ($\gamma_t^{\bar{y}}$) and a level-shock term ($\varepsilon_t^{\bar{y}}$). The potential growth (2. 3) evolves accordingly to an AR(1) process

[19] Authors show that the new block on labour force participation rate also enriches the model, but we do not see it as crucial as the block on capacity utilization. This is likely because the original model already included another labour market block.

with a growth-specific shock ($\varepsilon_t^{\gamma^{\bar{y}}}$) and autocorrelation term controlling the speed of adjustment ($\rho_{\bar{y}}$) to the steady-state growth path while enabling gradual changes in the trend growth. Furthermore, in order to address the economy openness and large share of export in domestic production, we augment the corresponding UCM output gap ($\hat{y}_t$) equation by the terms representing cyclical dynamics of the foreign demand[20] ($\hat{y}_t^*$; $\hat{y}_{t-1}^*$) and the real exchange rate (2. 8) fluctuations ($\hat{z}_t$). Finally, output gap equation (2. 4) is enriched by the economic sentiment indicator[21] cycle component[22] ($\hat{\theta}_t$) and the demand shock ($\varepsilon_t^{\hat{y}}$).

(GDP decomposition)	$y_t = \bar{y}_t + \hat{y}_t$	(2. 1)
(GDP level)	$\bar{y}_t = \bar{y}_{t-1} + \gamma_t^{\bar{y}} + \varepsilon_t^{\bar{y}}$	(2. 2)
(GDP trend growth)	$\gamma_t^{\bar{y}} = \rho_{\bar{y}}\, \gamma_{t-1}^{\bar{y}} + (1-\rho_{\bar{y}})\, \gamma^{\bar{y}} + \varepsilon_t^{\gamma^{\bar{y}}}$	(2. 3)
(Output Gap)	$\hat{y}_t = \rho_{\hat{y},1}\hat{y}_{t-1} + \rho_{\hat{y},2}\hat{y}_{t-2} + \alpha_{\hat{y}}^{\hat{y}^*,0}\hat{y}_t^* + \alpha_{\hat{y}}^{\hat{y}^*,1}\hat{y}_{t-1}^* + \alpha_{\hat{y}}^{\hat{z}}\hat{z}_t + \alpha_{\hat{y}}^{\hat{\theta}}\hat{\theta}_t + \varepsilon_t^{\hat{y}}$	(2. 4)

Concerning the three output-related shocks, we include a forward-looking Phillips curve for price inflation cycle component which links the dynamics of the unobserved output gap to the observable data on inflation. Moreover, as Slovakia's industrial sector is highly import-dependent, trading partners´ fluctuation in prices, captured by evolution of real exchange rate cycle component, affect its inflation dynamics. The trend term evolution is considered to be captured by a simple AR(1) process:

(Inflation decomposition)	$\pi_t = \bar{\pi}_t + \hat{\pi}_t$	(2. 5)
(Phillips curve)	$\hat{\pi}_t = \lambda\hat{\pi}_{t+1} + (1-\lambda)\hat{\pi}_{t-1} + \alpha_{\hat{\pi}}^{\hat{y}}\hat{y}_t + \alpha_{\hat{\pi}}^{\hat{z}}\hat{z}_t + \varepsilon_t^{\hat{\pi}}$	(2. 6)
(Trend inflation)	$\bar{\pi}_t = \rho_{\bar{\pi}}\bar{\pi}_{t-1} + (1-\rho_{\bar{\pi}})\,\bar{\pi} + \varepsilon_t^{\bar{\pi}}$	(2. 7)
(Real exchange rate change)	$\Delta z_t = \pi_t^* - \pi_t$	(2. 8)

Equations rendering an account of the unemployment rate development are further included to provide additional insights needed to estimate of the output gap. For the sake of simplicity we utilize the same „trend-cycle" setting to intercept the labor market movements. Following Alichi (2015) the unemployment gap ($\hat{u}_t$), defined as the difference between the actual unemployment rate and NAIRU follows an Okun's Law (2. 12), so its dynamics is affected not only by its past value but also by the

[20] Technically, we use weighted foreign imports of trading partners from the euro-area and V3 group. However, similar results are obtained when foreign output gap is employed instead. We assume that domestic economy business cycle is affected by both the past and current phases of the foreign business cycle.

[21] Economic sentiment indicator (ESI) for Slovakia published by Eurostat is considered being the leading harmonised composite indicator intercepting what is likely to happen in the short-term in domestic economy. Including sentiment indicator enables us to capture partially ongoing changes, market hysteresis and expectations about future development that have become important during 2008 - 2009 crisis recovery post - pandemic era and are even more relevant during ongoing inflation crisis (Agarwal (2022)).

[22] Measured in terms of the seasonally-adjusted deviation from its long-term mean scaled to 100. Thus, positive values indicate above-average economic sentiment and vice versa.

current phase of the business cycle. Time-varying part of NAIRU (ζ_t, (2.11)) - converges to its exogenously determined equilibrium value. It reflects gradual economy transition process, increased labor force mobility and impact of innovation that lead to increased dynamics over time (2. 10):

(Unemployment rate definition) $$u_t = \bar{u}_t - \hat{u}_t \qquad (2.9)$$

(NAIRU) $$\bar{u}_t = \rho_{\bar{u}}\bar{u}_{t-1} + (1-\rho_{\bar{u}})\,\bar{u} + \zeta_t + \varepsilon_t^{\bar{u}} \qquad (2.10)$$

$$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_t^{\zeta} \qquad (2.11)$$

(Okun's law) $$\hat{u}_t = \rho_{\hat{u}}\hat{u}_{t-1} + \alpha_{\hat{u}}^{\hat{y}}\hat{y}_t + \varepsilon_t^{\hat{u}} \qquad (2.12)$$

A gradual increase in labour market participation rate (q_t) reflecting deeper structural changes, economic transition and ongoing demographic changes is one of the reasons of a significant unemployment rate decline observed over the past decade (2. 15). Furthermore, gradually increasing participation rate contributes to labour market tightness[23] (denominator effect).

(Labour market participation rate) $$q_t = \bar{q}_t + \hat{q}_t \qquad (2.13)$$

(Participation rate trend) $$\bar{q}_t = \rho_{\bar{q}}\bar{q}_{t-1} + (1-\rho_{\bar{q}})\,\bar{q} + \varepsilon_t^{\bar{q}} \qquad (2.14)$$

(Participation rate cycle) $$\hat{q}_t = \rho_{\hat{q}}\hat{q}_{t-1} + \alpha_{\hat{q}}^{\hat{u}}\hat{u}_t + \varepsilon_t^{\hat{q}} \qquad (2.15)$$

Next, recalling Beneš et al. (2010) we extend the model by equations describing the evolution of capacity utilization (κ_t). Consistently with our UCM equations blocks we rely on „trend-cycle" formulation. Therefore, the negative cyclical component of the capital capacity utilization rate ($\hat{\kappa}_t$) indicates slack within firms (2. 18).

(Capacity utilization rate definition) $$\kappa_t = \bar{\kappa}_t + \hat{\kappa}_t \qquad (2.16)$$

(Capacity utilization rate trend) $$\bar{\kappa}_t = \rho_{\bar{\kappa}}\bar{\kappa}_{t-1} + (1-\rho_{\bar{\kappa}})\,\bar{\kappa} + \varepsilon_t^{\bar{\kappa}} \qquad (2.17)$$

(Capacity utilization rate cycle) $$\hat{\kappa}_t = \rho_{\hat{\kappa}}\hat{\kappa}_{t-1} + \alpha_{\hat{\kappa}}^{\hat{y}}\hat{y}_t + \varepsilon_t^{\hat{\kappa}} \qquad (2.18)$$

Finally, we decided to take ongoing changes in the foreign economy as given. Hence, rather than specifying the mutual relationship between foreign economy key variables (output and inflation) we let them to follow only simple AR(1) processes.

23 For detailed discussion see Zidong et al. (2021).

The model parameters are derived using Bayesian estimation techniques[24] (see Appendix 2, Figure 11, Figure 12**,** Figure 13**,** Table 4 and Box 1) and Kalman filter and smoother are used to decompose jointly key observable variables into their corresponding trend and cycle components (see Planas (2008)). Several observations arising from Bayesian estimation approach should be highlighted. Firstly, trend variables are very stable and have small variations, so the out-of-cycle stability is guaranteed also in the future. Next, unemployment gap is very persistent, which can reflect long-run impact of past structural changes that are only gradually transmitted to decline in both NAIRU and unemployment. Finally, strong vulnerability of Slovak economy and its dependence on foreign environment evolution is confirmed. More than half of the estimated output gap evolution is affected by changes in foreign economy.

Box 1: The SSM Bayesian Estimation Results

The SSM model parameters calibration relies on Bayesian approach using the Metropolis-Hastings algorithm.

Our estimation priors[25] are based on standard literature[26] (e.g. QPM model, Mucka (2016)), expert judgement and data vs. model moments comparison (see Figure 11 and Figure 12 in Appendix 2).

Resulting posterior distributions[27] are consistent with our expectations. There are several observations that are worthy of pointing out. Firstly, all parameters are well-estimated despite large trends present in data for unemployment, participation rate, capacity utilization rate and real GDP growth rate. Trend variables are persistent and have small variations with approximately 85 percent of information transmitted between periods, so the out-of-cycle stability is guaranteed. This result is particularly important as trends are not stationary due to undergoing structural changes occurring in the Slovak economy between 2006 and 2022. The changes are gradual and from historical shocks decompositions it follows that they are not reverted. Inasmuch as comparably large structural changes are not assumed in the subsequent years, currently estimated persistence and variance of trends define the corresponding lower and upper, respectively, bounds expected in the future.

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24 Bayesian approach is useful whenever prior information about parameter distribution (based on e.g. historical observation, expert judgement) is available and assesses estimates with the level of belief. Furthermore, it provides exact inferences (no need of asymptotic approximation) conditional on data and unlike classical inference it follows the likelihood principle. Moreover, resampling from posterior distributions allows to evaluate and communicate the level of uncertainty in model simulations. The posterior distributions are obtained using adaptive random-walk Metropolis-Hastings posterior estimator.

25 Individual prior distribution types arise from parameters domain and character with hyperparameters arising from the data vs. (calibrated) model moment comparison while keeping distribution supports wide in order to avoid too restrictive priors (see Table 4 in Appendix 2).

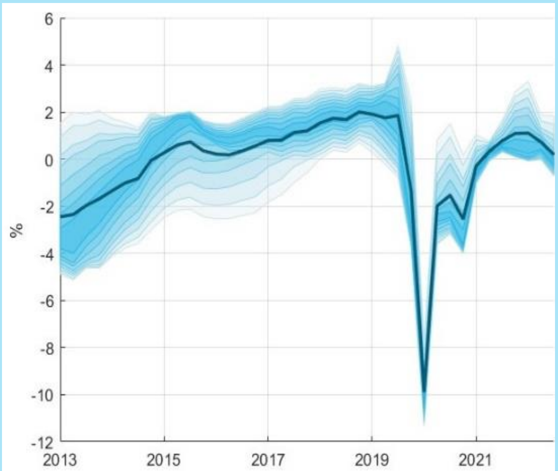
26 For instance QPM model (IMF 2023) or Mucka (2016).

27 The scale factor of the asymptotic covariance matrix and the corresponding decay rate were set to 2.25 and 0.75, respectively, to meet the required 0.24 acceptance rate assuming 100000 draws and 25% burning rate. The log posterior density attains 2346.22.

Larger deviations (see Figure 13 and Figure 14) are observed in real GDP evolution (standard deviation approximately 0.3 percent) and capacity utilization rate (1 percent) while only small variations are observed in trend inflation, unemployment rate and labor market participation rate. Estimated output gap has relatively low persistence and more than half of its evolution is affected by changes in foreign economy (output gap, real exchange rate) which are reflected by increasing pace in Slovak economy openness. The dominance of foreign environment is supported by Slovak economy openness, increased by one fourth to almost 200 percent of GDP between 2006 and 2022. Economic sentiment has only a minor role, possibly due to its partial incorporation in observed foreign data. Degree of inflation gap forward-lookingness (59 percent) is in-line with literature and is consistent with EU-SILC estimates on share of Ricardians in Slovakia.

Unemployment gap is very persistent, which can be partially explained by the influence of the participation rate gap and relatively low impact of the current business cycle phase. This could possibly reflect long-run impact of past structural changes that are only imperceptibly transmitted to decline in both NAIRU and unemployment, labor market inelasticity and tightness, or sizeable and steady structural unemployment limiting further unemployment decline without additional investment or innovations. Furthermore, high degree of unemployment persistency indicates that labor market reforms must be carefully designed as they have long-lasting impact on environment that might not be observed immediately after their implementation.

Figure 1: SSM Output gap estimate and confidence intervals (90%)



Source: CBR

Resampling from the Bayesian posterior distribution allows us to investigate periods of higher uncertainty in SSM estimate of output gap obtained using Kalman filter. The substantive degree of uncertainty in recent output gap estimates indicates strong impact of pandemics, ongoing energy and inflation crisis and economic downturn already expected in 2019 in foreign economies. This confirms high vulnerability and strong dependence of the Slovak economy on its foreign partners. Furthermore, it emphasizes the importance of careful model estimation and results interpretation especially during turbulent periods, when not only point estimates but posterior distributions, the degree of confidence, should be considered and communicated when assessing output gap estimates.

4.3. Modified Hamilton filter

The hope of progress for univariate filters from recent years lies in the models, that would be successfully minimizing the problems with the end-points (bias). After Hamilton (2017) criticism of Hodrick-Prescott filter, Quast and Wolters (2020) proposed slight modification of Hamilton original filter, that lies in replacing the 8 quarters ahead forecast errors of simple autoregressive process of real GDP with the mean of 4 to 12 quarters. Additional advantage of this approach should be in capturing turning points better, in comparison to basic trend-cycle filters. Therefore, we follow the strategy of Quast and Wolters and we try to fit the Modified Hamilton model on Slovak real GDP.

4.4. Principal Component Model (revisited)

Original PCA model (2014) has been routinely used in the CBR, but its results were at odds with other economic data after the outbreak of the Covid-19 pandemic in 2020. Therefore, we propose some refinements in the methodology. After testing various new variables and their performance in the model, we carefully choose proxies[28] for cyclical demand in services, industry, retail and construction in the domestic real economy. In the first step we make them time-varying based on their share in the real economy[29] (Figure 2). By standardizing them we create a set of 4 time-series jointly representing domestic demand pressures (Figure 3).

Figure 2: Weights of individual sectors in the economy (Value added approach)

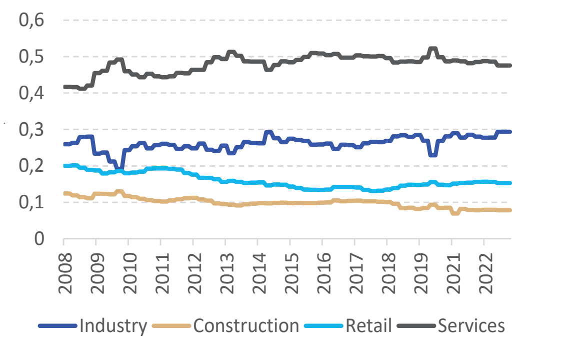
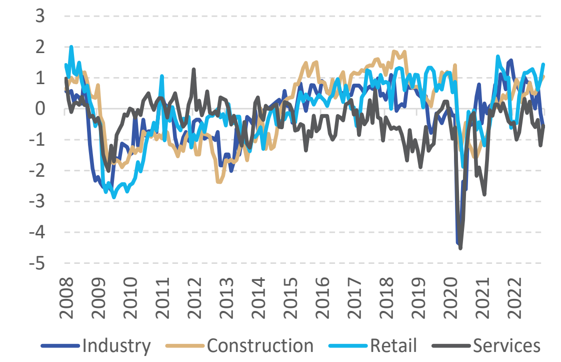


Figure 3: Weighted and standardized vintages of individual sectors in the economy



Source: CBR, Eurostat

Going further, current level of industry performance is covered by the capacity utilization rate[30]. In similar spirit as in the forward-looking UCM, economic sentiment indicator[31] is used as a composite leading indicator to proxy economic activity in the economies of key trading -partners. Finally, by incorporating the Index of Financial Stress, we plug-in the role of financial cycle and its expected growing importance in the future[32]. The final set of variables with their loadings is presented in Table 1.

28 More details are included in Appendix 4.

29 Weights are calculated based on value added shares.

30 Indicator is viewed as having additional information about the spare capacity in the sector, while variable industry relates only to the level of orders.

31 In this case Economic Sentiment Indicator for the EU27 is used, for more details see Appendix 4.

32 The FYWP (Ódor and Kucserová, (2014)), Melolinna and Tóth (2016).

Table 1: Correlations and loadings of PCA variables

Ordinary correlations:	ECONOMIC SENTIMENT INDICATOR	FINANCIAL STRESS	CAPACITY UTILIZATION	RETAIL	SERVICES	CONSTRUCTION	INDUSTRY	Principal component eigenvectors (loadings)
ECONOMIC SENTIMENT INDICATOR	1.00							0.42
FINANCIAL STRESS	0.31	1.00						0.26
CAPACITY UTILIZATION	0.71	0.33	1.00					0.44
RETAIL	0.61	0.33	0.77	1.00				0.41
SERVICES	0.20	0.49	0.15	0.13	1.00			0.19
CONSTRUCTION	0.64	0.22	0.73	0.72	0.15	1.00		0.41
INDUSTRY	0.83	0.46	0.72	0.58	0.41	0.68	1.00	0.44

Source: CBR, ECB, EC

In terms of estimation strategy, we are following the procedure proposed in the FYWP. The total variance explained by the first principal component is 54,4%. The model is estimated on quarterly basis, but as most of the time series are available on monthly basis, it brings additional benefit regarding real-time OG assessment in comparison to other models in this paper.

5. Results, weighting procedure and evaluation of model properties

In this section, a comparison of models' properties provides a starting point. As models score differently in various criteria, we follow with the step-by-step review of the process of deriving one central estimate from the "suite of model". Targeting a credible anchor, we define a strategy for weighting scheme of models' estimates based on five prominent forecasting institutions, to which we statistically link our estimate. The set of institutions is enlarged compared with the FYWP and includes not only the MoF, NBS and EC estimates, but also IMF and OECD estimates. We consider all these institutions particularly relevant as they regularly produce economic forecasts. The weight of each single CBR model is derived from the deviation from the institutional mean. Finally, we discuss our results and advocate the novel approach in detail.

5.1. Properties of Models

As a first step of our evaluation exercise, we check stability performance of the newly proposed models and compare their performance with the models from the FYWP. We partially follow the in-house approach suggested in the FYWP. The idea is to compare absolute difference between the estimate for year t compared to its estimate at time t+1 (average over horizon). Brief overview of models directly used in both papers[33] with corresponding abbreviations is provided in Table 2.

Table 2: Overview of models [34]

Model	Abbreviation	Paper
Hodrick-Prescott filter	HP (FYWP)	Finding Yeti (CBR, 2014)
Multivariate Kalman filter	MVKF (FYWP)	
National Bank of Slovakia (Multivariate Unobserved component model)*	NBS	
European Commision (Production function)*	EC	
Principal component analysis	PCA (FYWP)	
Ministry of Finance of the Slovak Republic (Production function)*	MoF	
Semi-structural model	SSM	Closer to Finding Yeti (CBR, 2023)
Principal component analysis	PCA	
Modified Hamilton filter	M. HAMILTON	
Unobserved Component model A	UCM A	
Unobserved Component model B	UCM B	

Source: CBR

[33] The weighting scheme in Closer to Finding Yeti paper is enriched also by model estimates from the IMF and the OECD. As we do not have historical vintages, they are not included in the evaluation of model properties.

[34] Models tagged by * are indirectly used also in Closer to Finding Yeti (weighting scheme). In the Finding Yeti working paper they are directly involved in output gap calculation.

Figure 4: Evaluation of ex post average revisions of output gap estimates for period 2013 – 2021 (2022, % of GDP) (*)[35]

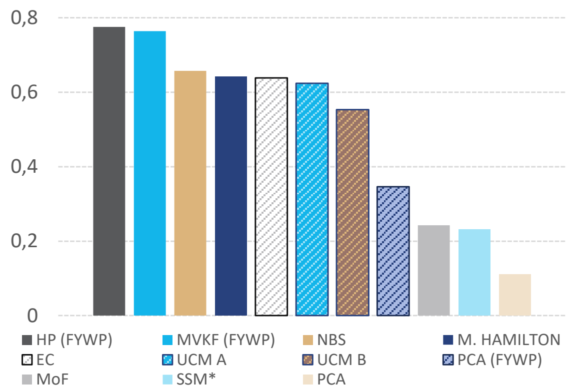
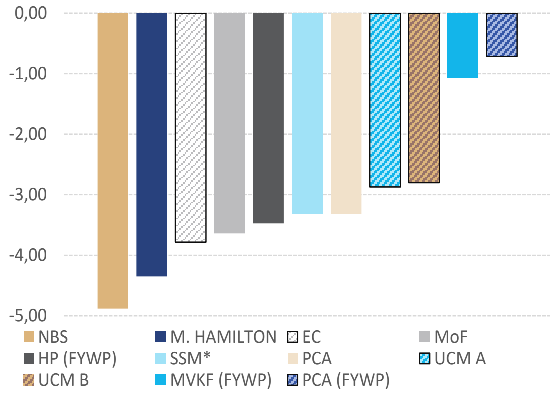


Figure 5: Point estimates of the output gap for 2020 (2021, % of GDP) (*)[35]



Source: CBR, NBS, MoF

In line with expectations[36], one year period revisions (Figure 4) are on average largest for univariate filters – Hodrick-Prescott and Multivariate Kalman Filter[37]. Newly adopted UCMs (UCM A and UCM B) score in the middle of the range in terms of ex-post revisions statistics, slightly exceeding 0,5 p.p revision from one year to another. The best performing methods in terms of revisions properties seem to be PCA model and SSM, where revisions of models are only somewhat above 0,1 percentage points on average.

One of the initial reasons to revise all in-house CBR's approaches were undervalued model estimates in 2020, as reported in Figure 5 for values produced by the MVKF and the old PCA model (FYWP)[38]. On the other hand, the lowest estimate in that year is now produced by the Modified Hamilton filter from CBR's models. New models (mainly PCA and SSM) are again scoring well, close to the middle of the range, indicating a plausible signal even in crisis period.

Additional important property to consider is the performance of models at turning points. A plausible model should be able to capture changes in business cycle dynamics even in its early stage. Vintages pictured in Figure 6 clearly identify superior models in terms of these trends. UCM A and UCM B score relatively well, in line with consensus trends, while results from both PCA models and SSM do perform

[35] Results for SSM should be viewed as inconclusive. While in all models we used historical real-time GDP, substantial explanatory power of SSM is grounded in external demand for which we do not have historical data.

[36] That these models incorporate very little of economic structure (similarly it applies to Modified Hamilton Filter).

[37] In the Figure 3 and 4 abbreviation "MVKF" (Multivariate Kalman Filter) from the FYWP is used.

[38] The original PCA model from 2014 served in the CBR service exceptionally well until the pandemic times (2020). The part of the economy that was hit the most – services, had not been originally included in this model, resulting in evidently underrated estimate for that year (Figure 5). A very similar story applies to the basic two-sided MVKF model. Underlying structure of this model simply could not capture and incorporate such a shock in its full scale.

weakly in terms of capturing the turning point[39]. Finally, one can easily spot the significantly displaced size (level) for the HP filter.

From a longer-term historical perspective, the proposed new CBR methods exhibit a well-behaved pattern. When compared with selected institutions (Figure 7), they seem to plausibly capture the main features of cyclical development in Slovakia. The great recession in 2009 is generally assessed to have led to a sizeable negative output gap, followed by only a temporary rebound. The sovereign debt crisis impact bottoming-out in 2013 is visible too, followed by cyclically more favourable years from 2015 onwards. From 2020, the cycle plunged again into negative territory followed by a recovery phase.

Figure 6: Performance of the models at the turning point (Autumn 2022, % of GDP)[40]

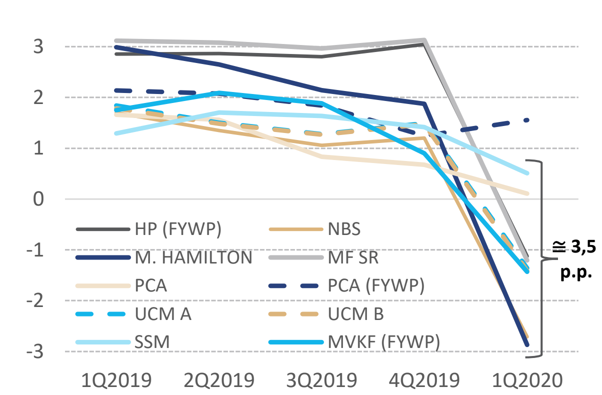
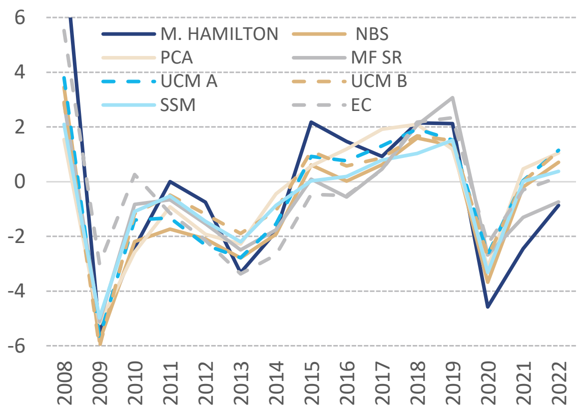


Figure 7: The output gap vintages by institutions (Autumn 2022, % of GDP)



Source: CBR, EC, MoF, NBS

Consequently, we are naturally converging to the question, what is "plausible" to consider a reliable estimate of the output gap. It is clearly observable and natural that different methods lead to different results (Figure 7). UCM A and B with their underlying economic structure give plausible result across vintages as well as at the turning point. However, revision properties are in favour of the SSM and the PCA model. Figure 8 shows a heatmap that provides a brief qualitative overview of the models' performance.

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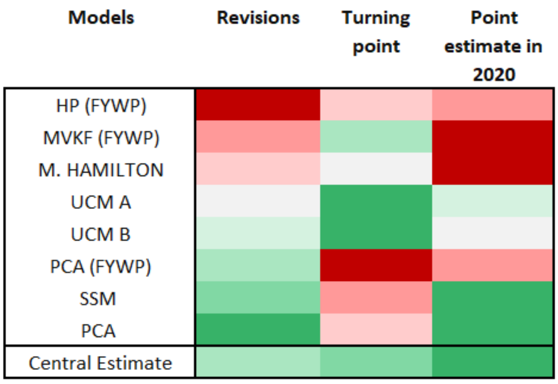
39 In regards to SSM, this feature was expected as an attribute of forward/looking modelling approach when using filters (Andrle (1998)).

40 EC projection is missing as we were unable to obtain quarterly data.

Figure 8 : Heatmap of models' performance



Source: CBR

5.2. From Group of Models to One Central Estimate

Standard model selection procedure[41] leads to choosing a single, most promising model, from the portfolio of data fitting models. However, this technique neglects the bias occurring from the model selection and may miss some useful information present in other models. Any model is designed as a reduction of a complex and dynamic world to a set of several variables with relationships (model data fitting) observed in past. However, since past relationships might not be valid in the future and features present in discarded models that were not relevant in past might gain importance over time, model averaging method[42] is a more suitable approach in delivering output gap estimates. Model averaging technique is often used to account for model uncertainty and to reduce the impact of model misspecification when a single model is used for inference.

Hence, in case of output gap estimation problem, instead of selecting a single model, model averaging considers a whole suite of models developed by the CBR for this purpose. This is done by weighting each model depending on an average relative difference between the model estimate and the average estimate of external institutions[43]. Notice that both models calibrations and averaging procedure depend on data set available at CBR output gap estimation date T, $\mathcal{J}_T$ (currently, data available at the end of T= 2022). Furthermore, averaging procedure is affected by our choice of moving window width, H, which we set to 10 consistently with EU IFI's recommendations[44]. Hence the final weighted CBR output gap is obtained conditionally on input data valid at time $T = 2022$ (i.e. taken from the

41 Build on e.g. information criterion (Akaike, 1973), or Mallows' C_p (Mallows, 1973).

42 Model averaging approach incorporates all available information and constructs a weighted average of the individual prediction from all potential models. Thus, it aims to achieve the best trade-off between bias and variance, and tends to perform better than the model selection estimator in finite samples (Zhang & Liu (2022)).

43 We follow Bayesian model averaging approach (Hinne (2020)) where the output gap estimate is obtained by averaging Appendithe estimates of the different models under consideration, each weighted by its model probability (so the model probability coincides with its estimator weight).

44 Our choice of moving window width, $H = 10$, arises as a compromise between estimation weights stability and ability to capture innovations in economy. Study on its optimal value is included in our future research plan.

information set $\mathcal{J}_T$) and moving window length $H = 10$. Therefore, for the sake of simplicity in what follows we drop subscripts H and $\mathcal{J}_T$ in the notation.

5.2.1. Algorithm Description

Taking into account all mentioned results from testing criteria, we decided to apply the following strategy to obtain a point estimate of the output gap from the suite of models:

1. In the first step, we calculate the annual arithmetic mean and standard deviation of five institutional output gap estimates ($N_{inst} = 5$)[45] for every year in the time horizon 2013 – 2022 given the information set available up to and including year 2022, $\mathcal{J}_T$. In general, we propose a moving window approach:

$$\mu_t^* \stackrel{\text{def}}{=} \overline{Institutional\ OG}_t = \frac{1}{N_{inst}} \sum_{i=1}^{N_{inst}} Institutional\ OG_{it} \tag{3. 1}$$

$$= \frac{MoF\ OG_t + NBS\ OG_t + EC\ OG_t + IMF\ OG_t + OECD\ OG_t}{N_{inst}}$$

where t = T-H+1, …, T and T = 2022 is the last year containing actual data. Notice that newest data are subject to future revisions. Next, we define moving window size H, the length of the horizon over which the central CBR output gap estimate is calculated and set it to 10. In this place it must be critically noted that the choice of window size is arbitrary at present, though features of economic development in Slovak economy do provide some guidelines about suitable parameter choices.[46]

Furthermore, we denote σ_t^* the standard deviation of five institutional output gap estimates at time t given the information set available up to and including time T, $\mathcal{J}_T$.

2. Afterwards we calculate for every period the distance between the output gap estimate of every new CBR model j (PCA, UCM A, UCM B, SSM, Modified Hamilton filter …, $N_{CBR} = 5$), $\hat{y}_{t,j}^{CBR}$ and value of the "institutional mean" in the same period:

[45] The estimates included are produced by the institutions from the FYWP - MoF, NBS, EC, plus we enriched this paper with IMF and OECD.

[46] When retrieving the past CBR output gap estimate, on the downside, the limit is given by precondition, that window length must achieve at least the stylized length of business cycle of circa seven years. In non-converging economy it would be optimal to take into account maximum of available historical data and that way, capture nature of most business cycles. However, such an approach would lead to caveats of biased future estimates in case of converging economy such as Slovakia. Owing the country-specific expertise, the maximum that could be considered for window size is 15 years. Our arbitrary choice of 10 should be viewed as a compromise that might undergo changes in future non-automatic revisions of the suite of models. Hence, the optimal setting of the moving window length will be determined utilizing machine learning approach in future (validation, supervised learning).

$$\delta_{t,j} \stackrel{\text{def}}{=} Model\ deviation_{t,j} = \left|CBR\ model\ OG_{t,j} - \mu_t^*\right| = \left|\hat{y}_{t,j}^{CBR} - \mu_t^*\right| \quad (3.2)$$
$$\forall t \in \{T-H+1, \dots, T\}, \forall j \in \{1, \dots, N_{CBR}\}$$

This way we want to identify how "far away" are different CBR methods from the hypothetical institutional mean value of the output gap. Notice that each CBR model estimate of output gap at time t depends on data set available at time T, $\mathcal{J}_T$.

3. Following the previous step, by calculating a p-specific metric mean of obtained deviations (distances) normalised[47] by the (time-varying) standard deviation of institutional gap estimates over H-year moving window, we arrive to central point indicator for each CBR method j :

$$Point\ indicator_j = \frac{1}{H}\left[\sum_{t=T-H+1}^{T}\left[\frac{\delta_{t,j}}{\sigma_t^*}\right]^p\right]^q, \quad (3.3)$$
$$\forall j \in \{1, \dots, N_{CBR}\}\ and\ q = \min\left\{1, \frac{1}{p}\right\}$$

At this stage we set p to 1 (so-called *Manhattan metric*) however, different calibrations aimed specifically at fulfilling the CBR role are available[48].

Additional step here involves obtaining the inverse of the point indicator. It subsequently serves as an input to our proposed weighting scheme:

$$Inverse\ point\ indicator_j = \frac{1}{Point\ indicator_j}, \qquad \forall j \in \{1, \dots, N_{CBR}\} \quad (3.4)$$

Notice that each (inverse) point indicator depends on data set available at time T, $\mathcal{J}_T$ and the moving window width, H.

4. Conditionally on $\mathcal{J}_T$ and H, in order to obtain final weights for each CBR model, we simply divide every single inverse point indicator of specific model, by the sum of all inverse point indicators[49]:

[47] Normalization of CBR models deviations by time-varying standard deviations of institutional output gap estimates enables us to emphasize the importance of CBR models fitting-in institutional output gap estimates when institutions are relatively consistent in their views. On the other hand, this approach allows us to be more flexible to CBR models whenever institution's views vary significantly. This is particularly useful in turbulent times and at the end of estimation horizon when revisions are often and larger, but also during more peaceful periods when exact estimation of output gap is due to fiscal consolidation need highly desirable. Effect of normalization on CBR models obtained weights is illustrated in Appendix 3.

[48] From the sake of CBR's mandate and its role in detecting periods when fiscal consolidation is achievable (not during turbulent times), a very precise estimate of the output gap that is very close to the institutional average is highly desirable.

[49] Hence, the weighting scheme prefers even more models with lower normalized errors.

(3.5)

$$\omega_j \stackrel{\text{def}}{=} Model\ weight_j = \frac{Inverse\ point\ indicator_j}{\sum_{i=1}^{N_{CBR}} Inverse\ point\ indicator_i}, \quad \forall j \in \{1, \dots, N_{CBR}\}.$$

Table 3: Obtained weights for period 2013-2022

PCA	UCM A	UCM B	SSM	M. Hamilton
0.13	0.29	0.21	0.20	0.17

Source: CBR

5. The final weighted output gap is then determined as inner product of a vector of model estimates and a vector of model weights given the information set available up to and including time T and moving window size H.

(3.6)

$$Output\ Gap_t = \langle \omega, \hat{y}_t^{CBR} \rangle, \quad \forall t \in \{T - H + 1, \dots, T\}.$$

Above, we denote vectors of model weights and CBR output gap estimates as

(3.7)

$$\omega = (\omega_1, \cdots, \omega_{N_{CBR}}), \text{ and } \hat{y}_t^{CBR} = (\hat{y}_{t,1}^{CBR}, \cdots, \hat{y}_{t,N_{CBR}}^{CBR}),$$

respectively.

Observe the impact of information set $\mathcal{J}_T$ and moving window width H on estimation of the final weighted output gap: while CBR output gap estimates, $\hat{y}_t^{CBR}$, obtained conditionally on information set available at time T, $\mathcal{J}_T$, *the* vector of CBR model weights, ω, considers both the information set $\mathcal{J}_T$ and the moving window width H.

5.2.2. Discussion

Time-varying property of estimated weights vector arises out of four factors:

- Past data revision (domestic and foreign real GDP) "rewrites" the history, so the information set available up to and including time T is important (Andrle (1998)).
- Models are re-estimated annually taking into account the newest available data ($\mathcal{J}_T$).
- Most recent institutional estimates of output gap are always used as inputs.
- Moving window embedded in the rule design that each year discharges the very first year considered in the previous year ($T - H$) while incorporating model estimates of recent year (T).

Hence, the final "central" output gap estimate acquired employing the weighting procedure introduced above is revised on annual basis over the whole estimation horizon (*H*). While changes in output gap estimate are small during the former years of the horizon (with increasing stability of past results), the last two years of the horizon account for the innovations stemming from newest available data and their revisions that makes the output gap estimate more volatile between revisions during last years of the horizon.

From a conceptual perspective anchoring and weighting is plausible as it is rule-based and transparent. We consider the advantage of weighting scheme not only in anchoring (and checking) to foreign institutions, but also in receiving information how individual models' perform on average. In addition, changes in weights might serve as an early warning indicator of ongoing structural changes in the economy[50].

From the technical perspective, the role of the CBR – to provide precise estimation of the output gap when it has small magnitude – is supported by several procedure features. Firstly, using inverse of the model-institutional average (i.e. point indicator) distance systematically downweigh models having large deviations from the institutional average – thus model response obtained in normal times[51] is more important than the one during large crisis when models typically fail working correctly. Furthermore, both normalization of the model-institutional gap by time-varying institutional standard deviation, so that crisis times do not drive our decision process, and setting $p < 1$ in p-metric, favor this objective.

5.3. Results

The final "central" output gap obtained from the suite of CBR models (CBR output gap estimate hereafter) by weighting procedure is pictured in the Figure 9, as well as the range implied. To provide a comprehensive information, we complement our results with the institutional estimates. Significantly wider ranges both of CBR models as well as institutional estimates in the last two years, will necessarily get narrower on the back of further data revisions and turning of short-term forecasts of observables into actual data[52].

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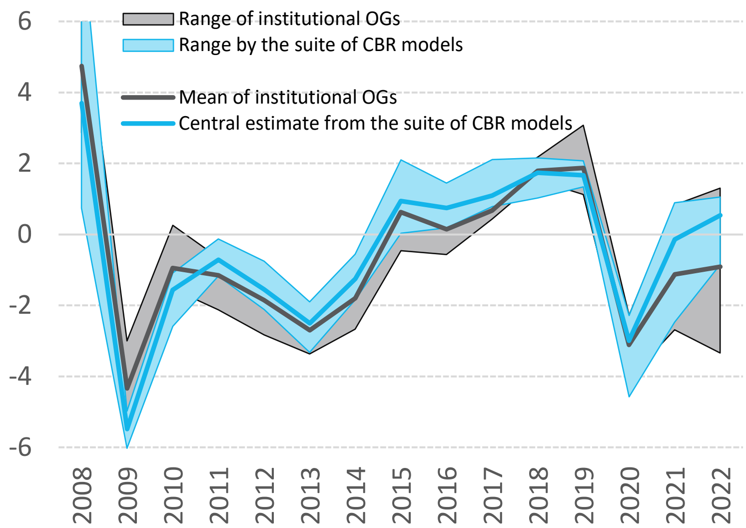
50 Decreasing weight of a particular model indicates that the model needs to be recalibrated/re-estimated or its structure does not capture well ongoing changes in the economy so it is outperformed by the other ones.

51 When output gap estimates of all models (institutional, CBR) have large magnitudes.

52 Latest institutional estimates operated with short-term forecasts for Q42022. Wider ranges thus as such implicitly include high uncertainty of forecasts during war in Ukraine and the outburst of inflation shock.

Figure 9: Central CBR estimate of the output gap



Source: CBR, MoF, EC, NBS, OECD, IMF

We see substantial and novel contribution of this approach in four areas:

1) By combining five different modern methods we do expect robustness in terms of revisions and precision of CBR estimate in time and over time. Even though the difference between ranges in the Figure 9 is evident, both output gap vintages are constrained within both ranges[53] simultaneously, what further decreases the uncertainty around the estimates.

2) From our point of view, the difference between vintages of mean of institutional output gaps and CBR output gap can be interpreted as a "rule-based" and transparent substitute to the expert judgement practice.

3) Next, as weights are generated on a moving window basis, we manage to capture new information available in the economy (in current year), to construct the central estimate in-time, while information from the past (more than ten years ago) is neglected. This allows elegantly "tracking" structural changes in a converging economy.

4) Finally, and most importantly, as individual weights of models are anchored to official estimates of prominent domestic and foreign institutions, and the whole procedure of estimation is transparent and replicable, we do consider this approach especially plausible and relevant for prudent fiscal policy assessment.

[53] The only point that is not within both ranges is institutional mean in the crisis year 2009.

6. Conclusions and further work

In this paper we introduce a robust and plausible central CBR estimate of output gap with clear and transparent methodology. Firstly, a "suite of models" is developed based on literature and recent institutional best practice. This specifically means two unobserved component models, one forward looking semi-structural model, a principal component model, as well as Modified Hamilton filter. As the results of the models vary significantly in their properties, the estimates obtained from these models are weighted. These weights are based on the (10 year) average relative differences between the CBR model estimates and the average output gap estimate of five distinguished external forecasting institutions. As a final step one central CBR estimate is derived and compared to mean institutional output gap.

Such an approach has a major advantage in its robustness, as estimates are obtained from various models that substantially differ in their architecture. Then, central CBR output gap estimate is plausible, as its calculation is anchored to output gap estimates of leading domestic and foreign institutions, what secures that it falls within a certain narrow range of consensual estimates. Anchoring central estimate might be viewed as rule-based quantitative substitute to expert judgement concept.

Obtaining one central estimate of output gap has direct practical implications on the processes in the CBR. The calculation of cyclically adjusted budget balance where historical and nowcasting estimates are needed, including notably an ex-post assessment of balanced budget rule (fiscal compact), expenditure ceilings and long-term public finance sustainability, will be fully based on the new central output gap estimate[54]. In addition, this new central estimate will serve as an input to structural forecasting model. As for the medium-term forecasting, the in-house procedure will continue to rely on the production function estimates as it incorporates the most relevant information and linkages from structural model (Appendix 1).

Regarding future work and novelty approaches the key challenge in Slovakia still remains to adequately incorporate financial cycle into output gap models. Similar problems might be tackled in future by various innovative machine learning or deep learning techniques as proposed by Coulombe (2022). Before that, it is crucial to conduct relevant and extensive research on domestic financial cycle, so we can better understand the behaviour of domestic financial cycle itself.

Furthermore, for Slovakia as a small and open economy, it will be crucial to consider and examine the topic of synchronizing business cycles among economies. Especially distinction of synchronization between cycles in core Eurozone member states and peripheral Eurozone member states (Arcabic, Panovska, Tica, 2022) may be relevant in case of Slovakia. We consider an equally important issue for Slovakia is a continuous discussion about determinants, and particularly role of FDI in business cycles synchronization (Stiblarova, 2021).

[54] The weighting procedure takes into account the standard deviations of institutional output gaps (EC, MoF, NBS), which enables to drop these methodologies from direct use in the suite of models as it was originally proposed in FYWP (2014).

For the "suite of models" and model averaging technique we are considering employing a machine learning approach to jointly determine the optimal estimation horizon length as well as the p-metric that would be able to deliver over-years stable output gap estimates with the lowest possible bias[55], especially in periods when fiscal consolidation is needed.

Finally, as already mentioned, new models with more sophisticated economic background (e.g. Quarterly Projection Model developed by the IMF) or estimation technique will be always in the cards and we must fully reflect and incorporate such innovations as Closer to Finding Yeti is still a marathon rather than a sprint.

[55] Measures as a time-specific distance of the CBR's central estimate from the institutional average when recent years deviations are more emphasized.

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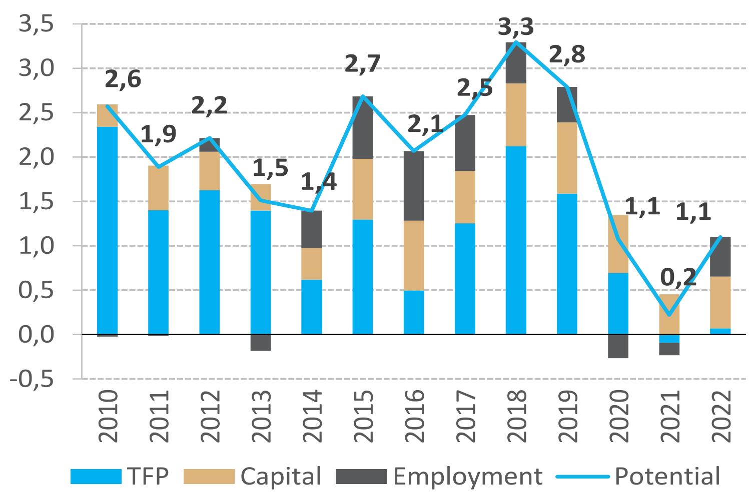
Appendix 1 - Production Function approach in the CBR forecasting process

Within the CBR official medium-term forecasts (four-year horizon), as well as long-term projections (50 -year horizon), the output gap estimation process necessarily differs from the above proposed methods. We follow the strategy of the EU IFI which outlines "a two way methodology" (EU IFI, 2022). The suite of models that includes univariate and particularly multivariate semi-structural models, is routinely used to extract historical output gaps, as well as real-time estimates (nowcasting) for the actual year. In the forecasting process, the production function approach relying on growth accounting is used following the FYWP. By adopting this method, we are able to address the need for reliable and consistent assumptions of long-term drivers of the supply side of the economy. These are an integral part of each forecasting exercise. The long-term trends of variables such as labour force, participation rates and hours worked, NAIRU, interest rates and trend total factor productivity are fully forecasted by the CBR.

In line with international practice, on a medium-term horizon a macroeconomic structural model[56] captures the demand-supply side interactions, where the long-term supply side variables enter exogenously. In addition, the forecasting process requires assumptions regarding exogenous (medium-term) demand variables (external demand, financial markets and monetary policy, fiscal forecast) as well as expert judgement when linking the short- to medium- term horizon forecasts. Over the very long-term horizon (50 years ahead), output gap is assumed to be closed, i.e. equal to zero. The economy is thus assumed to be on its potential growth path.

Figure 10: Potential growth and its components (in % of GDP)

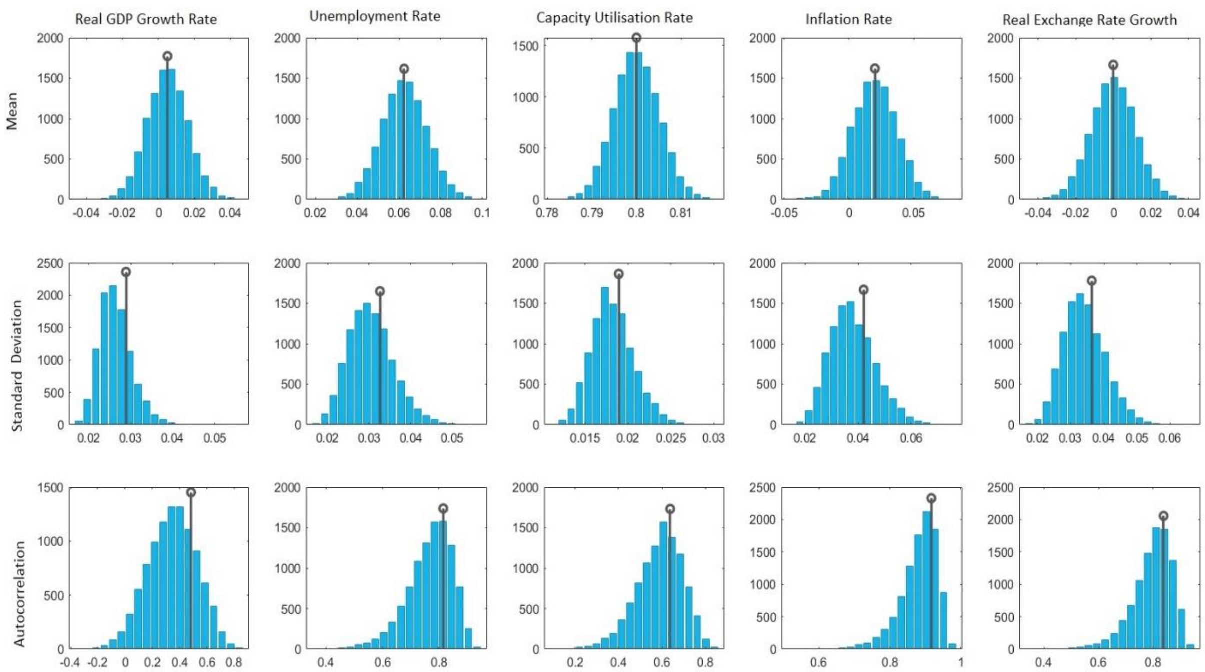


Source: CBR

56 The CBR uses a model based on the foundations from Reľovský and Široká (2009). The first few forecast quarters are typically calculated using an expert short-term forecast approach, based on yet further short-term forecasting tools. The details are out of scope of this paper.

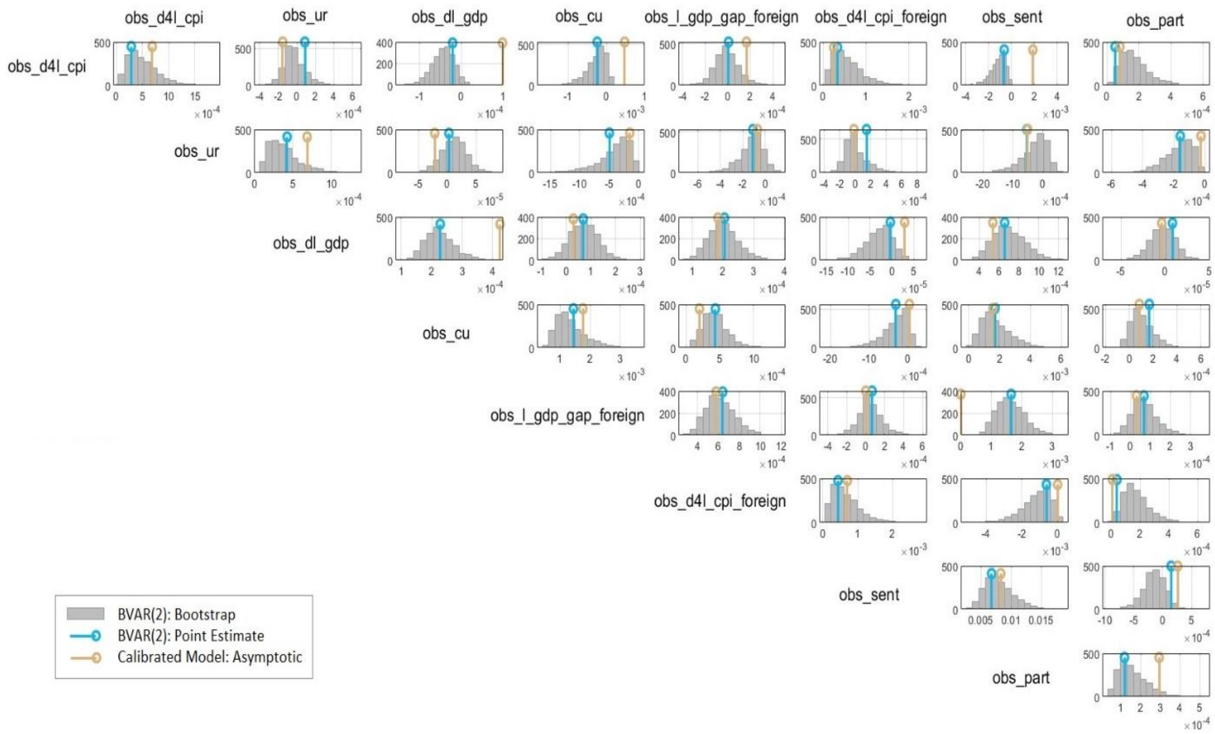
Appendix 2- Semi-structural Model Properties: Bayesian estimation of model parameters and shocks

Figure 11: Asymptotic properties of key SSM variables



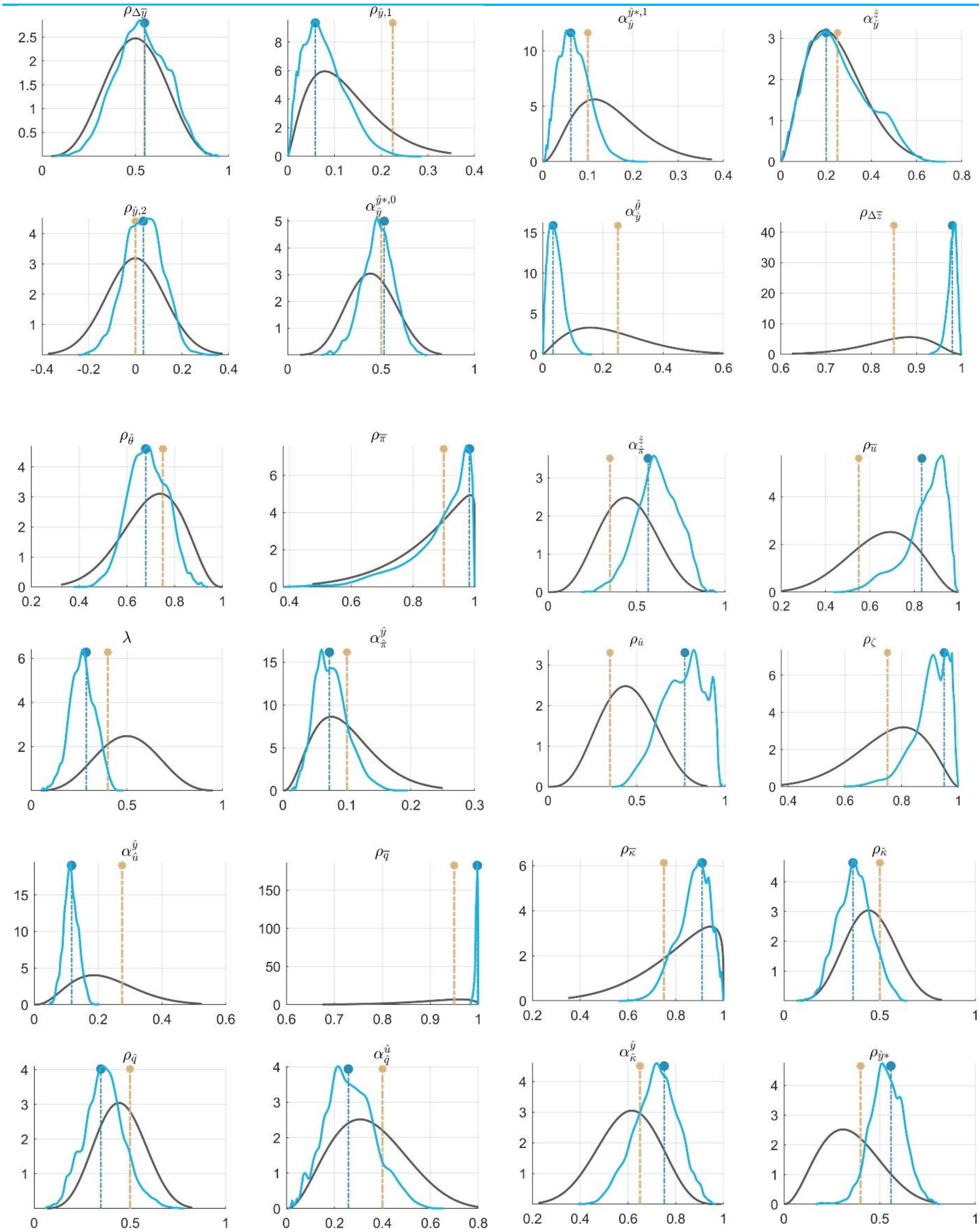
Sample mean, standard deviation and autocorrelation parameters calculated from the resampled SSM calibrated consistently with estimation priors. *Source: CBR*

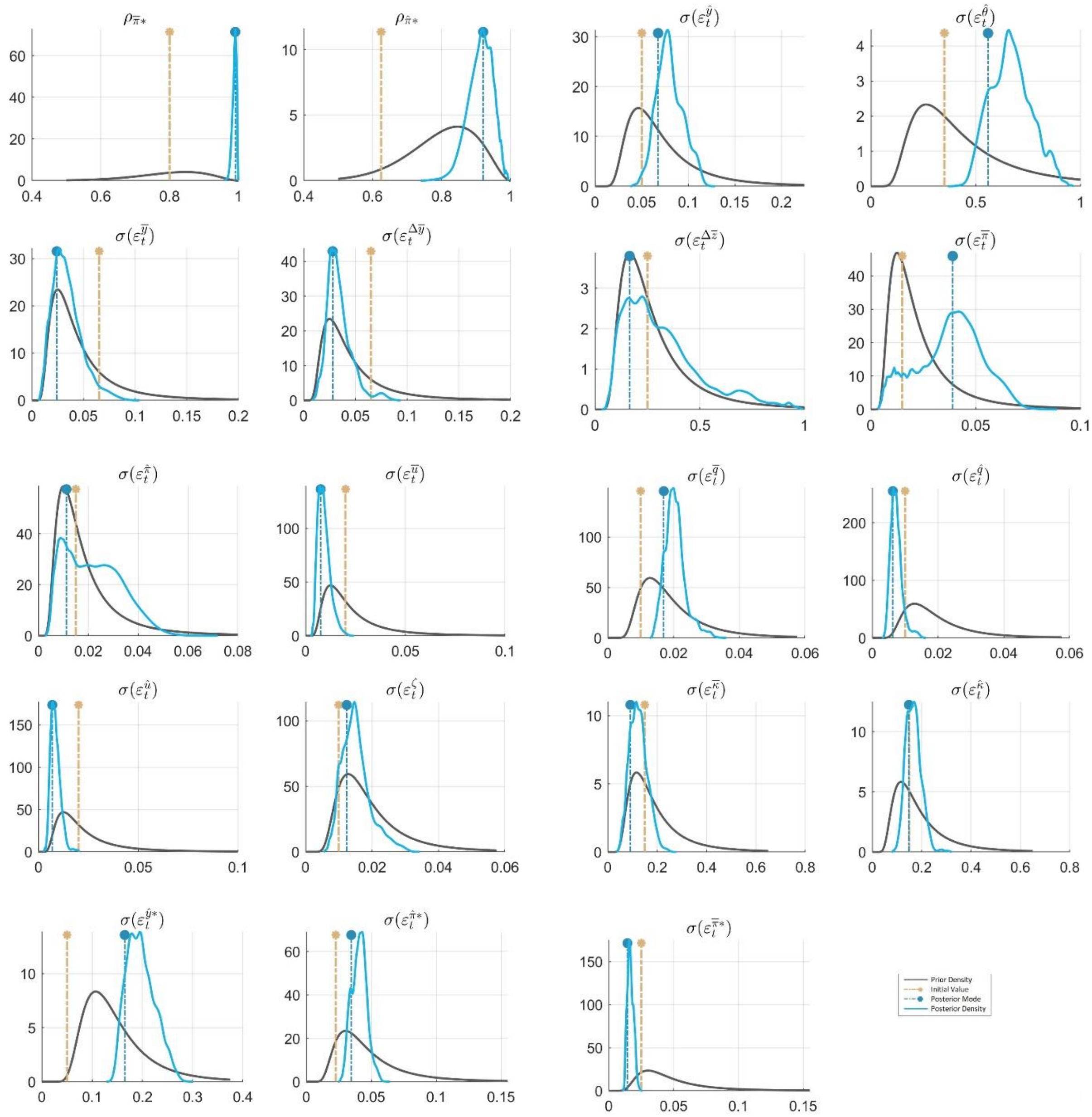
Figure 12: Estimated covariances, comparison of model and data properties



Source: CBR

Figure 13: Bayesian estimation, prior and posterior distributions of the SSM-model parameters and shocks standard deviations




Prior Density
Initial Value
Posterior Mode
Posterior Density

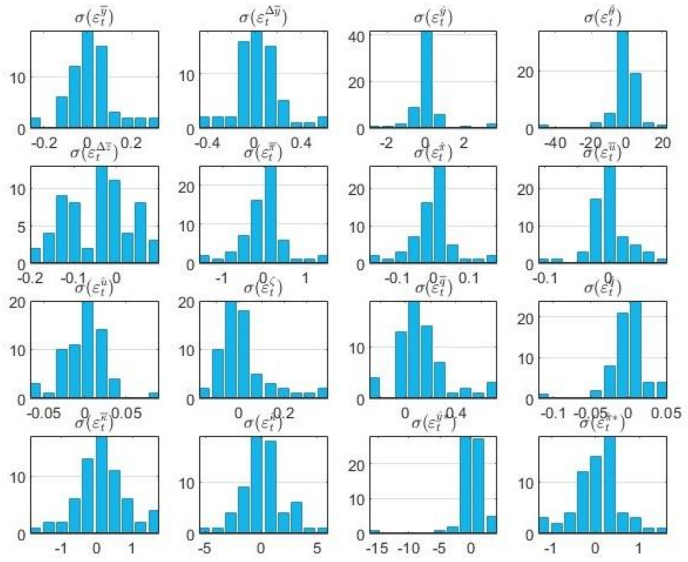
Source: CBR

Table 4: Bayesian estimation of the SSMA model: Priors and Posteriors statistics

	Prior				Posterior			
	Starting value	Distribution type	Hyperparameters		mean	std	Confidence Intervals (90%)	
			mean	std				
$\rho_{\bar{y}}$	0.55	Beta	0.50	0.15	0.50	0.13	0.29	0.69
$\rho_{\hat{y},1}$	0.23	Beta	0.13	0.08	0.09	0.05	0.01	0.17
$\rho_{\hat{y},2}$	0.00	Normal	0.00	0.13	0.04	0.07	-0.08	0.14
$\alpha_{\hat{y}}^{\hat{y}^*,0}$	0.50	Beta	0.45	0.13	0.46	0.08	0.33	0.59
$\alpha_{\hat{y}}^{\hat{y}^*,1}$	0.10	Beta	0.15	0.08	0.07	0.04	0.02	0.13
$\alpha_{\hat{y}}^{\hat{z}}$	0.25	Beta	0.25	0.10	0.25	0.10	0.07	0.38
$\alpha_{\hat{y}}^{\hat{\theta}}$	0.25	Beta	0.23	0.13	0.05	0.02	0.01	0.09
$\rho_{\Delta\bar{z}}$	0.85	Beta	0.85	0.08	0.98	0.01	0.97	0.99
$\rho_{\hat{\theta}}$	0.75	Beta	0.70	0.13	0.67	0.08	0.54	0.80
$\rho_{\bar{\pi}}$	0.90	Beta	0.85	0.13	0.82	0.16	0.54	1.00
λ	0.40	Beta	0.50	0.15	0.29	0.07	0.19	0.40
$\alpha_{\hat{\pi}}^{\hat{y}}$	0.10	Beta	0.10	0.05	0.07	0.02	0.04	0.10
$\alpha_{\hat{\pi}}^{\hat{z}}$	0.35	Beta	0.33	0.15	0.56	0.12	0.35	0.75
$\rho_{\bar{u}}$	0.55	Beta	0.60	0.13	0.76	0.10	0.61	0.92
$\rho_{\hat{u}}$	0.35	Beta	0.35	0.13	0.70	0.10	0.54	0.85
ρ_{ζ}	0.75	Beta	0.75	0.13	0.95	0.03	0.91	0.99
$\alpha_{\hat{u}}^{\hat{y}}$	0.28	Beta	0.23	0.10	0.12	0.02	0.08	0.16
$\rho_{\bar{q}}$	0.95	Beta	0.90	0.08	1.00	0.00	0.99	1.00
$\rho_{\hat{q}}$	0.50	Beta	0.45	0.13	0.40	0.12	0.19	0.59
$\alpha_{\hat{q}}^{\hat{u}}$	0.40	Beta	0.35	0.15	0.27	0.11	0.11	0.45
$\rho_{\bar{\kappa}}$	0.75	Beta	0.80	0.13	0.85	0.08	0.73	0.99
$\rho_{\hat{\kappa}}$	0.50	Beta	0.45	0.13	0.39	0.09	0.24	0.53
$\alpha_{\hat{\kappa}}^{\hat{y}}$	0.65	Beta	0.60	0.13	0.73	0.09	0.58	0.87
$\rho_{\hat{y}*}$	0.40	Beta	0.35	0.13	0.53	0.08	0.41	0.65
$\rho_{\bar{y}*}$	0.80	Beta	0.80	0.10	0.99	0.01	0.98	1.00
$\rho_{\hat{\pi}*}$	0.63	Beta	0.80	0.10	0.91	0.04	0.85	0.98
$\sigma(\varepsilon_t^{\bar{y}})$	0.07	Inv.Gamma	0.05	0.05	0.04	0.02	0.01	0.06
$\sigma(\varepsilon_t^{\gamma^{\bar{y}}})$	0.07	Inv.Gamma	0.05	0.05	0.03	0.01	0.02	0.05
$\sigma(\varepsilon_t^{\hat{y}})$	0.05	Inv.Gamma	0.08	0.05	0.08	0.01	0.06	0.10
$\sigma(\varepsilon_t^{\hat{\theta}})$	0.35	Inv.Gamma	0.45	0.35	0.67	0.08	0.53	0.78
$\sigma(\varepsilon_t^{\Delta z})$	0.25	Inv.Gamma	0.30	0.25	0.24	0.10	0.09	0.37
$\sigma(\varepsilon_t^{\bar{\pi}})$	0.02	Inv.Gamma	0.03	0.03	0.03	0.01	0.01	0.05
$\sigma(\varepsilon_t^{\hat{\pi}})$	0.02	Inv.Gamma	0.02	0.02	0.02	0.01	0.01	0.04
$\sigma(\varepsilon_t^{\bar{u}})$	0.02	Inv.Gamma	0.03	0.03	0.01	0.00	0.00	0.01
$\sigma(\varepsilon_t^{\hat{u}})$	0.02	Inv.Gamma	0.03	0.03	0.01	0.00	0.00	0.01
$\sigma(\varepsilon_t^{\zeta})$	0.01	Inv.Gamma	0.02	0.01	0.02	0.00	0.01	0.02
$\sigma(\varepsilon_t^{\bar{q}})$	0.01	Inv.Gamma	0.02	0.01	0.02	0.00	0.02	0.02
$\sigma(\varepsilon_t^{\hat{q}})$	0.01	Inv.Gamma	0.02	0.01	0.01	0.00	0.01	0.01
$\sigma(\varepsilon_t^{\bar{\kappa}})$	0.15	Inv.Gamma	0.20	0.15	0.13	0.03	0.08	0.18
$\sigma(\varepsilon_t^{\hat{\kappa}})$	0.15	Inv.Gamma	0.20	0.15	0.17	0.03	0.11	0.21
$\sigma(\varepsilon_t^{\hat{y}*})$	0.05	Inv.Gamma	0.15	0.08	0.20	0.03	0.15	0.23
$\sigma(\varepsilon_t^{\hat{\pi}*})$	0.02	Inv.Gamma	0.05	0.04	0.04	0.00	0.03	0.05
$\sigma(\varepsilon_t^{\bar{\pi}*})$	0.03	Inv.Gamma	0.05	0.04	0.02	0.00	0.01	0.02

Source: CBR

Figure 14: Estimated shocks histograms (2007-2022)



Source: CBR

Appendix 3 - Weighting Procedure Sensitivity Analysis

Impact of P-Metric on CBR Output Gap Estimate

Choice of $p > 0$ value in metric used to measure relative deviations between each individual CBR model output gap estimate and average institutional estimate reflects CBR preferences in model error perception. Generally, values greater than one are associated with large errors aversion (i.e. larger errors are penalized relatively more than smaller) while metric with $p < 1$ are aimed more at model precision. That said, among two models having the same average error is preferred the one fitting better the institutional average over time. Hence, the winner delivers the path very close to the institutional average although, rare exceptions are allowed.

Table 5: Obtained weights for period 2013-2022 for various values of P

	PCA	UCM A	UCM B	SSM	M.Hamilton
P = 0.5	0.16	0.25	0.20	0.20	0.19
P = 1	0.13	0.29	0.21	0.20	0.17
P = 2	0.12	0.29	0.23	0.18	0.17

Source: CBR

However, moving window technique and averaging approach over the horizon H impact of p on model weights (Table 5) and on resulting output gap (Figure 15) is negligible. Stability of estimated weights over time.

During the post-crisis period we observe a gradual decline in UCM A – model weight while univariate filter (Modified Hamilton filter) or semi-structural model gain importance (Figure 16).

Figure 15: Output gap sensitivity for different values of P

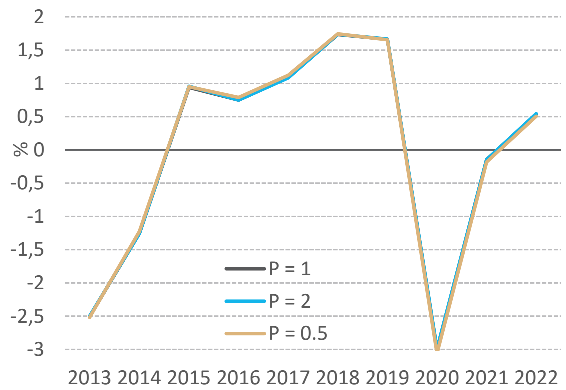
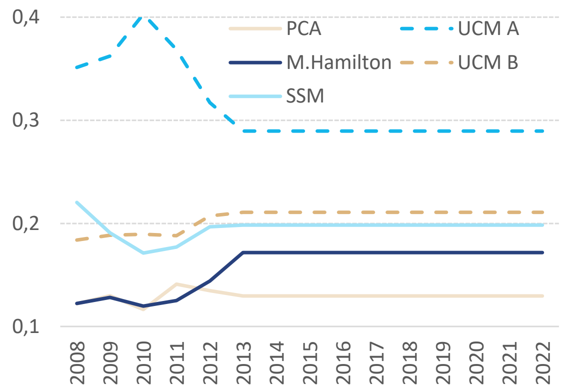


Figure 16: Weights of individual models



Source: CBR

Appendix 4 - Data description

Alias	Description	Frequency	Range	Source	Transformation	Model
y	Gross Domestic Product in Constant Prices, SA, mil. EUR	Q	2000Q1 - 2022Q3	SO SR	Natural Logarithm	UCM (A,B), MH, PCA, SSM
U	Unemployment rate (Labour Force Survey), SA	Q	2000Q1 - 2022Q3	SO SR		UCM (A,B), SSM
π (1)	Consumer Price Index (CPI), NSA	Q	2000Q1 - 2022Q3/4	NBS		UCM (A,B), SSM
π*	Consumer Price Index (CPI), NSA for 27 EU countries, NSA	Q	2000Q1 - 2022Q3	Eurostat	Natural Logarithm	SSM
q	Participation rate (15-64 years), SA	Q	2000Q1 - 2022Q3	NBS		SSM
κ	Current level of capacity utilization, SA	Q	2000Q1 - 2022Q3	Eurostat	Standardized for PCA model	UCM (A,B), PCA, SSM
y*	Foreign demand (WED)	Q	2002Q1 – 2022Q3	Eurostat, CBR	Natural Logarithm	SSM
ESI	Economic sentiment indicator (EU 27), SA	M	2000Q1 - 2022Q3	DG ECFIN	Standardized	PCA
ESI_SK (θ)	Economic sentiment indicator, long-term average (SK)	M	2001Q1 – 2022Q3	SO SR	Standardized	SSM
Fin Stress	Country-Level Index of Financial Stress (CLIFS) Composite Indicator, Index	M	2000Q1 - 2022Q3	ECB	Multiplied by minus one, Standardized	PCA
Retail	Business activity (sales) development over the past 3 months, SA	M	2000Q1 - 2022Q3	Eurostat	Standardized	PCA
Retail	Value added (for weighting). Retail and wholesale trade including motor vehicles (Interpolated from annual data on detailed branches value added), SA	Y/Q	2000Q1 - 2022Q3	Eurostat		PCA
Services	Evolution of demand over past 3 months in services, SA	M	2000Q1 - 2022Q3	Eurostat	Standardized	PCA
Services	Value added (for weighting). Interpolated from annual data on detailed branches value added), SA	Y/Q	2000Q1 - 2022Q3	Eurostat		PCA
Construction	Order books in construction	M	2000Q1 - 2022Q3	Eurostat	Standardized	PCA
Construction	Value added (for weighting), SA	Q	2000Q1 - 2022Q3	Eurostat		PCA
Industry	Order books in industry	M	2000Q1 - 2022Q3	Eurostat	Standardized	PCA
Industry	Value added (For weighting) manufacturing, SA	M	2000Q1 - 2022Q3	Eurostat		PCA

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